Overturning and Dissipation Caused by Baroclinic Tidal Flow near the Sill of a Fjord Basin

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ABSTRACT

Dissipation time series and moored velocity and density time series on the inner slopes of the Gullmar Fjord sill showed that the internal tides generated at the sill radiated to the head of the fjord, were reflected, and then radiated back to the sill, where they dissipated their energy mainly below sill level. A large amount of the dissipation was caused by a transitional flow at a particular phase of the internal tide, when the bottom layer descended down the sill slope and had to pass a constriction set up by a submarine hill. The inward, baroclinic bottom-layer flow transformed into a supercritical bottom jet, which separated from the bottom just downstream of the constriction. A large fraction of the dissipation took place in the successive rebounding region (the hydraulic jump) above the bottom jet, where overturns of the same size as the vertical extent of the rebounding region were observed. More than half of the dissipation was happening in the bottom boundary layer below the jet. During the transitional flow, there were clear pulsations of the jet with periods of about 15 min. The amount of diapycnal mixing caused by the turbulence was reduced by the large fraction of dissipation within the bottom boundary layer and perhaps also by the high-buoyancy Reynolds numbers within the rebounding region. When using a relatively new parameterization of mixing, the mixing was significantly reduced compared to using the traditional constant mixing efficiency method.

1. Introduction

The transfer of energy from barotropic tides to baroclinic tides and from there into turbulence and mixing is believed to be one of the main sources of diapycnal mixing in the deep ocean (Sjöberg and Stigebrandt 1992; Munk and Wunsch 1998) and in many deep fjord basins (Stigebrandt and Aure 1989; Arneborg and Liljebladh 2001). The energy transfer takes place when tidal motions over steep topography cause horizontal density gradients, which set up internal wave motions. Today, several theoretical models exist for describing the energy loss from barotropic tides to internal tides (e.g., Baines 1982; Llewellyn Smith and Young 2002, 2003). However, much less is known about how and where this energy finally ends up as dissipation and mixing. Such knowledge is necessary, for example, to obtain a proper description of deep ocean circulation in large-scale ocean circulation models.

In the case of strong barotropic flow, where the barotropic velocity is comparable to the phase velocity of the first baroclinic mode, lee waves are formed on the downstream side of a ridge, with supercritical bottom jets, hydraulic jumps, and bores that propagate toward the ridge as the tide slackens. This case has been intensively investigated in fjords (e.g., Farmer and Armi 1999; Klymak and Gregg 2004; Inall et al. 2005). In this type of energy transfer, much of the energy withdrawn from the barotropic tide goes more or less directly into local turbulence in the vicinity of the sill. Nevertheless, it turns out that this type of tidal energy transfer is not very efficient in mixing the waters below sill level (Stigebrandt and Aure 1989).

More common in the deep ocean, and probably also in fjords, is the case in which the barotropic velocity is weaker than the phase velocity of the first baroclinic mode. Then the tidal motion causes internal waves of tidal frequency that propagate away from the topographic feature. In that case it is less clear how the energy transforms into turbulence and mixing. In the Hawaiian Ocean Mixing Experiment (HOME) it was found that about 15% of the energy lost from the barotropic tides at the Hawaiian Ridge dissipates close to
the ridge, whereas the rest propagates away as internal tide energy (Klymak et al. 2006). Some of this will probably be lost to the continuous internal wave field via nonlinear interactions before it finally goes into turbulence. However, a large fraction radiates over large distances with unchanged amplitudes (e.g., Ray and Mitchum 1997) and eventually has a high probability of coming into contact with rough topography over the midocean ridges. Naturally, this chance is much larger for internal tides generated over the midocean ridges. Very little work has been done on the influence of rough topography on the dissipation of internal waves.

Gullmar Fjord is a 30-km-long and 1–2-km-wide fjord on the Swedish west coast with a maximum depth of 120 m and a sill depth of 43 m. The water below sill level is mainly renewed during late winter and early spring, whereas it is isolated from advective exchanges during late spring, summer, fall, and early winter. During this period, oxygen levels in the deepest parts often decrease below levels that are healthy for living creatures. To slowly facilitate a new inflow, the density in the basin is continuously decreased by diapycnal mixing caused by baroclinic tides, baroclinic seiches, and internal waves of baroclinic seiche frequency (Arneborg and Liljebladh 2001). Even though the main players of this mixing are known, the processes are not, and therefore a detailed mapping of the dissipation rates in the deep basin was performed in 2001, which showed that about 80% of the mixing below sill level happened close to the sill (Arneborg et al. 2004). This was not a trivial result because many other fjords tend to show a large fraction of the barotropic energy loss radiating into the fjord via internal tides (e.g., Inall et al. 2004; Klymak and Gregg 2004), necessitating dissipation also in the inner fjord parts. One speculation about the focusing of dissipation in the sill region in this particular fjord is that the sill region is near-critical to internal tide reflection, whereas most of the remaining fjord is either too gentle or too steep to cause energy loss at reflection. However, the time resolution of that study was too low to look at detailed processes, and therefore a new cruise was performed in September 2004, focused solely on the sill region. It is some of the results of this latest cruise that will be presented in the present paper.

The paper is organized as follows: The moorings and instrumentation and some of the data analysis methods are presented in section 2. The dataset is presented in section 3. In section 4 the focus is specifically on the dissipation mechanism below sill level. The mixing caused by this dissipation is estimated in section 5, and an overall tidal energy budget is put forward in section 6, leading to the summary and discussion in section 7.

2. Instrumentation and methods

a. Instrumentation

Three moorings were placed within the sill region of Gullmar Fjord from the beginning of September to the beginning of October 2004, one with eight Microcat CTD recorders just outside the main entrance, one with an upward-looking, bottom-mounted, 300-kHz ADCP in the narrowest entrance and one with two 300-kHz ADCPs and 16 Microcat CTDs at the 80-m isobath inside the sill (Fig. 1). On the inner mooring, one ADCP was bottom mounted, whereas the second was mounted at about 30-m depth. Both were upward-looking. All Microcats were storing data with 5-min intervals. Bottom-mounted ADCPs were configured with 2-m bins and 50 pings per ensemble stored each 10 min. The middepth ADCP was configured similarly but with 1-m bins.

During the end of the mooring deployment an intensive study was performed with R/V Skagerak and the smaller work boat Alice. Working around the clock from 0700 UTC 28 September to 0800 UTC 31 September, microstructure casts were taken from Alice at three stations inside the sill (Fig. 1). A burst of four casts was performed at each visit of a station, starting at station 1 (60 m), continuing to station 2 (70 m), and ending at station 3 (80 m). The repeat time of such a cycle was about 2 h. Casts were taken while drifting freely with the boat. During the strongest wind and current conditions, the vessel was repositioned after the second cast. The microstructure profiler was an MSS-90L with two shear probes, one microthermistor, one turbidity sensor, one accelerometer, two tilt sensors, and standard CTD sensors. All sensors were sampled at 1024 Hz. Profiles were generally obtained from a few meters below the surface to the bottom, which means that the microstructure sensors stopped about 10 cm from the sediment–water interface.

b. Methods

After despiking all microstructure profiler (MSS) channels, the pressure was low-pass filtered in order to determine the falling velocity of the probe. The average falling velocity was about 0.6 m s$^{-1}$. This was subsequently used first when converting from shear probe voltage to transverse velocity $U$ and later to transform the time derivative $\partial U/\partial t$ into vertical derivative $\partial U/\partial z$, using Taylor’s frozen turbulence hypothesis. Finally, the dissipation rate of turbulent kinetic energy (TKE) was calculated as

$$\varepsilon = 7.5\nu \frac{\partial U}{\partial z},$$

assuming isotropic turbulence, where $\nu$ is the kinematic viscosity. The shear variance was calculated by integrating.
the shear spectrum from 2 to 30 cycles per meter (cpm) and correcting for missing variance by assuming a Na-smyth spectrum. The calculation was performed in 50% overlapping segments of 512 data records. These were then binned into fixed 0.5-m depth bins.

To obtain dissipation rates as close as possible to the bottom, a separate analysis was performed there. The analysis was similar to the fixed depth bin analysis, except that the last 512 data records ended exactly before the bottom hit and the dissipation rates were binned in 0.5-m depth bins relative to the bottom.

3. General dataset
   a. Mooring data

The mooring data for the whole deployment period are shown in Fig. 2 together with wind data from Måseskär, which is located about 10 nm south of the fjord entrance. The intensive measurement period is indicated with shading in Fig. 2f. The surface elevations inside and outside the entrance vary to a large extent and are clearly dominated by semidiurnal tides, although some lower- and higher-frequency variations may also be seen. Phase
and amplitude elevation differences caused by choking cannot be seen but will be evaluated more carefully in section 6. Above sill level (43-m depth) the mean stratifications inside and outside the fjord are similar (Figs. 2d,g). Below sill level at the inner mooring there are locally increased vertical gradients in salinity and temperature between 43 and 60 m, both contributing positively to what will be termed the sill-level pycnocline. Stratification changes above sill level (43 m) are dominated by time scales of several days, and these variations are similar inside and outside the fjord (Figs. 2c,f), as reported elsewhere (Arneborg 2004). The velocities above sill level are also dominated by these low-frequency fluctuations, where coastal upwelling forces coastal bottom waters into the intermediary layers of the fjord and surface waters out of the fjord and vice versa. This “baroclinic pumping” is the main mechanism of water exchange above sill level (Arneborg 2004). During the last 8 days the coastal pycnocline is less pronounced than in the beginning of the period, and the variations in stratification between sill level and surface becomes much smaller. Correspondingly, the subtidal current fluctuations become less pronounced during the last 8 days, with a general estuarine circulation pattern only broken once by a strong inverse exchange. The winds are also weaker during the last 8 days than during the first period, except at the inverse exchange.
event where strong eastward winds force surface water into the fjord.

The velocities are generally much smaller below sill level than above. This is especially pronounced at the entrance mooring, which is placed just inside the sill. However, there are some relatively strong inward current peaks close to the bottom of the 80-m mooring, with amplitudes up to 0.2 m s$^{-1}$. These are associated with downward movements of the near-bottom isopycnals, as one would expect if the near-bottom currents were parallel to the bottom. The near-bottom isotherms show large vertical movements with peak-to-peak values of up to 15 m. During the last 8 days, both current and isotherm fluctuations below sill level show a clear tidal signal, whereas this is less clear during the preceding part, where the amplitudes are larger and more irregular. We expect that the first period is mainly dominated by internal seiching, whereas the last period is dominated by internal tides. Such changes in regimes have been seen also in earlier studies in the fjord (Arneborg and Liljebladh 2001).

Finally there are also some high-frequency oscillations of the current, seen most clearly in the entrance mooring near day 256. These are caused by the barotropic seiche, which has a period of about 2 h (Arneborg and Liljebladh 2001).

**b. MSS data**

The burst average turbulent kinetic energy dissipation rates and isotherms are shown in Fig. 3 as functions of time and depth for the three stations. The varying maximum depth at station 3 is caused by drifting of the vessel along the fjord in combination with a relatively steep bottom gradient at that location.

There are generally increased dissipation rate levels between sill level and the upper pycnocline during days 272 and 273. This is probably related to the inflow of intermediate water below the pycnocline (see also Fig. 2). It is also interesting that the dissipation rate

![Fig. 3. Burst averages of dissipation rates of TKE and isotherms as function of time and depth for stations (a) 1, (b) 2, and (c) 3. The color scale is logarithmic. The temperature interval between isotherms is 0.25$^\circ$. The black dashed lines indicate the approximate time of minimum isothermal displacements at station 3 as a function of depth.]
levels are largest at the station closest to the entrance. These elevated dissipation rates are interesting, but in order not to spread the focus of this paper too much we will defer a more detailed analysis of this phenomenon to a later publication. In the present paper we will focus on what happens below sill level.

Below 40-m depth the isothermal movements are clearly influenced by semidiurnal tides at all three stations, although the influence is most pronounced at the shallowest station. We will present a detailed analysis of the tides in section 6.

The isotherms below 40 m also have a clear subtidal component with an amplitude that seems to increase with depth. When looking at around 60-m depth, the largest negative displacements of the isotherms occur around day 273 at all stations. The largest negative displacements do, however, occur earlier at larger depths and later at shallower depths, as indicated by the dashed lines in Fig. 3. This corresponds to an upward phase propagation, which could be caused by an oblique internal wave with a downward component to its group velocity. Such oblique low-frequency motions with upward phase propagation have also been observed for low-frequency motions during earlier observations, at stations further away from the sill (Arneborg and Liljebladh 2001).

The dissipation rates below sill level show a clear enhancement toward the bottom during episodic events, when they rise toward $10^{-7}$–$10^{-6}$ W kg$^{-1}$ from background levels of $10^{-9}$–$10^{-8}$ W kg$^{-1}$. The peaks show a tidal (semidiurnal) behavior, such that they tend to occur at the time of largest negative displacements of the isotherms. In addition, the largest peaks at all three stations occur when the low-frequency isothermal variation shows the largest negative displacement. In other words, the maximum tidal dissipation rate peaks, which during each tide occur at lowest isothermal elevation, are modulated by the low-frequency fluctuations so that the maximum peaks occur at the time of lowest isothermal elevation.

4. Overturning events

a. Flow field and dissipation rates

Focusing in on the two tidal periods with largest dissipation rates below sill level at station 3, including the mooring data at that station, reveals the following picture of the flow (Fig. 4): During inward (downslope) flow below sill level, the isotherms are generally displaced downward, but with larger downward displacement close to the bottom than further up. The flow is bottom-intensified during the whole inward phase, but it includes more and more isotherms, leaving a thicker and thicker, nearly homogeneous layer above. The inward flow is abruptly ended by the entrance of denser water at the bottom, shifting the isotherms and the inward jet quickly upward, away from the bottom. Similar patterns are seen during many of the other tidal periods during the last 8 days of the mooring deployment (Fig. 7).

The bursts with largest dissipation rates occur during the bottom-enhanced downslope flow, before entrance of the denser water. The largest dissipation rates are clearly separated into bottom-enhanced bottom boundary layer (BBL) turbulence and a secondary maximum above the bottom current. There are also enhanced dissipation rates after the entrance of denser water, but not as large as before the entrance.

Even more detailed information can be found by zooming in on the individual profiles of the burst just before day 272.5 (Fig. 5). It turns out that the first and third profiles are similar to each other but very different from the second and fourth profiles, which are also similar to each other. The reason is a relatively strong drift of the vessel, which made us perform the first profile inside of the mooring, at about 85-m depth, drifting out toward shallower water where the second profile was performed at about 75-m depth. Then the vessel was repositioned to the first position and drifted while the third and fourth casts were performed.

Both of the deep density profiles have a large unstable region between 65- and 75-m depth and a denser and almost homogeneous bottom layer from 79-m depth down to the bottom, whereas the shallower profiles are more continuously stratified without any large unstable regions. The largest dissipation rates, up to $10^{-6}$ W kg$^{-1}$, occur in the unstable region and in the BBL of the deep profiles. The Thorpe displacements, which are the vertical movements needed to obtain a stable profile of the given density observations, are on the order of 10 m within the high dissipation region. The shallow profiles generally have lower dissipation rates, especially in the 65- to 75-m-depth region, where they are one to two decades smaller. The shallow and deep profiles are only separated by about 200 m and still there is this large difference from location to location but almost none from time to time at similar locations. This suggests that a localized region of overturning with a horizontal extension on the order of $10^2$ m is occurring at a certain phase of the tide.

Around the same time, the mooring data in Fig. 5f show a relatively quick spreading of the isotherms between 63 and 65 m into a 10-m-thick, nearly homogeneous layer with stagnant and even negative (outward) flow, whereas the isotherms below 65 m dive down into a swift bottom current down the slope (inward). After the
isothermal spreading, there are signs of high-frequency oscillations at the interface between the homogeneous layer and the bottom current. These oscillations are visible in both isotherms and velocities and will be treated in more detail below. There are no evident unstable events in the CT logger data, but such events cannot be ruled out with the given spacing and time resolution of the CT loggers.

Assuming that the density field is stationary in time during the MSS burst, one can calculate the isopycnal depths at each profile and at the mooring during the same time; one can then connect these depths with lines for a quasi-synoptic view of the spatial density variations (Fig. 6). Data from a Scanfish transect have been added inside and outside the MSS region, but these data are not available very close to the bottom. Velocities at the mooring and MSS dissipation levels are also shown. It is seen that the nearly homogeneous region with high dissipation rates is an oblique wedge pointing toward the shallow region. Moving from the shallow region, the lowest isopycnals dive down along the bottom and then rebound through the homogeneous region one by one, creating a horizontal density gradient in the homogeneous region. This is very similar to what has been observed in Knight Inlet, where the highest dissipation levels were also observed in the homogeneous rebounding region (Klymak and Gregg 2004). It is also similar to the breaking region observed during severe downslope windstorms in the atmosphere (Lilly 1978). Contrary to these two examples, “our” flow separates from the bottom just downstream of the mooring. Such a separation is also observed during the early phases of the flow in Knight Inlet (Farmer and Armi 1999) and in Loch Etive (Inall et al. 2005). Accordingly, what we see is probably a transitional flow, where the inward flow becomes supercritical just upstream of the mooring and rebounds toward the downstream state in a hydraulic jump. We will discuss this possibility further below, but first we will zoom out our view again, and see if this feature is a one-time phenomenon or if it occurs regularly.
When inspecting the last eight tide-dominated days of the mooring data (Fig. 7), flow situations similar to those on day 272 occur during several of the tidal periods, with strong, bottom-intensified downslope currents below stagnant, vertically strained layers. The maximum velocities tend to occur a little before the maximum flood barotropic tide, but they are coincident with the maximum mean velocities below 60-m depth. The latter are seen to be much larger than the barotropic velocities. The barotropic velocities are calculated from the surface elevation time series as

$$u_0 = \frac{A_f \, \partial \eta}{A \, \partial t},$$  

where $A_f$ is the surface area of the fjord inside the mooring, $A$ is the cross-sectional area at the mooring, and $\eta$ is the surface elevation. The assumptions behind (2) are that the local surface elevation is dominated by the barotropic contribution, that the fjord is much shorter than the barotropic wavelength, and that there is no significant choking inside the mooring. The dominance of baroclinic over barotropic velocities, as well as the strict tidal periodicity, indicates that the dissipation mechanism we observe here is related to the baroclinic tidal motions.

There may be more events than those marked in Fig. 7 if they have thinner jets and are therefore not captured by the lowest ADCP bin. One indication of this is that the depth-integrated dissipation rates below 60-m depth at station 3 show clear peaks at the same phase of the tide during all tidal periods, even though there are no observed bottom jets.

When looking at Fig. 2, one sees even stronger such events (e.g., on days 254 and 261). But these occur on a less regular basis and are probably connected with the internal seiche motions rather than the tidal motions or maybe, even more realistically, by the superposed internal tidal and internal seiche bottom-layer velocities. We will return to these events below.

**Fig. 5.** Individual profiles of the day 272.45 MSS burst (zooming in from Fig. 4) showing (a) $\sigma_\theta$, (b) Thorpe displacements, (c) dissipation rates of turbulent kinetic energy, and (d) buoyancy Reynolds number, for the first (blue), second (green), third (red), and fourth (cyan) profile of the burst. (e) Positions and (f) times of the profiles are shown. (f) Also shown is the time series of velocity and isotherms, as in Fig. 4a.
For a hydraulic transition to take place, the internal wave pattern downstream of the point of transition needs to have a horizontal relative phase speed similar to the layer velocity at the point of transition. The horizontal relative phase speed for a long wave is

\[ c_p = \frac{N \lambda}{2\pi}, \]

where \( \lambda \) is the vertical wavelength and \( N \) is the buoyancy frequency. The largest vertical wavelength for an internal wave mode restricted to a waveguide below 60 m is twice the layer depth (i.e., \( \lambda = 40 \) m at the 80-m mooring). The average value of \( N^2 \) below 60-m depth is \( 5 \times 10^{-3} \) s\(^{-2}\). This gives a phase velocity \( c_p = 0.045 \) m s\(^{-1}\). The mean velocities below 60-m depth are about 0.04–0.05 m s\(^{-1}\) (Fig. 7) when the bottom-intensified jets occur. This supports the interpretation above, namely that the observed flow pattern is caused by a hydraulic transition.

\[ b. \text{ Pulsations} \]

Figure 5f shows high-frequency oscillations of isotherms at about 70-m depth and corresponding velocity pulsations. A spectrum of the temperature time series at 71 m with mean velocities below 60-m depth, one finds that high variance levels are significantly correlated with large inward velocities. The correlation coefficient between the velocity time series and the variance time series is 0.26. A higher correlation coefficient, 0.50, is obtained by taking the logarithm of the variance time series. An even higher correlation coefficient, 0.74, is obtained by restricting the analysis to the maximum temperature variance and maximum inward velocity within each tidal period and taking the logarithm of the temperature variance. This shows that there is a strongly nonlinear increase in the pulsation amplitude with increasing downslope velocities.

Similar pulsations are observed in severe downslope windstorms in the atmosphere (Neiman et al. 1988), cascading cold waters in lakes (Fer et al. 2002), and laboratory experiments of dense gravity currents (Cenedese et al. 2004), and they seem to be a general feature of supercritical bottom jets. The physical explanation differs from case to case. Based on high-resolution numerical simulations, Afanasayev and Peltier (1998) argue that
Pulsations in atmospheric downslope windstorms are caused by Kelvin–Helmholtz instabilities in the shear region above the jet, whereas Fer et al. (2002) and Cenedese et al. (2004) argue that pulsations in supercritical gravity currents are of roll wave type (i.e., caused by bottom friction). The resolution vertically and in time of our velocity and density profiles is not good enough to obtain reliable estimates of Richardson (Ri) numbers, so we will not be able to resolve the mechanism behind these fluctuations or determine their importance for the observed overturning with the present dataset. However, the observed overturning above the jet indicates that Kelvin–Helmholtz or Holmboe instabilities may be of importance.

5. Mixing

When determining mixing from the dissipation rate data, we want to distinguish between dissipation that happens within and outside the bottom boundary layer, since turbulent motions very close to the bottom cannot be expected to mix fluid in the same manner as turbulent overturns far from the bottom. There are seldom any clear homogeneous boundary layers to see in the density data, which makes it difficult to define a boundary layer based on the density profile. We therefore choose to use another method based on the dominant turbulent length scales. Close to the bottom, the law-of-the-wall length scale is $\kappa z$, where $\kappa = 0.4$ is the von Kármán constant, and in the stratified region the natural vertical length scale is the Ozmidov scale $L_o = e^{1/2} N^{-3/2}$. Near the wall the Ozmidov scale goes toward infinity. Therefore we define the BBL as the dissipation bins closest to the bottom where $\kappa z < L_o$. The results tend to be rather insensitive to the exact definition of the bounding length scale. Increasing this by a factor of 2 increases the mean boundary layer dissipation rates by less than 10%.

Even though the observations end only about 10 cm from the bottom, a significant fraction of the dissipation can be expected to take place in those missing centimeters. According to the law of the wall, the dissipation rate is inversely proportional to the distance from the bottom. One can show that the missing dissipation can be written as

$$\int_{z_0}^h \varepsilon \, dz = \frac{\ln(h/z_0)}{\ln(z/h)} \int_{z_0}^h \varepsilon \, dz,$$

(Eq. 4)
where $z_0$ is the bottom roughness and $h$ is the thickness of the missing layer. Taking $z_0 = 10^{-3}$ m, $h = 10^{-1}$ m, and $z = 0.6$ m, where $h$ and $z$ correspond respectively to the lower and upper boundaries of the lowest bin, we find that the missing dissipation is 2.6 times the observed dissipation within the lowest bin. This decreases dramatically with increasing bottom roughness, but without knowing the bottom roughness, $z_0 = 0.001$ m is the best guess we can make.

The average dissipation rates within and outside the BBL are shown for stations 1–3 in Fig. 9 with 95% bootstrapped confidence bands; the mean BBL thickness is also indicated as the height of the BBL boxes. The depth-integrated dissipation rates below 50 m inside and outside the BBL are given in Table 1. It is seen that about 59% of the dissipation below sill level happens within the BBL.

The average dissipation rates below sill level are in the order of $0.5 - 1 \times 10^{-8}$ W kg$^{-1}$ at stations 2 and 3 and about $2 \times 10^{-8}$ W kg$^{-1}$ at station 1, whereas they are in the order of $2 - 5 \times 10^{-7}$ W kg$^{-1}$ within the boundary layer (Fig. 9). For comparison, the dissipation levels away from the sill region found during the 2001 experiment (Arneborg et al. 2004) were on the order of $3 \times 10^{-9}$ W kg$^{-1}$. Exactly at the sill-level pycnocline, there is no significant difference between the three stations, since the station 1 dissipation rates decrease toward $10^{-8}$ W kg$^{-1}$ at that depth.

The diffusion coefficient for vertical diffusion of buoyant kinetic energy, using the expression

$$K_z = \frac{\langle \Gamma \varepsilon \rangle}{\langle N^2 \rangle},$$  \hspace{1cm} (5)

where $\Gamma$ is the ratio of buoyancy flux to dissipation rate, also called the mixing efficiency. This is usually set to 0.2 (Osborn 1980), but a recently proposed parameterization based on laboratory and direct numerical simulations (Shih et al. 2005) suggests that it should be a decaying function of the buoyancy Reynolds number,

$$\text{Re}_b = \frac{\varepsilon}{\nu N^2},$$  \hspace{1cm} (6)

for $\text{Re}_b > 100$. We have calculated the values of $K_z$ both with the constant mixing efficiency and with the Shih et al. (2005) parameterization. Some care needs to be taken in the averaging procedure. We have used instantaneous values of the buoyancy Reynolds number to calculate instantaneous mixing efficiencies, which are multiplied with the dissipation rates to obtain the buoyancy fluxes. These are then averaged in time and divided by the average buoyancy frequency squared, as in (5), to obtain the diffusion coefficient.

The diffusion coefficients (Fig. 9c) range from about $10^{-7}$ m$^2$ s$^{-1}$ in the sill-level pycnocline to $4 - 20 \times 10^{-5}$ m$^2$ s$^{-1}$ farther down, depending on which mixing efficiency model is used. The diffusion coefficients are generally about 5 times larger at station 1 than at stations 2 and 3 at the same depths. At these depths (50–65 m) there are no big differences between the two diffusion coefficient estimates. Farther down at station 3, however, the Osborn model tends to give up to 5 times higher diffusion coefficient values than the $\text{Re}_b$-dependent model. Above sill level the two models also differ with up to a factor of 5 at stations 1 and 3. The reason lies in the buoyancy Reynolds number, which is based on average dissipation and buoyancy frequency values (Fig. 9d). This “average” buoyancy Reynolds number is larger than 100 in the depth intervals where there is a difference between the two models. Close to the sill-level pycnocline, the average value is so small (<10) that one would not expect the turbulence to be isotropic, which is one of the main assumptions behind (1). However, the main dissipation happens at much larger values of instantaneous buoyancy Reynolds numbers, which means that we expect the dissipation rates to be reasonably correct, though maybe slightly overestimated.

Now, the bulk mixing efficiency defined as the integrated buoyancy flux divided by the integrated dissipation rates can be calculated. These values are given in Table 1 for depths below 50 m, with and without the bottom boundary layer dissipation rates included. In these estimates it is assumed that there is no work against buoyancy within the BBL, which may be a somewhat conservative assumption. One could also use the Shih et al. (2005) model in the BBL, but because that
model is based on shear-generated turbulence far from boundaries we do not want to do this. It can be seen that the bulk mixing efficiency including the BBL is about 0.07 when using the Shih et al. (2005) model. This is similar to values obtained from energy budgets both in the same fjord (Arneborg and Liljebladh 2001) and in other fjords (e.g., Stigebrandt and Aure 1989) but is much smaller than the value of 0.2 in the Osborn (1980) model. It is also seen that not only the BBL but also reduced mixing efficiencies in interior large-\(Re_p\) regions cause this reduction in bulk mixing efficiency. In addition, it is unclear whether the decrease in potential energy caused by collapsing localized mixing regions (Arneborg 2002) is taken into account in the Shih et al. (2005) parameterization, whereas the dashed lines are based on the Osborn (1980) model.

![Graph](image)

**FIG. 9.** Average profiles of (a) \(N^2\); (b) \(\varepsilon\); (c) \(K_{\alpha}\); and (d) \(Re_p\), at stations 1 (blue), 2 (green), and 3 (red). The shaded regions are the bootstrapped 95% confidence intervals. The lines are based on data outside the BBL; the boxes in (b) are based on BBL data. The thicknesses of the boxes are the mean BBL thicknesses. The full lines in (c) are based on the Shih et al. (2005) parameterization, whereas the dashed lines are based on the Osborn (1980) model.

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<th>Table 1. Depth-integrated dissipation rates of turbulent kinetic energy below 50-m depth inside and outside the bottom boundary layer, and bulk mixing efficiency (\Gamma) below 50-m depth with and without including the bottom boundary layer.</th>
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<td>Integrated dissipation rates (mW m(^{-2}))</td>
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<td>Outside BBL</td>
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<td>Average</td>
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* The average mixing efficiency is calculated as the integrated rate of work against buoyancy fluxes at all three stations divided by the integrated dissipation rates.
localized mixing regions, and large BBL dissipation rates, one may need to measure buoyancy fluxes rather than dissipation rates to obtain reliable estimates of mixing.

6. Tidal energy budget

a. Tidal analysis

The results of tidal analysis for days 271–274 are shown in Fig. 10 for the mooring and Fig. 11 for the MSS data. The amplitudes and phases are obtained by least squares fitting a sine function with semidiurnal ($M_2$) period, amplitude $a$, and phase shift $\varphi$ to a time window of length 37.2 h (three semidiurnal periods). This window is then moved through the time series with a time step of one semidiurnal period. Then the average amplitude and phase are found as the amplitude and phase of $\langle a \exp(i\varphi) \rangle$, where angle brackets denote time averaging. Finally, the average phase shift of the barotropic velocity is subtracted to obtain the phase shift relative to the flood tide. A positive phase shift denotes that the tidal maximum occurs prior to the flood tide.

Tidal analysis is performed on the baroclinic velocities, baroclinic pressure, and vertical displacements. The baroclinic velocities are the observed velocities minus the barotropic velocities calculated using (2). The vertical displacements are calculated as

$$VD = \rho' \left( \frac{d(\rho')}{dz} \right)^{-1},$$

(7)

where $\rho'$ is the perturbation density ($\rho' = \rho - \langle \rho \rangle$). The baroclinic pressure is

$$p_{bc}(z) = p_s + \int_z^\eta \rho' g \, dz,$$

(8)

where $p_s$ is the baroclinic surface pressure calculated as

$$p_s = -\int_{-H}^\eta \rho' g \, dz,$$

(9)

where $H$ is the depth. Calculated this way, the depth-averaged baroclinic pressure is zero (e.g., Kunze et al. 2002). The reason for this definition of the baroclinic pressure is that the total energy flux can be separated into a purely barotropic component, which is the product of the depth averages of the perturbation velocity and the perturbation pressure, and a baroclinic component, which is the product of the baroclinic perturbation velocity and baroclinic pressure.

The baroclinic semidiurnal velocity has almost two-layer characteristics with clearly larger amplitudes below sill level than above, and an almost $180^\circ$ phase shift at about 45-m depth (Fig. 10a). Below sill level the phase is nearly constant. The bottom-layer baroclinic velocities

Fig. 10. The $M_2$ amplitudes (solid) and phases (dotted) at 80-m mooring of (a) baroclinic velocities, (b) baroclinic pressures, and (c) vertical displacements. Dashed lines in (c) are model results based on (6).
are about 45° (1.5 h) earlier than the barotropic. This may even be seen in Fig. 7c. Above sill level there are somewhat larger variations in both phases and amplitudes, but the main phase variations are later than the barotropic velocity within the interval 90°–180°.

The baroclinic pressures show an even stronger two-layer behavior than the velocities, with nearly constant phases above and below sill level, a 180° phase shift just below sill level at the sill level pycnocline, and a minimum amplitude near sill level (Fig. 10b). The phase of the bottom-layer baroclinic pressure is about 165° earlier than the barotropic velocity, which means that it is about 120° earlier than the bottom-layer baroclinic velocity. A progressive internal wave radiating away from the sill would have a bottom-layer baroclinic pressure in phase with the bottom-layer baroclinic velocity. For a progressive internal wave radiating toward the sill, the phase difference would be 180°. For a pure standing wave with a node placed further away from the sill, the baroclinic pressure would be 90° earlier than the baroclinic velocity. Therefore, the observed phases indicate that the baroclinic motions are a combination of a standing mode and a wave propagating toward the sill. Similar results have been reported in earlier observations (Arneborg et al. 2004).

When looking at the tidal results for the baroclinic pressures at the three MSS stations (Fig. 11), one sees that the amplitude above sill level is close to constant at the three MSS stations, whereas it increases toward the sill below sill level. The phase decreases toward the sill from about 165° at station 3 to 135° at station 1. This indicates a phase propagation toward the sill. The phase change is about 30° (~1 h) from station 3 to station 1, which are located 3.6 km from each other. This gives a phase velocity of about 1 m s⁻¹. The first baroclinic mode has a phase velocity of 0.72 m s⁻¹, so the phase difference is difficult to explain with a pure propagating internal wave. However, a standing wave would give zero phase difference and infinite phase velocity using the same arguments, so the observations could be caused by a partly standing wave with the wave propagating toward the sill having a somewhat larger amplitude than the one propagating away. This is in accordance with the results based only on the mooring data in Fig. 10.

A standing internal wave is in accordance with earlier observations by Arneborg et al. (2004), who find very little dissipation in the main and inner parts of the fjord, indicating that the internal tides are reflected from the inner parts almost without energy loss. However, one would still expect a small progressive component into
the fjord rather than toward the sill. The fact that there seems to be a small energy component toward the sill will be discussed below. But first we will look at the vertical displacements in Fig. 10c.

The vertical displacements (Fig. 10c) show a strong increase in amplitude toward the bottom. The phase is about 180° out of phase with the barotropic velocity below sill level, so that lowest isopycnal levels are obtained around flood tide. One explanation for the large vertical displacements could be the convergence of the bottom layer velocities because of the bottom slope. A simple model of this process is that the velocity vectors are horizontal at a certain level \( z_0 \) and increase their slope linearly toward the bottom, so that they are parallel to the bottom at the bottom. This gives the vertical velocity

\[
    w = \frac{\partial V D}{\partial t} = -s(z_0 - z)U(z) \left( \frac{z_0 + H}{z_0} \right),
\]

where \( s \) is the bottom slope. The amplitude and phase of the modeled vertical displacements are shown in Fig. 10, using the observed velocities and taking \( z_0 \) to be \(-50 \text{ m} \) (about the level of the sill level pycnocline), with the bottom slope being 0.02. This forcing is caused by a low-mode internal tide rather than by the barotropic tide, which would have had \( z_0 = 0 \) and much smaller velocity amplitude. It is seen that this forcing mechanism of the vertical displacements explains the observed amplitudes and phases relatively well. There is a tendency that the observed displacements occur earlier than the modeled. However, given that the model does not take into account the baroclinic response to the horizontal density gradients that are set up, the question is rather why it works so well than why there are some discrepancies. Probably the reason is that internal waves below the sill-level pycnocline have phase velocities of the same order of magnitude as the flow and cannot freely propagate away and modify the stratification. This is in accordance with the results of section 4.

b. Baroclinic tidal energy fluxes

The baroclinic tidal energy flux per unit area,

\[
    F_{bc} = (u_{bc} p_{bc}),
\]

where \( u_{bc} \) and \( p_{bc} \) are the baroclinic velocity and pressure respectively, can be calculated as

\[
    F_{bc} = \frac{1}{2} a_u a_p \cos(\varphi_u - \varphi_p),
\]

where \( a_u \), and \( a_p \) (\( \varphi_u \) and \( \varphi_p \)) are the tidal amplitudes (phases) of the baroclinic velocity and pressure. The results for the 80-m mooring, averaged over days 271–274, are shown in Fig. 12. There is a general net energy flux toward the sill below sill level, which is a consequence of the slightly more than 90° phase shift between baroclinic velocities and pressures seen in Fig. 10. Thus, as discussed in the previous section, the baroclinic motion is a standing wave, caused by reflection from the inner parts of the fjord, with a small component toward the sill. Integrating the energy flux over the whole cross section, assuming that the energy flux is uniform across the width of the fjord, one obtains a net energy flux of 0.68 kW toward the sill. The energy flux toward the sill below sill level is a rather rigid result, but the fluxes above sill level are less certain. There is a significant low-frequency component caused by the relatively strong exchange currents and the rising halocline, and the results show relatively large variations depending on the method used to remove them from the tidal analysis. It is therefore possible that there are inward fluxes above sill level that compensate for those below sill level.
An alternative method is to fit normal modes to the tidal amplitudes and phases (e.g., Webb and Pond 1986). This is a less exact method than the one used above, since normal modes are not valid on sloping bottoms. When least squares fitting the first two dynamic modes to the observed tidal amplitudes and phases (Fig. 10), one obtains a net energy flux toward the sill of 1.6 kW consisting of 2.3 kW radiating into the fjord and 3.9 kW radiating outwards. One could also fit more modes, but generally the fit does not improve considerably, and the result for the net flux remains in the vicinity of 1–2 kW toward the sill. Considering the results of the direct tidal energy flux calculation and the modal decomposition, we estimate the energy flux past the 80-m mooring to be 0.7 ± 1.0 kW.

The radiation of internal tide energy can be compared with that calculated from simple linear theory, assuming the sill to be a step, and neglecting energy radiation on the shallow side of the step. Under these approximations, and inspired by the two-layer model of Stigebrandt (1976), Stacey (1984) developed an expression for continuous stratification:

\[ F_{IT} = \frac{1}{2} \rho g B_s U_s \sum_{n=1}^{\infty} \frac{|W_n(z_s)|^2}{\int_{-H}^{0} (dW_n/dz)^2 \, dz}, \tag{13} \]

where \( F_{IT} \) is the total internal tide energy flux away from the sill, \( B_s \) is the width of the fjord near the sill, \( U_s \) is the barotropic velocity amplitude over the sill, \( z_s \) is the sill depth, and \( W_n \) is the \( n \)th vertical velocity mode. Using (2) to calculate the barotropic velocity over the sill and tidal analysis to determine the semidiurnal amplitude, an average value of 3.2 kW radiating into the fjord is obtained. This fits well in magnitude with the 2.3 and 3.9 kW found above to radiate inwards and outwards past the mooring.

### c. Energy budget

A rough estimate of the integrated dissipation rate of kinetic energy below 50-m depth can be found by extrapolating the results of Table 1 to the whole region between the entrance and the inner mooring. The depth-integrated dissipation at these stations is approximately 0.7 mW m\(^{-2}\) and the area is 4.8 km\(^2\), which gives an integrated dissipation rate of 3.4 kW.

The energy loss from the barotropic tidal current can be calculated from the phase difference of pressure records inside and outside the sill. If there is no phase difference, the barotropic response is a standing wave without energy loss. For small phase differences the energy loss between the pressure recorders can be written as (Stacey 1984)

\[ E_{BT} = \frac{1}{2} \rho g A_f \omega \sigma^2 (\phi_i - \phi_f) \left( 1 - \frac{2 \Delta A}{A_f} \right), \tag{14} \]

where \( A_f \) is the surface area of the fjord inside the outer pressure recorder; \( \Delta A \) is the surface area between the pressure recorders; \( \sigma \) is the tidal surface elevation amplitude at the outer recorder; \( \phi_i \) and \( \phi_f \) are the tidal phases at the inner and outer recorders, respectively; and \( \omega \) is the tidal frequency. The main assumptions behind (14) are that the fjord is short relative to the barotropic wavelength and that the surface elevation is dominated by the barotropic component.

Tidal analysis is performed on the bottom pressure records at the outer and inner moorings in a manner similar to that used on the inner mooring data. The average energy loss from the barotropic semidiurnal tides over days 271–274 is 14 kW. The energy loss from the barotropic tide is seen to be about 5 times larger than the integrated dissipation below sill level and the inward internal tide energy flux.

One does not have to look far for possible energy sinks of the barotropic energy. The average dissipation between 10- and 50-m depth at station 1 is \( 8.7 \times 10^{-8} \) m\(^2\) s\(^{-3}\), which means that 4 km\(^2\) with such dissipation rates are needed to obtain 14 kW. This is not unreasonable, considering that there seems to be a strong increase in dissipation rates within this depth range toward the entrance. Some of the barotropic energy loss could also be lost to outward-radiating internal tides on the seaside of the sill. However, the depths there are similar to the sill depth in a large region, and it is difficult to see that region as a strong internal tide generation region.

The total semidiurnal tidal energy budget as we see it is sketched in Fig. 12. The barotropic semidiurnal tide loses 14 kW in the vicinity of the sill. About 3 kW is lost to internal tides, which radiate past the inner mooring, are reflected in the inner parts of the fjord, and return to the sill region. We expect that the net energy flux toward the sill obtained from our mooring data is an artifact of uncertainties mainly in the upper part of the water column and that the real net energy flux is close to zero. About 3 kW dissipates below sill level near the sill, and the remaining 11 kW dissipates above sill level near the sill. It may be a coincidence that the energy dissipation below sill level and the inward internal tide radiation are similar, but the dissipation below sill level is clearly tidally driven, and the result fits perfectly with the ideas of Stigebrandt (1976) that most of the energy put into internal tides dissipates below sill level.

Considering the large fraction of dissipation that is concentrated in the bottom boundary layer, some of the bottom-layer dissipation may be caused by frictional
dissipation of the barotropic tide. However, because bottom friction dissipation is proportional to the bottom velocity cubed, and the baroclinic velocity is about 5 times larger than the barotropic (Fig. 5c), the barotropic contribution to the BBL dissipation is less than 1% of the baroclinic. Therefore, we think that the 3 kW dissipating below sill level at the sill is mainly taken from the internal tides.

7. Summary and discussion

Turbulent dissipation rate measurements and mooring data near the sill of Gullmar Fjord were examined with emphasis on the turbulence processes below sill level and the overall tidal energy budget. The highest dissipation rates below sill level, on the order of $10^{-6}$ W kg$^{-1}$, were observed during the tidal phases with maximum negative isopycnal displacements. At the deepest station these high dissipation rates occurred within a hydraulic jump and in the bottom boundary layer of a supercritical jet downstream of a submarine constriction.

During transitional flow there were clear pulsations of the jet and density fluctuations at its upper boundary with a period of about 15 min. These pulsations were generally present during inward flow but were increasing dramatically with increasing intensity of the bottom jet. Even though our dissipation rate measurements were performed during a period when tides were dominating, mooring data from earlier periods with strong internal seiche motions indicate that exactly the same dissipation mechanism acts on the internal seiche.

The new thing relative to earlier observations of hydraulic jumps in fjords (e.g., Klymak and Gregg 2004; Inall et al. 2005) is that this fjord does not have barotropic tidal velocities large enough to cause supercritical conditions for the lowest internal wave modes. It is the baroclinic response to the barotropic forcing that cause supercritical conditions for high internal wave modes in the deeper parts of the fjord, where the stratification is weaker. It is not totally clear from the present dataset what causes the enhanced bottom-layer velocities, but two relevant explanations are (i) the near-critical slope of the bottom toward reflection of internal semidiurnal tides and (ii) the constriction of the bottom layer set up by the submarine hill located at the 80-m mooring (Fig. 1). In the present case it is probably a combination of these two conditions that causes the enhancement of velocities necessary for a hydraulic transition.

More than half of the dissipation below sill level was found to happen in the bottom boundary layer. This turbulence cannot be expected to contribute much to mixing. Two different models were used to obtain buoyancy fluxes from dissipation rates outside the bottom boundary layer. One was using the traditional constant mixing efficiency and the other assumed a decreasing mixing efficiency with increasing buoyancy Reynolds number according to results of direct numerical simulations (Shih et al. 2005). These two models gave very different results mainly within the hydraulic jump region where the buoyancy Reynolds number was large. If the Shih et al. (2005) model is correct, the combined effect of high-buoyancy Reynolds numbers in the hydraulic jump and the large fraction of dissipation happening within bottom boundary layers may explain the low bulk mixing efficiencies obtained in previous fjord studies (e.g., Stigebrandt and Aure 1989). However, direct observations of buoyancy fluxes within hydraulic jumps would be necessary before drawing such conclusions. A recently published tracer study (Inall 2009) indicates that similar low mixing efficiencies are obtained for internal tide dissipation on the inner slopes of a fjord, and in that case it is probably mainly the concentration of the diapycnal transports to the bottom boundary layer that causes the decrease in mixing efficiency.

Tidal analysis of mooring data inside the sill showed that the main baroclinic response had no net energy flux away from the sill; rather, it consisted of a standing internal wave pattern with about 3 kW propagating in and out of the fjord. The total energy loss by the barotropic tide, estimated from pressure time series inside and outside the sill, is 14 kW. Because there is no net energy flux away from the sill, this amount of energy must be lost to turbulence and dissipation near the sill. We estimate that 3 kW dissipates below sill level, leaving the remaining 11 kW to be dissipated above sill level near the sill. We think that the 3 kW below sill level is mainly caused by decay of the reflected internal tides. The mechanisms behind the large energy loss above sill level is beyond the scope of the present study, but lee waves and three-dimensional effects such as vortex shedding from the fjord sides are potential explanations.

One may speculate as to whether the tides are in resonance or not and whether this is important for the present results. In the case of resonance, one would expect large energy fluxes in and out of the fjord compared with the rates of energy transformation from barotropic to baroclinic energy, and from baroclinic energy to turbulence and dissipation. In the present case the best estimates of tidal transformation, baroclinic energy fluxes, and dissipation rates are all in the same range of about 3 kW. This indicates that most of the reflected internal tide energy is dissipated without participating constructively in the generation of transmitted
internal wave energy. This, in turn, indicates that resonance is not important for the present results.

Another relevant question is whether the hydraulic transition depends critically on the fact that the baroclinic response is of a standing rather than a progressive nature. In other words, the question is whether a progressive internal wave with a velocity magnitude similar to that of the present standing wave would cause a similar response. One may want to remember that the low-mode internal tides that cause the velocity field are not supercritical, since the phase speed is about 10 times larger than the layer velocities. The question is similar to asking whether an internal hydraulic jump set up by a barotropic tide is dependent on the tide being standing or progressive. The answer naturally is no, since it is the velocity field that causes the transition, whereas the surface elevation may be seen as a rigid lid. In our case the pycnocline at 50 m could just as well be a rigid lid (see also Fig. 10c); it does not influence the isopycnal positions at larger depths. A progressive internal wave with similar velocity amplitudes in the lower layer would therefore, most likely, cause exactly the same response.

These fjord results may have relevance even in the deep ocean. It has been found that most of the dissipation in the deep ocean happens within flank canyons and rift valleys on the midocean ridges (e.g., St. Laurent et al. 2001). The two governing explanations for these high dissipation levels are the breaking of topographically generated internal waves (St. Laurent et al. 2001) and overflow-related mixing (e.g., hydraulic jumps) caused by along-valley mean flows (Thurnherr 2006). With the present results in mind, it is not unreasonable to think that hydraulic jumps are generated also by the baroclinic response to the barotropic tidal forcing. With the huge amount of irregular crests and sills within the canyons on the midocean ridges, it may in fact be that our fjord observations are highly representative for the deep ocean.

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