The Physical Oceanography of Fjords

DAVID M. FARMER and HOWARD J. FREELAND

Institute of Ocean Sciences, Sidney, B.C., V8L 4B2, Canada

(Received 1 December 1982)

CONTENTS

1. Introduction 149
2. General Description and Oceanographic Classification of Fjords 151
3. Theoretical Approaches to Estuarine flow 156
3.1. K-models 159
3.2. Layered models 160
4. Exchange Processes 166
4.1. Tides in fjords 167
4.1.1. Barotropic tides 167
4.1.2. Internal tides 170
4.2. Deep water renewal 174
4.3. Shelf processes 177
5. Mixing Processes in Fjords 178
5.1. Wind in fjords 181
5.2. Tidal mixing 183
5.2.1. Mixing by internal tides 183
5.2.2. Sill processes 185
5.2.3. Nonlinear internal wave trains 195
5.3. Lateral variability; mixing in river plumes 201
5.4. Thermal convection 203
5.5. Double diffusive effects 204
6. Concluding Remarks 207
6.1. Energy constraints on mixing 207
6.2. Summary 213
References 215

Symbols used - The list of symbols below is not meant to be exhaustive. In the interests of brevity we list only symbols that are used at two or more distinct areas of the text. Symbols that are used for one paragraph only, and are defined locally within the text, are not included in this list.

\( A \) Surface area of a fjord.
\( A_v \) Vertical eddy diffusivity of momentum.
\( B \) Inlet breadth.
\( B_f \) A buoyancy flux.
\( B_m \) \( B \) evaluated at the mouth.
\( B_o \) Surface buoyancy flux.
\( B_s \) \( B \) evaluated for a sub-basin.
\( b_0 \) The buoyancy of sea-water = \( g(p - \rho_0)/\rho_0 \).
\( c_n \) Eigenvalues of the vertical modal equation for internal waves, wave speed for internal Kelvin waves.
\( c_p \) The specific heat of water.
\( F_e \) An estuarine Froude number.
\( F_n \) n'th internal Froude number, \( F_n = u/c_n \).
\( F_{b}, F_{H} \) Fluxes of salt and heat due to the salt fingering instability.

\( f \) The Coriolis parameter.

\( g, g' \) The acceleration due to gravity and reduced gravity.

\( H \) Total fluid depth.

\( H_{t} \) Heat flux through the free surface.

\( H_{m} \) \( H \) evaluated at the mouth.

\( h_{1}, h_{2} \) Thicknesses of layers 1 and 2, respectively.

\( L \) The Monin-Obukhov length scale.

\( N \) Brunt-Väisälä frequency.

\( \mathbf{n} \) Unit outward normal to the surface of a control volume.

\( P \) Energy dissipation rate, power.

\( p \) Pressure.

\( Q_{f} \) Fresh water discharge.

\( q_{1} \) Surface layer transport/unit breadth.

\( q_{f} \) \( Q_{f} \) per unit breadth.

\( R \) The Rossby radius of deformation.

\( R_{f} \) A flux Richardson number.

\( R_{1} \) A Richardson number.

\( R_{n} \) The \( n \)'th internal Rossby radius of deformation, \( R_{n} = c_{n}/f \).

\( R_{p} \) Salinity versus temperature stability ratio.

\( S_{0} \) Depth and tidally averaged salinity.

\( s \) Mean circulation part of salinity.

\( U_{f} \) Cross-sectionally averaged river flow velocity.

\( U_{*} \) Circulation velocity measured at the surface.

\( u_{w} \) The surface friction velocity.

\( (u, v, w) \) Along-inlet, across-inlet and vertical components of fluid velocity.

\( (u', v', w') \) Turbulent parts of \( (u, v, w) \).

\( (u_{T}, v_{T}, w_{T}) \) Tidal parts of \( (u, v, w) \).

\( (\bar{u}, \bar{v}, \bar{w}) \) Mean circulation parts of \( (u, v, w) \).

\( W_{e} \) The vertical entrainment velocity.

\( (x, y, z) \) Along-inlet, across-inlet and upward directions.

\( \alpha \) The thermal expansion coefficient for water.

\( \dot{\beta} \) Rate of change of density with salinity.

\( \gamma \) A wind mixing efficiency factor.

\( (\Delta S_{v}, \Delta S_{h}) \) Vertical and horizontal salinity contrasts.

\( \Delta \rho \) Density difference between upper and lower layers.

\( c \) A tidal phase shift.

\( \epsilon_{i}(\omega) \) Internal wave energy flux as a function of frequency.

\( \xi \) Tidal elevations.

\( \xi_{e} \) Equilibrium tidal elevations.

\( (\xi_{1}, \xi_{o}) \) \( \xi \) measured inside or outside of a constriction, respectively.

\( \eta \) Non-dimensional interface location.

\( \Omega \) Non-dimensional salinity contrast between upper and lower layers.

\( \rho \) Fluid density, subscripts connote layer numbers.

\( \sigma \) Root-mean-square turbulent velocity.

\( \tau \) Vector stress.

\( (\tau_{x}, \tau_{y}, \tau_{z}) \) \( (x, y, z) \) components of \( \tau \).

\( \tau_{b} \) Bottom stress.

\( \omega \) A tidal frequency, \( \omega = 2\pi/\text{period} \).

\( \nabla \) Vector gradient operator.

\( \nabla_{h} \) Horizontal projection of \( \nabla \).
1. INTRODUCTION

THE PHYSICAL processes occurring in fjords have attracted oceanographers for over a century. Early observations in Scandinavian fjords by EKMAN (1875), GRAU (1900), HELLAND-HANSEN (1906), GRUND (1909) and others demonstrated the fundamental mode of circulation, with a brackish outflow on the surface above a deeper compensation current. These observations stimulated a number of attempts to describe theoretical aspects of fjord circulation, starting with W. EKMAN's (1899) simple barotropic model and KNUDSEN's (1900) application of continuity considerations to layered flow and later to studies of deep water exchange (GAARDE, 1916; SAELEN, 1950) and other processes including the properties of internal tides (FJELSTAD, 1933, 1964). Fjord research remained almost entirely a Scandinavian domain until TULLY's (1949) study of Alberni Inlet and the theoretical contributions of STOMMEL and FARMER (1952a,b). Studies of fjords of the North East Pacific developed with the synoptic surveys by PICKARD (1961, 1963, 1967, 1973, 1975) and PICKARD and STANTON (1980) and his students, and theoretical work by CAMERON (1951), RATTRAY (1967) and others. Tully's study was indicative of a new motivation for estuarine oceanography provided by concern over the consequences of human activity. The same concern has led to numerous other fjord studies both in Scandinavia and North America, including the very comprehensive survey of the Oslofjord by GADE (1970)\(^1\). Fjord research is now recognized as an active branch of coastal oceanography, of practical importance to all nations having deeply indented coastlines and providing a focus for studies by many scientists with a common interest in the conditions provided by this special class of estuary.

This recognition led to a NATO Advanced Studies Workshop held in Victoria, British Columbia in 1980 (see FREELAND, FARMER and LEVINGS, 1980; and FARMER and HUPPERT, 1980), from which the present review has to some extent evolved. There have been a number of recent advances in the physical oceanography of fjords and it seemed useful to present a general review at this time. Some aspects of fjord research, for example the problem of deep water renewal and barotropic forcing, were comprehensively reviewed during the NATO workshop, and we shall refer to such articles without much additional discussion, as appropriate.

Excellent descriptive accounts of the water characteristics of both North and South American and New Zealand fjords are given by PICKARD and STANTON (1980), who also include many earlier references. Many of the Norwegian fjords have been similarly described by SAELEN (1950, 1967). The purpose of this review will be to place some of the recent theoretical and observational studies in perspective and to indicate some of the problems and opportunities in the study of fjords that make it such a challenging and intriguing aspect of oceanographic research. While no attempt will be made to reference inclusively the many descriptive accounts of fjords, when discussing specific processes we will use examples drawn from our own observations and in particular from Knight Inlet, B.C. This choice does not reflect any particular bias and the processes occurring in Knight Inlet are certainly not representative of all fjords, but they do provide ready examples illustrating the mechanisms under discussion. Knight Inlet is a useful example from another point of view in that it has become a testing ground for several different theories of fjord circulation; such comparisons are more easily made using data from a single fjord.

To assist the reader detailed maps of Knight Inlet are shown in Fig. 1 which identify the principal features of the inlet referred to elsewhere in the text.

\(^1\)Gade (1970), in addition to providing a review of many aspects of fjord oceanography, also includes an historical account of fjord research, from which some of the earlier references were taken.
FIG. 1(a). Knight Inlet and its immediate vicinity. The principal landmarks cited in the text are located on this diagram. Two cross-sections of the inlet, including the nearby orography, along the lines marked A-B and C-D are shown without vertical exaggeration. A depth profile along the talweg is shown in Fig. 16.
2. GENERAL DESCRIPTION AND OCEANOGRAPHIC CLASSIFICATION OF FJORDS

The term fjord, from the old Norse fjórh, has been rather loosely applied to geological structures developed by glacial erosion and partly filled with seawater. Its original Norwegian usage also included freshwater lakes and more recently the term 'fjord-lake' has been used to describe lakes in glacially carved valleys, but we shall here be strictly concerned only with semi-enclosed coastal inlets. The same coastal structures have been alternatively called sounds, inlets or arms. Given this rather general nomenclature it is not surprising to find a broad variety of topographical features covered by the same heading, including complex, interconnected channels. Nevertheless, we can identify several features characteristic of most fjords: they are usually long relative to their width; they are steep sided and deep (often deeper than the adjacent continental shelf); they typically possess one or more submarine sills which define the deep basin(s) of the fjord and which may be remnant moraines and there is usually a river discharging into the head (Fig. 2). The term head is used to describe the inland termination of the fjord; the mouth is its seaward opening.

Since fjords are associated with glacial carving, they occur at higher latitudes where there are mountainous coasts. The principal areas are the western coasts of North and South America (above about 45° latitude), the Kerguelen Islands and parts of Kamchatka, the western coasts of Europe and Britain (north of 56°N in Scotland) the coasts of Spitsbergen, Iceland and Greenland and the islands of the Canadian Arctic Archipelago, the coasts of Labrador and Newfoundland, the southwest coast of South Island, New Zealand, the open coasts of Antarctica and of South Georgia and other high latitude islands.

There have been several attempts at classification of the oceanographic conditions encountered in fjords. PICKARD (e.g. PICKARD, 1961; PICKARD and STANTON, 1980) has used a purely descriptive scheme which seeks to place different vertical profiles of salinity and other properties in recognizable categories [Fig. 4(a)]. Though not based on any physical concepts, the scheme has the special merit of reflecting observed profile shapes. Typical profiles in a fjord can be expected to change from one type to another, both with respect to position in the fjord and time of year (Fig. 3).
Fig. 2. Schematic diagram of a fjord, showing sills and deep, intermediate and surface water layers. In reality the oceanography of a fjord is constantly changing, the transient effects often dominating the mean so that a simple representation of this type can be misleading. Moreover the distinction between different layers is seldom as clear-cut as implied in the figure. The slope of the free surface, typically of order 1 cm per 10 km, provides the hydraulic head that drives the brackish layer seaward. Other factors influencing the circulation include wind effects, tides and changes in the outer boundary conditions caused by fluctuations in the coastal density structure.

One of the more frequently quoted classification schemes is that of HANSEN and RATTRAY (1966). It is derived under certain mathematical constraints on the hypothesized salinity and velocity profiles, and on assumptions about vertical and horizontal diffusion, so as to fit their similarity solutions. The scheme is based on two parameters, representing the circulation and the stratification. Different estuarine regimes occur for different values of the parameters. The classification is shown in Fig. 4(b), where the ordinate denotes the ratio of the tidally averaged salinity difference $\Delta S$, from bottom to surface, to the depth-and-tidally averaged salinity $S_0$ at a given location; the abscissa denotes the ratio of tidally averaged net circulation velocity at the surface $U_s$, to the cross-sectionally averaged net river run-off flow velocity $U_f$. The surface velocity $U_s$ is difficult to measure accurately, especially in the presence of surface waves and wind effects; nevertheless, the scheme does serve to distinguish the deep fjord estuaries with relatively thin and thus relatively highly sheared near surface circulation (Type 3) from the shallow, partially mixed estuary (Type 2) and the arrested salt-wedge (Type 4). Of course, it is possible for an estuary to move from one classification to another depending upon seasonal changes in river discharge, wind mixing and tidal conditions. For fjords the change is most likely to be between Type 3a and 3b as the stratification is modified by run-off changes. A criticism of this scheme is that it does not incorporate information on the scales or intensities of turbulent processes or of the sources of energy for mixing which ultimately determine the nature of circulation and stratification. In discussing the scheme, RYDBERG (1981) suggests an alternative approach based on salt and mass transports expressed as functions of salinity. While this scheme appears to have interesting possibilities it has yet to be applied to fjord flows.

A quite different scheme based on the assumption of steady hydraulic control at a constriction was proposed by STIGEBRANDT (1981). As discussed in Section 3.2, the concept of hydraulic control in an estuary, first presented by STOMMEL and FARMER (1952a), applies specifically to simple layered flow. Under steady conditions it can be shown that there is an
The physical oceanography of fjords

FIG. 3. Representative Temperature, Salinity and Buoyancy profiles for different seasons taken near the head (station Knight-8) and near the mouth (station Knight-2) of Knight Inlet. The profiles were obtained in spring (Sp, 24/5/78), summer (Su, 25/7/78) and winter (W, 19/1/78) and demonstrate the changes that occur in stratification with different river discharge and meteorological conditions. In this inlet high discharge occurs in summer, leading to a sharp pycnocline near the head. In winter the stratification is almost nonexistent. Note change of scale at 50 m depth.

The physical oceanography of fjords

The upper limit to the two-way transport capacity of a constriction. At the constriction the control condition can be expressed as

$$\bar{F}_1^2 + \frac{\rho_1}{\rho_2} \bar{F}_2^2 = 1 \tag{2.1}$$

where $\bar{F}_1 = \bar{U}_1 (g' h_1)^{-1/2}$ and $\bar{F}_2 = \bar{U}_2 (g' h_2)^{-1/2}$ are similar to densimetric Froude numbers for the upper and lower layers respectively. $\bar{U}_1, \bar{U}_2$ are the sectionally averaged flow speeds and $h_1, h_2$ are the thicknesses for the upper and lower layers respectively. The parameter $g' = g(\rho_2 - \rho_1)/\rho_2$ is the reduced gravity where $\rho_1, \rho_2$ are the corresponding densities of each layer. The maximum two-way transport is achieved approximately when $h_1 = h_2$, for which the estuary is said to be overmixed. When the constriction is deep relative to the upper layer thickness, (2.1) reduces to

$$\bar{F}_1^2 \approx 1 \tag{2.2}$$

Stigebrandt refers to such a fjord as ‘normal’. In equation (2.2) $\bar{F}_1$ approximates the densimetric Froude number for the flow, since $(g' h_1)^{1/2}$ is the interfacial wave speed when the lower layer is very deep. However, in view of the range of possible internal Froude numbers in the continuously stratified example we leave their formal definition to Section 5. Given the con-
dition (2.1), together with the requirement of salt and mass conservation and an equation of state, it is possible under steady conditions to determine the relative depth of the interface at the constriction for given mixing rates and fresh water discharge.

Figure 4(c) shows the relative interface height

\[ \eta = \frac{h_2}{h_2 + h_1} \]

and salinity difference

\[ \Pi = \frac{S_2}{S_2 - S_1} \]

at the constriction or fjord mouth due to mixing for different values of an Estuarine Froude Number defined as

\[ F_e = \frac{Q_f g^{1/2}}{B_m H_m^{3/2}} \]

where \( Q_f \) is the fresh water discharge, \( B_m \) the channel breadth and \( H_m = h_1 + h_2 \) the total depth at the mouth. For given fresh water discharge, increased mixing by whatever mechanism
The physical oceanography of fjords

Fig. 4(b). Estuarine classification scheme due to Hansen and Rattray (1966). Relative, tidally averaged salinity difference from bottom to surface is plotted against the ratio of tidally averaged net circulation velocity at the surface to the sectionally averaged net river run-off flow velocity. The location of a point on the plot depends on both location in the estuary and time of year. The entries KI₁ and KI₂ are August and February positions (respectively) on the plot for a station about 30 km from the head of Knight Inlet. These represent approximate extreme positions for Knight Inlet, a station would generally lie somewhere between these two points. Other points are plotted for: Mississippi River mouth (M); Columbia River estuary (C); James River estuary (J); Juan de Fuca Strait (JF); narrows of the Mersey estuary (NM) and Silver Bay (S). Subscripts h and l refer to high and low river discharge. Fjords fall into region 3, partially mixed estuaries into 2, salt wedges into 4 and well mixed estuaries into region 1.

increases the interface depth. The maximum mixing condition (minimum vertical salinity difference at the mouth) occurs when the two layers are of equal thickness, as shown by Stommel and Farmer (1952a), corresponding to the overmixed or 0-Type fjords. It is important to keep in mind that this classification scheme is based on a steady, two-layer approximation to the flow and on the assumption of hydraulic control either at the mouth or at a constriction. No assumptions are made about the detailed dynamics within the fjord nor is

Fig. 4(c). Fjord classification scheme due to Stigebrandt (1981). Relative interface location \( \eta \) is plotted against relative salinity difference \( \Pi \) for different values of the estuarine Froude Number (see text). For N-Fjords the interface is shallow with respect to the total depth at the mouth. The scheme is based on the assumption of steady internal hydraulic control of a two layer flow at the estuary mouth.
the classification applicable downstream of the region in which the control occurs. Stigebrandt makes a further distinction between wide fjords and narrow fjords. A wide fjord is one in which transverse density gradients due to the estuarine circulation are negligible: in a narrow fjord the dynamics are controlled by hydraulic conditions within the fjord and significant property gradients occur in the surface layer. Of course, a fjord may also be wide or narrow with respect to rotation effects; horizontal flows will generate significant transverse gradients if the fjord is wider than the internal Rossby deformation radius \( R = \sqrt{gh/f} \), where \( f \) is the local value of the Coriolis parameter.

Stigebrandt's hydraulic scheme [Fig. 4(c)] is particularly simple and also satisfying in that the physics is clearly stated. It must be kept in mind however that it will break down or be modified if (i) conditions either at the mouth or in the interior are strongly time dependent or (ii) a two-layer flow condition does not occur. Fjords subject to rapidly changing meteorological forcing or to tides cannot be presumed to satisfy (i) or (ii) in a simple time averaged sense. Thus, while the scheme is undoubtedly of interest and points strongly at the role of the outer boundary conditions, in any practical application we must look closely at the kind of processes and scales of variability actually occurring in nature before deciding on the validity of a particular classification. This point is taken up again later in Section 6.1.

3. THEORETICAL APPROACHES TO ESTUARINE FLOW

The development of our perception of fjord circulation has closely followed the development of observational techniques; these in turn have stimulated new theoretical approaches which have served, with variable success, as a framework for the interpretation of new data. Extensive measurements in Alberni Inlet led TULLY (1949) to make some interesting observations on the relationship between surface salinity, river discharge and tidal action. He found an almost linear relationship between tidal amplitude and the dilution of river water with salt water. Both the brackish layer depth and salinity decreased with increasing river discharge to a certain minimum value and then increased with increasing river discharge. Comparisons between theoretical models and data have superficially seemed quite successful, despite the fact that the modelling approaches have differed substantially. This may be a consequence of the fact that the most readily measurable quantity, the density profile, does not provide a very sensitive test of a model's representation of the underlying physics.

Since fjord circulation represents the consequences of mixing fresh and salt water within a partly confined space, an important distinction between different modelling approaches lies in the representation of turbulence processes; a second key distinction lies in the choice of an outer boundary condition. Two general types of model have evolved: (i) models that represent mixing of mass and momentum by smoothly varying eddy coefficients, which we refer to as K-models, and (ii) models in which mixing is assumed to be rapid and complete over a surface brackish layer which entrains salt water from below, which we will call layered models.

The K-models have the ability to represent processes leading to continuous velocity and density distributions, which are indeed observed in nature, but they require the somewhat arbitrary specification of eddy functions. Since these can be adjusted at will so that the model solutions fit the observations, it appears that further insight into the underlying physics must be derived from an interpretation of the required eddy functions themselves, a procedure which is probably premature in the light of available knowledge of turbulence in fjords. The layered models are mathematically much simpler, readily allow the inclusion of suitable outer boundary
conditions and have proved useful in analyzing fjords with Type 1a, and to a lesser extent 1b, salinity stratification [Fig. 4(a)]. Their limitation lies partly in the representation of a continuous density and velocity stratification that may extend over a few tens of metres, by a single sharp pycnocline and partly, as with the K-models, in the assumption of steady state and with a grossly simplified parameterization of the physics associated with mixing and entrainment. It is important to emphasize that these modelling approaches concentrate on the isolated problem of movement out of a fjord and mixing of freshwater discharged from a river, with consequent replacement of salt water from outside. They do not address the exchange of deep water discussed in Section 4.2 or the exchange of intermediate water discussed in Section 4.3.

In some fjords, especially during low run-off, the estuarine flow is weak and the circulation is dominated by deep and intermediate exchanges. We shall briefly sketch these different modelling efforts before examining in detail some of the processes observed in fjords.

A useful starting point in considering different models is obtained from the perturbation equations derived for estuarine flow. By representing the flow as a sum of three parts, i.e. \( u = \bar{u} + u_T + u' \), \( w = \bar{w} + w_T + w' \), etc., corresponding to mean flow, tidal components and turbulent fluctuations, substituting into the Navier Stokes equations, making use of the incompressibility condition and taking the time mean values, PRITCHARD (1956, 1971), SVENSSON (1980) and others derived equations applicable to various types of estuarine flow. It is assumed that the turbulent fluctuations are unrelated to the tidal flow, so that cross-terms of the form \( u_T w' \), \( u_T u' \) are neglected. The time average implicit in these equations is a tidal average. However the mixing processes associated with tidal effects are nonlinear, so that many interaction terms evolve from different components of tidal forcing. Specifically, as we shall discuss in Section 4.1.2, mixing due to effects of the M2 and S2 tides results in a measurable signal at the MSf frequency (period = 14.7 days) and thus serves as a sensitive probe of turbulence and mixing in a fjord. Likewise wind mixing effects are subject to similar averaging inconsistencies. Moreover, since the spectrum of wind energy is typically broad with significant low frequency components, application of steady state assumptions to real data requires special care.

A further simplification usually applied to fjord models is the neglect of the Coriolis terms, leading to the following equation, in curvilinear co-ordinates, for flow along the channel:

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{w} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} + u_T \cdot \nabla u_T &= - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\partial}{\partial x} (u'u') - \frac{\partial}{\partial y} (u'u') - \frac{\partial}{\partial z} (u'u') - X. \\
\end{align*}
\]

In all models steady solutions are sought and so term (i) is set equal to zero. Various combinations of terms (ii), (iii) and (iv) are retained or dropped by different authors. Term (v) may be important close to a sill or constriction, but is usually neglected, and will be neglected in the subsequent development, as are the body forces (x) associated with channel curvature. The body forces are of course absorbed into the inertial terms if the equation is written in Cartesian co-ordinates.

A similar development for the salinity equation yields:

\[
\begin{align*}
\bar{u} \frac{\partial \bar{s}}{\partial x} + \bar{w} \frac{\partial \bar{s}}{\partial z} &= - \frac{\partial}{\partial x} (s'u') - \frac{\partial}{\partial y} (s'v') - \frac{\partial}{\partial z} (s'w') \\
\end{align*}
\]

and likewise for continuity

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial y} = 0. 
\]
All of the dynamical theories impose some constraints on the transverse terms or on the transverse gradients. A key assumption incorporated in all models of fjords is that in contrast to shallow estuaries, the horizontal exchange is strictly advective, with the term \((u'u')\) assumed negligible. McAlister, Rattray and Barnes (1959) attempt to justify this assumption on the basis of estimated horizontal salt fluxes across a section of Silver Bay, Alaska, derived from current profiles taken from an anchored ship. However, as will be shown with specific examples in Section 5.2 below, the problem may be much more complicated than this, with horizontal intrusions of locally mixed fluid moving along the fjord axis intermittently.

Under the assumption of zero horizontal eddy exchange together with the simplifications indicated above, Rattray (1967) and Winter (1973) use the equations:

\[
\begin{align*}
\frac{\partial}{\partial x} (Bu) + \frac{\partial}{\partial z} (B\bar{w}) &= 0, \\
\frac{\partial}{\partial x} (Bu') + \frac{\partial}{\partial z} (B\bar{w}') &= -\frac{\partial}{\partial z} (\bar{w}'z'),
\end{align*}
\]

(3.4)

together with the constraint that the integral mass flux through a section is equated to the cumulative run-off.

In the development of the layered, hydraulically controlled models of Stommel and Farmer (1952b), Long (1975b) and others, we may write the momentum equation, directly from (3.1) with similar simplifications i.e. neglect of terms (i), (v), (vii) and (x), as

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nabla \cdot \mathbf{\tau},
\end{align*}
\]

(3.5)

where \(\mathbf{\tau} = (\tau_x, \tau_y, \tau_z) = -(0, 0, \bar{u}'w')\), where the horizontal eddy flux terms \(u'u', u'v'\) are set equal to zero as in (3.4). In the above equation we have dropped the overbar symbol introduced earlier to represent the time averaged mean field quantities and consider the implications of vertical and transverse averaging. We define an averaging operator by the integral

\[
\bar{u}^2 = \frac{1}{Bh} \int_0^B \int_0^h u^2 \, dz \, dy
\]

(3.6)
in which the integrals are taken over breadth \(B\) and thickness \(h\) of an hypothesized upper layer. Through this manipulation we do not explicitly assume that transverse or vertical gradients vanish, but the dependence on the transverse and vertical co-ordinates is removed. Applying the operator (3.6) to equation (3.5) together with the Boussinesq approximation yields:

\[
\frac{1}{B} \frac{d}{dx} (u^2 Bh) = \frac{d}{dx} \left( \frac{1}{2} g h^2 \Delta \rho \right) - \tau_b + \bar{u} W_e,
\]

(3.7)

where \(\tau_b\) is the stress at the base of the upper layer and \(W_e\) is the vertical velocity of water entrained at that depth. The horizontal averaging implicit in equation (3.7) is only useful if one has information on the horizontal structure of the depth mean flow. This information is notably lacking; however Long argued that \(u^2 = \xi \bar{u}^2\) with \(\xi\) expected to lie between 1 and 2.

The continuity equation may be similarly integrated over the upper layer yielding

\[
\frac{d}{dx} \int_0^B \int_0^h u \, dz \, dy = -\int_0^B \int_0^h \frac{dw}{dz} \, dz \, dy
\]
The physical oceanography of fjords

159

FIG. 5. Observed salinity as a function of depth (●) for four stations in Knight Inlet, B.C., compared with the similarity solution to a theoretical model (Adapted from WINTER, 1973).

or,

$$\frac{d}{dx} (Bhu) = BW_e. \quad (3.8)$$

3.1. K-models

Formally it is possible to define eddy coefficients $A_v, K_v$ for momentum and salt diffusivity that are related to the appropriate stress and salt flux terms in the governing equations:

$$-\bar{u}'w' = A_v \frac{\partial u}{\partial z} \quad -s'w' = K_v \frac{\partial s}{\partial z}. \quad (3.9)$$

Following an early attempt by CAMERON (1951) incorporating constant coefficients, RATTRAY (1967) transformed the conservation equations for momentum, mass and volume into a set of two coupled, non-linear ordinary differential equations and a definite integral. Using a technique previously applied to shallow estuaries Rattray found similarity solutions for exponentially decaying functions for $K_v(z)$, subject to the constraint that the fresh water run-off is a small fraction of the total circulation. This assumption is equivalent to requiring that the gravitational circulation be dynamically more important than run-off in determining the near surface streamline configuration, which would not be valid near the head of the fjord but might apply to some central section. WINTER'S (1973) extension of this approach incorporated coefficients based on field observations reported by GADE (1968) and TRITES (1955), but retained an exponential depth dependence. The relative salinity defect and stream functions are assumed similarly expressible as powers of exponentials.

Winter compared this similarity solution to density data observed in Knight Inlet; the comparison is shown in Fig. 5. The observed density profiles certainly bear some similarity to the modelled results. However, it must be kept in mind that the theoretical development is based on a tidal average while the density profiles were observed at discrete times. More recent observations in Knight Inlet, discussed later, have demonstrated the short term variability due to tides which may make single observations unrepresentative, especially near the sill. A more fundamental criticism of this approach however is that eddy coefficients, which represent the essential physics of mixing and momentum transfer, are specified as independent functions. Yet the turbulence processes they represent must depend upon the basic flow field and density field they are supposed to model. Moreover the similarity assumption constraining channel breadth and run-off can only be valid over a limited stretch of the fjord; it certainly breaks
down at the head and at the mouth. If the flow in some central segment of the fjord is subcritical, it follows that information in the form of baroclinic adjustments must be free to propagate back up the channel from the mouth and thus control the flow throughout the subcritical portion. The existence of critical conditions near the mouth, at which these models must break down, was recognized by CAMERON (1951) and field observations of Tully indicated a "breakdown of estuarine structures near the mouth of certain fjords" (PRITCHARD, 1952). However, it seems highly unlikely that the critical conditions toward which the similarity solutions inevitably converge at some point in the inlet, would correspond to actual boundary conditions in nature. Thus the *apriori* specification of appropriate boundary conditions would appear to be an essential component of fjord circulation models.

The restrictions imposed by similarity constraints can partly be removed using alternative, numerical techniques. PEARSON and WINTER (1975) have shown how the method of weighted residuals can be used to solve the same system of equations used in the similarity theory, but with more general specification of the eddy coefficients and topography. Figure 6 shows calculated and observed profiles for the Knight Inlet stations used in the similarity comparison.

3.2. Layered models

The existence of a thin, well defined brackish surface layer in many fjords during periods of high run-off has motivated the development of layered models in which the surface and deeper layers are assumed to be vertically homogeneous, being separated by a sharp pycnocline. Since the lower layer is deep and has a salinity that changes little, its velocity and its horizontal density gradient are usually considered negligible. The problem then reduces to the dynamics of a single layer, subject to entrainment of salty water through its lower interface together with appropriate boundary conditions.

STOMMEL (1951) and STOMMEL and FARMER (1952a, b, 1953) appear to have been the first to develop a dynamical model of two layered stratified flow applicable to fjord circu-
lation. An important contribution was the concept of internal hydraulic control that occurs at an abrupt change in cross-section of a channel. They showed that the two way flow was subject to a maximum transport condition represented by equation (2.1), which imposes an upper limit on the salinity of the surface layer in the estuary. The salinity of the surface layer in this 'overmixed' condition depends on the shape of constriction, the fresh water discharge and the salinity of inflowing sea-water, but for steady conditions there is a maximum surface layer salinity that is independent of mixing intensity. This 'overmixed' condition may not be too common in deep fjords, but the assumption of a control of the type (2.2) is fundamental to the analysis of STOMMEL (1951), STOMMEL and FARMER (1952a, b) and others.

A second assumption commonly used in this approach is that there is no horizontal pressure gradient in the lower layer. This will be true if the lower layer velocity is zero, and the depth mean will tend to zero for a very deep layer. However, as pointed out by LONG (1980), this does not place restrictions on the distribution of velocity within the lower layer. In their derivation, Stommel and Farmer neglect the shear stress $\tau_b$ in (8) and also the entrainment term $\bar{u}W_0$, although they discuss the latter in their analysis. Thus a balance is assumed between the inertial terms and the pressure gradient. An interesting consequence is the relationship between surface layer thickness $h_1$ and surface layer transport per unit breadth $q_1$:

$$\frac{dh_1}{dq_1} = \frac{2h_1(F_1^2 - \frac{1}{4})}{q_1(F_1^2 - 1)}, \tag{3.10}$$

where $F_1 = \bar{u}_1/\sqrt{g'\bar{h}_1}$ is the densimetric Froude number for the surface layer. Thus the surface layer thickness is predicted to increase along the channel to a maximum at $\bar{u}_1 = \frac{1}{4}(g'h_1)^{1/2}$, for which $F_1 = \frac{1}{2}$, and thereafter decrease.

Further progress depends upon the specification of an entrainment function. STOMMEL and FARMER (1952b) used KEULEGAN's (1949) laboratory result, that above a certain critical value, entrainment is proportional to shear. If, as is often observed over some section of a fjord, the surface layer thickness changes little, then this particular entrainment assumption implies a relative salinity difference $(S_2 - S_1)/S_2$ that increases exponentially, which Stommel found to be in reasonable agreement with Tully's data. The outer control condition (2) at the mouth ($x = x_m$) can also be applied to obtain the salinity and layer thickness along the inlet as well as the following relationship between river discharge per unit breadth $q_t$ and relative salinity difference at the mouth:

$$q_t^2 = g\beta S_2 C x_m [(S_2 - S_1)/S_1]^3 \tag{3.11}$$

where $C$ is the entrainment constant and $\beta$ is a constant of proportionality relating salinity to density. Equation (3.11) predicts a decreasing salinity with increasing discharge, consistent with Tully's findings in Alberni Inlet for smaller discharges. This early work was therefore generally supported by observations in Alberni Inlet.

More complicated, though not necessarily more physically realistic, representations of the exchange processes and of the shear stress between layers have been tried. For example, STOMMEL and FARMER (1952b) allowed for a two-way exchange of fluid across the interface in addition to entrainment. PEARSON and WINTER (1975) took a quite different approach by developing a two-layer 'diagnostic' model, for use as a numerical tool to analyze salinity and run-off data. As with any application of layered models to real data, a choice must be made as to the mean surface layer salinity, but given this choice the scheme allows calculation of various flow properties including the entrainment rate and sea surface slope as a function
of position. The scheme also allows frictional effects to be included as well as variable channel topography. Figure 7 shows an example of this calculation applied to data from Knight Inlet.

LONG (1975b) extended Stommel's original model to include a frictional term \( \tau_b \) in (8), which he assumed quadratic \( \tau_b = k\bar{u}^2 \), a channel of variable width and an entrainment condition specified in terms of turbulence intensity using a relationship derived in the laboratory. He did however neglect the entrainment term \( \bar{u} \bar{W}_e \) in (8) in his initial derivation (see FREELAND and FARMER, 1980; and LONG, 1980). It is still not clear whether these modifications have resulted in a significantly better model of fjord circulation. LONG (1975b) considered a drag coefficient of about 1 between the layers to be appropriate so that the momentum balance was between the pressure gradient and the horizontal stress with the inertial terms playing a minor role, but this value seems far too high to allow agreement with observations. GADE and SVENDSEN (1978) estimated the drag in the Sognefjord at around \( 10^{-3} \) and FREELAND and FARMER (1980) found that in Knight Inlet, the model fitted the data best in the inviscid case (Fig. 8). Thus failure to include frictional effects does not appear to be a serious defect of Stommel's model, at least for these examples. In a summary of several

![Figure 7. Observed (●) and modelled (---) salinities for two months in Knight Inlet, B.C., using a two layer 'diagnostic' model (adapted from PEARSON and WINTER, 1975). Given a surface layer salinity, fresh water discharge and geometrical properties, dynamical and continuity constraints are used to define the resulting flow field.](image)

![Figure 8. Relative surface layer depth observed in Knight Inlet, plotted against relative density difference (●), compared with solutions for a two-layer model due to LONG (1975). The curves are derived for different values of the friction coefficient. The closest fit is for the inviscid solution (outermost curve). Figure adapted from FREELAND and FARMER, 1980.](image)
fjord models, PEDERSEN (1978) has emphasized that the interfacial shear stress must be balanced by pressure gradients in the deeper counterflow; but this lack of balance is not significant if friction is very small. This feature was included in a model by OTTESEN-HANSEN (1975). It is harder to assess the parameterization of the entrainment process. LONG's (1975b) model requires the specification of a r.m.s. turbulence velocity, but as yet this must be a rather uncertain estimate based on wind or tidal energy levels.

A key element of Stommel's original two-layer model as well as the more recent adaptations of it, is the critical condition (2.2) assumed to occur at the estuary mouth. It is this condition that provides upstream control of the surface layer properties. There is a difficulty in testing this condition in a fjord, since typically the outer reaches do not follow the simple two-layer structure applicable near the head. Moreover it is not always possible to identify the sudden broadening associated with the transition. FREELAND and FARMER's (1980) observations in Knight Inlet showed that in the outer reaches of the inlet, 100 km from the head, the surface layer Froude Number inferred from observed salinity distribution and continuity considerations for a two-layer model, was still less than 0.3. Nevertheless there are undoubtedly cases, especially for deep silled fjords not subject to strong tidal forcing, where a steady two-layer representation is satisfactory and equation (2.2) might be expected to apply.

A limitation of all the vertically integrated models described thus far is that they do not address the problem of recirculation within the surface layer. STIGEBRANDT (1981) has presented an innovative approach that does tackle this problem while yet retaining the simplicity of a simple layered model. He argues that within the brackish surface layer of a narrow fjord there is a vertical density gradient due to overturning caused by the longitudinal density gradient. He further assumes that the pycnocline can be considered dynamically passive 'like a rigid porous bottom'. The circulation is then controlled by a sequence of internal hydraulic transitions at one or more constrictions along the fjord. The control condition $F_e = 1$, [equation (2.3) within the brackish layer, with $H$ now taken as the layer thickness], defines the flow at each constriction. The interface between fresher and saltier water in the upper layer, slopes upward toward the fjord mouth. Thus the vertical density difference $\Delta \rho / \rho_0$ within the brackish layer is also the horizontal density difference in the brackish layer along the fjord axis in the neighbourhood of the control section. Upstream of each control section mixing proceeds by some unspecified mechanism, but the permissible horizontal gradient, for given conditions, is defined by the requirement $F_e \approx 1$.

The thickness of the surface layer in each sub-basin between control sections is assumed proportional to the thickness of the basin nearest the mouth; the latter thickness is determined from the control condition (2.3) and the ratio $\phi$ of layer thickness of the adjacent basin to that in the mouth. $\phi$ is thought to lie between $3/2$ and $\sqrt{3}$ depending upon the importance of friction. The horizontal salinity difference $\Delta S_h$ between two sub-basins is then shown to be

$$\Delta S_h \propto \Delta S_v \phi^{-3} \left( \frac{\Pi_s}{\Pi} \right)^2 \left( \frac{B_m}{B_s} \right)^2$$

(3.12)

where $\Delta S_v = S_2 - S_1$ is the vertical salinity difference between brackish $S_1$ and salt $S_2$ water at the mouth and $\Pi_s, \Pi$ are the 'mixing factors' for sub-basin and outermost basin respectively defined in terms of vertical salinity differences: $\Pi = S_2/(S_2 - S_1)$. $B_m, B_s$ are the respective breadths at the mouth and in the sub-basin. It is further assumed that $\Pi_s$ and $\Pi$ are proportional; summing the horizontal density difference over all possible constrictions, Stigebrandt thus finds
FIG. 9. Horizontal salinity differences along the Nordfjord, plotted against vertical salinity differences between layers at the mouth, based on monthly values observed over several years. Hysteresis in the resulting curve is attributed to the transient response of the fjord to fluctuations in freshwater discharge. (Adapted from STIGEBRANDT, 1981)

\[
\Delta S_h = \Phi (\Delta S_v - S_2/\Pi)
\]

where \(\Phi\) has a unique value for each fjord.

Thus the horizontal gradient along the inlet is predicted to be proportional to the vertical salinity difference at the mouth. The results should be easy to test against available data and Stigebrandt shows remarkable agreement with monthly values of \(\Delta S_h\) and \(\Delta S_v\) from the Nordfjord (Fig. 9). Small discrepancies are interpreted in terms of the delay in response of the circulation to changes in river discharge.

Since significant vertical gradients occur in the surface layer, some care is required in the definition of ‘horizontal’ and ‘vertical’ salinity differences. A self-consistent and objective scheme for representing continuous density profiles by a two-layer approximation can be based on the joint conservation of (i) potential energy, (ii) phase speed of a mode 1 long internal wave and (iii) surface layer mass, between the real and the model profiles. Density profiles from Knight Inlet obtained between 1977 and 1981 were transformed to a two-layer representation under these constraints, in order to derive values of \(\Delta S_v\) and \(\Delta S_h\) for comparison with Stigebrandt’s model. The Nordfjord data were averaged over many years, but in Fig. 10 we have plotted separate values for Knight Inlet based on individual cruises. There is considerable scatter in the data. For example although four August values are fairly closely spaced, a fifth occurs for which \(\Delta S_h\) is very small. This point is based on data coinciding with a period in which river discharge increased rapidly. There are also a few points in which a small vertical, but large horizontal gradient exist. However a majority of the points do fit into a broad band implying that for at least some of the time \(\Delta S_h\) does increase with \(\Delta S_v\), roughly as predicted by Stigebrandt’s model. The detailed physical processes contributing to this relationship are almost certainly not the same as those dominant in the Nordfjord and the strong time dependence of river discharge and tidal forcing does not justify simple averaging in order to arrive at a steady state representation, but in one respect Stigebrandt’s model appears peculiarly appropriate to Knight Inlet and to other inlets with similar circulation. As we shall describe later, a fjord subject to strong mixing near the sill must experience spreading and recirculation of the mixed water just beneath the fresher surface layer, in a way not dissimilar to the ‘overturning’ process described by Stigebrandt.

In this sense Stigebrandt’s model may represent some components of the observed physical processes in Knight Inlet. A steady state hydraulic transition (i.e. \(F_e = 1\)) cannot occur for such a tidally pulsed source of mixing, but evidently the recirculation occurs in such a way that \(\Delta S_h\) and \(\Delta S_v\) are positively correlated for part of the year.
Before concluding this section we compare the models of STOMMEL and FARMER (1952a, b, 1953), and two models for wide fjords, free of horizontal gradients developed by LONG (1975a) and STIGEBRANDT (1981). An elementary consequence of mass conservation and hydraulic control at the mouth (2.2) is the surface layer thickness relation:

$$h = q_{t}^{2/3} (\beta \delta) \frac{S}{\Delta S_{h}}. \quad (3.14)$$

Stommel and Farmer’s use of Keulegan’s entrainment assumption leads to a decreasing salinity and increasing thickness with increasing discharge. The overmixing condition (2.1) provides a limit to this relationship, but for simplicity consider only the case of a deep mouth (i.e. N-fjords in Stigebrandt’s classification). Thus for large fresh water discharge the surface layer is almost fresh and the layer thickness varies as $q_{t}^{2/3}$. LONG (1975a) replaced Keulegan’s entrainment assumption with a wind-mixing model. For small discharge in the ‘wide’ fjord case he found

$$h = \frac{AK\sigma^{3}}{b_{0}Q_{t}}. \quad (3.15)$$

where $A$ is the area subject to wind mixing, $b_{0}$ is the buoyancy of seawater, $K$ is a parameter determined by the mixing efficiency and $\sigma$ is the r.m.s. turbulence velocity, presumably related to the surface friction velocity $u_{*}$. Thus the layer depth for low discharges varies inversely with fresh water discharge. In the absence of wind and for sufficiently large discharges, the surface

FIG. 10. Plot of horizontal salinity difference between head and mouth, of the brackish layer in Knight Inlet, B.C. against the vertical salinity difference between brackish and salty layers at the mouth. The numbered data points are identified by month, but are derived from cruises between 1977 and 1981. The definition of brackish layer salinity is based on a two layer approximation to the original profile subject to dynamical and mass conservation constraints, as described in the text. This plot may be compared with the similar one in Fig. 9 derived from Nordfjord data.
layer remains fresh and Long's (1975a) model converges to Stommel and Farmer's result (3.14).

Stigebrandt (1975, 1981) developed a solution applicable to wide fjords that is essentially the same as Long's (1975a) wide fjord model, but his derivation leads to a clear distinction between wind mixing and hydraulic control effects:

$$h = \frac{\gamma W^3 A}{Q_{1/g} \beta S_2} + \phi \frac{Q_f^{2/3}}{(g \beta S B^2)^{1/3}}. \quad (3.16)$$

The first term on the right-hand side describes wind mixing effects, with $W$ the wind-speed and $\beta$, $\gamma$ constants. This term is essentially identical to (3.15) and corresponds to a Monin-Obukhov length for the basin where the surface buoyancy flux arises from the fresh water discharge. The second term describes the influence of the hydraulic control with $h \propto Q_f^{2/3}$ as defined by Stommel and Farmer, except that the constant $\phi$ is included to allow for the shoaling of the interface at the mouth.

Thus Long and Stigebrandt showed that for given wind conditions the surface layer thickness increases for both low and high discharge. This result is just that found by Tully (1949) in Alberni Inlet and by Welander (1974) in a laboratory study. Alberni Inlet is not a 'wide' fjord in the sense prescribed by Stigebrandt, but evidently the two quite different effects due to mixing and hydraulic control govern the surface layer behaviour at extreme ranges of fresh water discharge.

All of the models described above are highly idealized in that mixing is represented by some continuous process along the fjord and it is presumed that flow conditions are steady. They have been useful representations that permit the testing of various physical concepts. Nevertheless we are struck by the enormous variability in water properties and flow conditions actually encountered in fjords; as we have indicated earlier, simple time averaging breaks down under such conditions and a deeper physical insight can best be gained by more detailed observation of natural flows. In the following sections we focus on specific processes in fjords and suggest ways in which they might contribute to the larger circulation.

4. Exchange Processes

In this section we discuss the means by which water is exchanged between the bulk interior of a fjord and the outside environment. To some extent exchange processes are inextricably related to mixing processes, but it will be more convenient to defer discussion of mixing to the subsequent section. This general problem is of considerable importance for the ecology of fjord systems since the manner and intensity of exchange determines how well ventilated the fjord will be; poorly ventilated fjords can become oxygen deficient very rapidly. Fjord exchange processes have a good analogy on a much larger scale. The Atlantic Ocean experiences mid-water column renewal via the Mediterranean outflow and deep water replacement via the Weddell Sea; this is a well ventilated ocean and has a deep oxygen minimum of about 4 ml/l at 50°N. At the same latitude (Station Papa) in the Pacific Ocean a deep oxygen minimum occurs at about the same depth but with only 0.44 ml/l of dissolved oxygen. The Pacific, of course, has no marginal sea equivalent to the Mediterranean or Weddell Seas. Poorly ventilated fjords show a very low dissolved oxygen concentration and frequently H$_2$S bearing water occurs at depths that have remained stagnant for a long period of time.

There are many mechanisms by which water can be exchanged between a fjord and the outside environment, including barotropic effects, especially tides, which force a cyclic exchange
over the sill but also meteorological forcing, mixing that generates horizontal density gradients across the sill, direct shelf processes that can drive dense water up to sill level and the deep gravitational circulation. It is frequently difficult to classify a particular observation of deep basin ventilation, but for convenience we discuss the subject under three headings: tides, deep water renewal and shelf processes. This separation is determined solely by a difference in the physics which can operate in various cases; actual examples of deep basin ventilation are likely to include all three physical processes.

4.1. Tides in fjords

4.1.1. Barotropic tides. The barotropic tide can be a highly energetic component of fjord circulations. In certain locations it may be possible to extract energy from the tide and make that energy available for mixing. Hence, the surface tide can affect, or possibly even drive, the general gravitational circulation of the fjord. Also, turbulence generated via tidal processes can mix surface waters downwards predisposing a deep basin to undergo a major renewal event. Hence it is useful to understand the origin and nature of the tide in a fjord.

Consider an inlet of arbitrary shape that connects to the ocean via an identifiable mouth. The Laplace Tidal Equations can be written

\[ \frac{\partial u}{\partial t} + 2\Omega \times u + g \nabla (\xi - \xi_e) - F = 0 \]  
and

\[ \frac{\partial \xi}{\partial t} + \nabla \cdot (Hu) = 0 \]

where \( u(x, t) \) and \( \xi(x, t) \) are velocities and elevations as functions of position, \( x \) and time, \( t \). The functions \( H \) and \( f \) are the depth and Coriolis parameter respectively and \( \xi_e \) is the equilibrium tide. \( F \) is a function (in general non-linear) that describes energy losses in the inlet.

The tidal energy balance of a gulf was analyzed by GARRETT (1975) and his development is instructive. Scalar multiplying (4.1) by \( Hu \) and (4.2) by \( g\xi \) and adding, we derive an energy equation which we average over a tidal cycle to obtain:

\[ \frac{\partial \bar{E}}{\partial t} + \nabla \cdot (\rho g H u \bar{\xi}) - \rho g H u \bar{\xi} \cdot \nabla \bar{\xi}_e = \rho E \cdot \bar{u} \]

where \( \bar{E} = \frac{1}{2} \rho H u^2 + \frac{1}{2} \rho g \xi^2 \). If we presume that \( \partial \bar{E}/\partial t = 0 \) and integrate over the surface area of the inlet we easily derive (using Gauss's theorem to convert areal integrals to line integrals around the perimeter of the inlet)
neglected by TAYLOR (1919) which complicates rather than simplifies the balance. The two middle terms can be collected together to yield

$$\int_{\text{Area}} gH u \cdot \nabla \xi \, dA$$

(4.5)

and it is easily seen that for inlets of dimensions much less than the radius of the earth, such as fjords, this term is negligible and the dominant balance is

$$- \int_{\text{Mouth}} gH u \xi \cdot n \, ds = \int_{\text{Area}} F \cdot u \, dA$$

(4.6)

For purely sinusoidal motions elevations and velocities must be in quadrature in the absence of dissipation. Note that we have not specified the form of $F$ or even the specific processes it represents to induce tidal energy loss; it can in general represent internal friction, side walls and bottom drag, or even internal wave drag. Furthermore, it is only necessary that the inlet be subject to linear physics at the section where the left hand side of (4.6) is evaluated. The control volume up-inlet of that section can include highly non-linear processes, around the sill for example, since $F$ is not specified.

If at some arbitrary section elevations are given by $\xi = \xi_0 \sin(\omega t)$ and $u = u_0 \cos(\omega t + \epsilon)$, where $u$ is the along channel flow, then the net inward flux of tidal energy, which is equal to the total tidal energy dissipation up-inlet of the control section, is $\frac{1}{2} \rho g A u_0 \xi_0 \sin(\epsilon)$. For a rectangular section the cross-sectional area $A$ at the control section is $A = H_0 B$ where $H_0$ is the total field fluid depth and $B$ the channel breadth. If $\epsilon$ is small then by continuity $u_0 \approx \frac{\xi_0 \ell \omega}{h_0}$ where $\ell$ is the length of the inlet and hence the net energy dissipation $P$ is

$$P = \frac{1}{2} \rho g \xi_0^2 \omega B \ell \sin(\epsilon).$$

(4.7)

This equation was used by FREELAND and FARMER (1980) who also show that the phase angle $\epsilon$ can be related to changes in the phase of the surface tide along the inlet.

In the particular example of Knight Inlet it was shown by Freeland and Farmer that the dominant process by which the tide loses energy is internal wave drag at the sill; a quadratic drag law for bottom friction requires a drag coefficient that is too large for geophysical conditions. Moreover careful measurements of $\epsilon$ show a close relationship to the seasonally varying stratification, which is interpretable in terms of internal wave drag, but not in terms of bottom friction. This is of significance in Knight Inlet, and probably elsewhere, since sufficient energy is put into intense wave-like disturbances and hydraulic jumps to dominate the mixing processes in the inlet. The measurement of $\epsilon$ can thus serve as a probe for the investigation of the location and seasonal variation of energy extraction from the tide.

The linear barotropic response for fjords is relatively simple. A typical depth for fjords of the British Columbia (B.C.) coast would be about 300 m which implies a wavelength at $M_2$ of around 2500 km. Hence, barotropic resonance to tidal forcing cannot occur. The tide propagates down the inlet strictly as a Kelvin wave. However, since the external Rossby radius (ca. 2000 km) is much larger than the typical fjord width (ca. 5 km) the tide can be treated as a plane wave. The case of slowly varying width and depth was first solved by GREEN (1837). The results are discussed by LAMB (1932, §185) who shows that tidal wave amplitude is proportional to $B^{-1/2} h^{-1/4}$. For rapid changes in depth and width and also to accommodate channel bends and twists, numerical models are required, (cf. JAMART and WINTER, 1978, 1980; PEARSON and WINTER, 1977). A limitation of barotropic models is that no account is taken of the important internal response which may be critical over the sill, except by fitting
appropriate frictional factors that simulate the total drag. However, we suspect that detailed 2-dimensional models may prove useful for studying the internal response. For example JAMART and WINTER (1980) demonstrate considerable complexity in the barotropic flow over the Knight Inlet sill, a 3 km section of which they simulate with over 1000 elements in their finite element modal analysis. An acceptable procedure for analyzing the internal response would be to force a suitable internal model with the solution to the 2-dimensional barotropic model.

The applicability of linear dynamics to the prediction of tidal elevations and transports in fjords can be assessed for specific examples. Knight Inlet has a sill depth of around 60 m, distant 75 km from the head. The dominant $M_2$ tide varies in amplitude from 1.527 m at Montagu Point (near the mouth) to 1.562 m at Siwash Bay and finally to 1.582 m at Wahshihlas Bay near the head. For a linear normal mode response in a constant depth inlet, amplitude will vary along the inlet as $a = 1.582 \cos \left( \frac{2 \pi x}{\lambda} \right)$ where $x$ is distance from the head of the inlet and $\lambda$ is the wavelength ($\lambda = M_2$ period $\times \sqrt{gH} = 2400$ km). Thus we predict amplitudes of 1.563 m at Siwash Bay ($x = 60$ km) and 1.528 m at Montagu Point ($x = 100$ km), in agreement to within 0.07% of the normal mode of a simple linear barotropic response. The reason for the good fit can be found by estimating the Froude number at the sill: $Fr = \frac{u_0}{\sqrt{gH}} \approx 0.008$, so that linearity is no great surprise. Note, however, that the nonlinear effects will be more profound in the detailed tidal velocity structure, which is not scaled by the Froude number. Deep sills are typical of the major B.C. and Alaskan fjords. However, some Scandinavian and B.C. fjords combine very shallow sills with large tides and under these circumstances non-linearity can become significant. An alternative example is the Oslofjord which has a Froude number over the Drøbak sill about 10 times that in Knight Inlet (GLENNE and SIMENSEN, 1963); for this case choking is sufficient to produce a small decrease in amplitude and the linear approximation will be less satisfactory than it is across the Knight Inlet sill.

For large Froude numbers a quite different balance of terms occurs. Just upstream, i.e. seaward of the sill on a flood or landward on an ebb tide, the balance is principally between field accelerations and the horizontal pressure gradient, thus approximating potential flow. On the lee of the constriction a jet develops with the balance being between acceleration and Reynolds stresses. Frictional effects can also occur within the constriction and will be important for longer channels. STIGEBRANDT (1980) developed a model incorporating both frictional and gradient effects, thus including the frictionless model of MCCLIMANS (1978) and the purely frictional model of GLENNE and SIMENSEN (1963) as special cases. Combining these effects he arrives at the following equation which must be solved numerically:

$$\frac{d\xi}{dt} = I(\xi_0 - \xi) / |\xi_0 - \xi|^{1/2} + S_f$$

where $\xi_0$, $\xi$ are the outside and inside sea-levels, $I$ is a geometrical term which depends on the effective cross-sectional area and $S_f$ defines the relative significance of fresh water discharge. For small constrictions and large discharge the effect of fresh water input is to impose a net sea level difference across the constriction thus lengthening the duration of the ebb relative to the flood.

Aside from the navigational problem, these highly nonlinear barotropic "choking" effects are important because they constitute a source of energy for mixing. In effect the jet that forms on the lee of the sill on each flood tide is dissipated and some of this energy can be recovered as potential energy (see MCCLIMANS, 1978 and 1981 for a more detailed account). In discussing the energetics of such a system the net energy flux $E$ is given as
\[ E = \frac{1}{2} \rho \int_0^r \delta \frac{Q^3}{A^2} \, dt \]  
(4.9)

where \( \delta = 1 \) for a net transport \( Q \) through the constriction into the fjord \( (Q > 0) \) and \( \delta = 0 \) for \( Q < 0 \), (i.e. STIGEBRANDT, 1980). However equation (4.9) does not appear to have been derived from the Laplace Tidal Equations and care should be used in its interpretation. As it stands it does not include the potential energy flux through the constriction, which may be appreciable. At most (4.9) can only be considered as an upper bound on the kinetic energy flux into the fjord.

There have been relatively few observational studies of such inflowing jets but measurements in Rupert Inlet (STUCCHI and FARMER, 1976; and STUCCHI, 1980) emphasize the extreme sensitivity of the jet's structure to the relative density differences between inflowing and ambient water. There is a need for more detailed studies of such jets in order to resolve their structure as a function of space and time and to assess the extent to which they contribute to mixing and deep water renewal under different oceanographic conditions.

4.1.2. **Internal tides.** Observations of the depth of isopycnal surfaces in a fjord will invariably show the presence of internal oscillations at tidal frequencies. This is certainly not surprising since we regularly observe a strong interaction between the stratified fluid, forced by the barotropic tide and the sills characteristic of classical fjords. STIGEBRANDT (1976, 1980) has suggested that the breaking of internal tides can supply turbulent energy for mixing in fjords. In this section we consider the propagation and generation of internal tides in detail.

Following the development in Section 5.2 of PHILLIPS (1966) but including the Coriolis force we derive an equation for the vertical velocity in a stratified fluid:

\[ \frac{\partial^2}{\partial t^2} (\nabla^2 w) + N^2(z) \nabla^2 w + f^2 \frac{\partial^2 w}{\partial z^2} = 0. \]  
(4.10)

If we look for wavelike separable solutions of the form \( w(x, y, z) = R_e \{ G(x, y)Z(z) \exp \left( -i\omega t \right) \} \) we find

\[ \nabla^2 G/G = \left( \frac{\omega^2 - f^2}{N^2 - \omega^2} \right) Z_{zz}/Z = -\mu^2 \]  
(4.11)

where \( \mu^2 \) is a separation constant. Then postulating that the horizontal variability is wavelike, i.e. \( G(x, y) = \exp \left( i(kx + ly) \right) \), we find a modal equation and a dispersion relation:

\[ \frac{d^2 Z}{dz^2} + \left[ N^2 - \omega^2 \right] \frac{c^2}{c^2} Z = 0 \text{ and } \omega^2 = f^2 + c^2(k^2 + l^2). \]  
(4.12)

With appropriate boundary conditions at the top and bottom the modal equation describes a standard Sturm-Liouville problem and admits solutions for a countable infinity of eigenfunctions \( Z_n, n = 1, 2, 3, \ldots \) and associated eigenvalues \( c_n, n = 1, 2, 3, \ldots \) which have the dimensions of speed. A complementary description of the internal tide can be carried out in terms of ray theory (see Section 5.2.1) which may be advantageous, for example when analyzing the propagation of energy into a fjord and the presence of shadow zones. CUSHMAN-ROISIN and SVENDSEN (1982) have analyzed current meter data using both methods and showed that together the two approaches provide a good description of the internal tide.

For free waves the dispersion relation limits \( \omega \) from below and the modal equation limits \( \omega \) from above, \( f^2 < \omega^2 < \max (N^2) \). Referring again to our example of Knight Inlet, \( f = 1.13 \times 10^{-4} \text{ s}^{-1} \) and waves propagating at tidal lines \( P_1 \) and \( M_2 \) have angular velocities \( \omega = 7.3 \times 10^{-5} \text{ s}^{-1} \)
The physical oceanography of fjords

and $1.405 \times 10^{-4}$ s$^{-1}$ respectively. Hence, it is possible to propagate free semi-diurnal tides, but not free diurnal tides. (This is also true for the barotropic case.) However, trapped waves can escape the low frequency limit; if we set $f^2 + c_n^2 l^2 = 0$, or $l = \pm i f/c_n = \pm i/R_n$ then we have a non-dispersive relationship $\omega = \pm c_n k$ and a shape in the horizontal plane of $G(x, y) = \exp(-y/R_n) \exp(ikx)$, the Kelvin Wave. These waves decay exponentially from a rigid wall, with scales given by the Rossby radii $R_n$ and particle motion is parallel to the wall. Hence, this wave is appropriate to describe the propagation of tides down a channel such as a fjord. For these waves the eigenvalues $c_n$ become the wave speeds (phase or group). For summer stratification in Knight Inlet $c_1$ is about 1.0 m/s, so the cross-inlet decay scale $R_1$ is about 8 km. In winter we would expect a value nearer 4 km which implies significant cross-inlet variability in the internal tide. For the barotropic case $c_0 \approx \sqrt{\gamma H}$ and $R_0 \approx 500$ km which exceeds the inlet width by two orders of magnitude, thus justifying our earlier representation of that tide by a plane wave.

Suppose that in an inlet the tidal frequency oscillations are represented by the barotropic and one internal mode only, which need not necessarily be the first internal mode. Let us consider the result of adding the baroclinic mode to the zeroth or barotropic mode. At a particular frequency we can represent the barotropic tidal velocities by an amplitude and phase, or equivalently as the vector $\mathbf{B}$ on the Argand diagram of Fig. 11. Near the surface we add a velocity and phase $\mathbf{R}$ characteristic of the baroclinic tide producing the vector $\mathbf{S}$. As we move down the water column from position (a) the amplitude of the added baroclinic vector decreases (b) but phase does not change until we cross a zero of the mode (c) when phase changes by 180 degrees. In the deep water below the first zero crossing (d to e) the observed tide is represented by the vector $\mathbf{D}$. Hence, if internal motions are dominated by a single internal mode the tip of the amplitude/phase vector describes a straight line on the Argand diagram. This type of behaviour has been observed in Knight Inlet and it can be exploited in an interesting way.
During the summer of 1977 two moorings were deployed in Knight Inlet [Fig. 12] near Tomakstum Island about 14 km up-inlet of the sill and 7 km seaward of Sallie Point. Since the latter landmark represents the end of the straight section of the inlet and the start of the sinuous section, it can be represented rather roughly by a right-angle turn in the inlet. The moorings were deployed by Dr. G. CANNON (P.M.E.L., Seattle) and the data kindly made available to us. Each current meter record was subjected to harmonic analysis, the M\textsubscript{2} components of which are shown in Fig. 12. The moorings are labelled N and S (North and South) and carried Aanderaa RCM4 current meters at 75 m and 300 m depths, indicated by subscripts "s" and "d" (shallow and deep) respectively. Since at each site the amplitudes and phases must lie on a straight line, the point of intersection yields the amplitude and phase of the barotropic tide, B; the vectors S and N then represent the amplitude and phase of the internal tidal signal at the south and north sites respectively. The intersection point has a bearing of 248.2° which we claim must be the bearing of the barotropic tide. A nearby tide gauge at Siwash Bay was analyzed over exactly the same time period and yielded a phase for barotropic tidal velocity (i.e. the phase of elevation plus 90°) of 249.1° corresponding to a discrepancy of only 0.9°. Confidence intervals estimated in the usual way, using chi-squared statistics, indicate that discrepancies of the order of 10° should be expected. However, tides are not chi-squared variables; rather they are narrow-band coherent signals. Phase estimates of tides are always much more stable than chi-squared statistics would imply. If we represent the two signals by a superposition of an incoming and an outgoing Kelvin wave we find that the outgoing wave has an amplitude of 0.93 x the amplitude of the incoming wave.

The question naturally arises as to where in Knight Inlet the internal tide is being reflected. Two obvious candidates are (i) the first bend, i.e. at Sallie Point, and (ii) the head of the inlet. In the latter case it would be surprising if an internal tide could propagate so far around a complex of bends and turns with so little energy loss. If the reflection were from the first bend then the energy loss (about 14%) presumably represents a partial transmission around the corner. In support of this hypothesis we cite two sets of measurements. Firstly, at the same
time as the current meter observations, measurements with a thermistor chain moored 30 km from the head showed only a very weak internal tide; secondly, observations by S. POND (U.B.C., personal communication) at several locations landward of Sallie Point suggest that the internal tide is indeed restricted to the straight section of the inlet. We conclude that an internal Kelvin wave will undergo almost specular reflection at a right angle bend in a channel.

Abrupt corners of this type are a common feature of fjords, so there is some interest in resolving the propagation of internal tides around them. The closest theoretical analysis relevant to this problem is the study of the propagation of a Kelvin wave around a corner in an infinite half plane, BUCHWALD (1968). For the frequencies of interest he predicts 95% transmission with the remainder scattered into cylindrical Poincaré modes. However, the presence of the second boundary changes the physics substantially since only then is the return Kelvin wave permitted; also for Knight Inlet parameters all Poincaré modes are evanescent. We would like to see Buchwald's calculation repeated for the case of a bend in a channel, although an alternative and perhaps more appropriate approach would be the collocation method described by BROWN (1973).

STIGEBRANDT (1976) carried out a similar calculation and reported computations indicating that none of the internal tide generated near Drøbak sill in Oslofjord is reflected. However, his analysis neglects rotation and it is instructive to consider the influence it might have on his data. His observations were at two sites, one near an island about 2 km landward of Drøbak and one 7 km further into Oslofjord at Langaara. The ratio $c_1/f$ is about 8 km, and the Channel is about 1 km wide at the first location, with the observing site on the west side of the channel; the channel is about 4 km wide at the second site on the east side. A variation in amplitude of the internal tide across a fjord leads, of course, to a variation in the observed phase of tidal velocities across the inlet which is used in the separation of the internal and barotropic modes. With these parameters the incoming Kelvin wave will have an amplitude on the west side of Langaara of about $\exp(-\frac{1}{2}) = 0.61$ of that on the opposite bank and thus have an appreciable influence on calculated phase angles.

The importance of these observations arises from a calculation of the energy transmitted in these waves. STIGEBRANDT (1976, 1979) has shown that if the waves are totally absorbed then only 5% of the energy released is needed to maintain the observed mixing in the deep water of the inner Oslofjord. Of course internal wave energy propagating out of the fjord will not necessarily escape over the sill and may still be available for mixing. The point to be emphasized is that analysis of internal tides, taking due account of topographic effects and rotation, can help to establish the nature and location of their decay and thus their possible contribution to mixing.

Internal tides can be generated whenever tidally driven stratified fluid oscillates over irregular topography, subject to the frequency bounds already discussed. BELL (1975) has shown how not only the fundamental mode is generated in this way, but also the harmonics, whose contribution depends on the value of the corresponding spatial derivatives defining the topography. Such topographic irregularities abound in fjords, with sills and the sloping bottom near the head being particularly favourable areas of generation. STIGEBRANDT (1976, 1980) has discussed the linear problem of internal tide generation over a sill for both two-layer and uniform stratification and both he (1979, 1980) and PERKIN and LEWIS (1978) provide evidence supporting his model. The linearity constraints are quite strong, however, and they will not be met in any fjord whose tidally generated sill flow approaches critical conditions. Nonlinear effects need not limit the generation of internal tides but they will move energy into the harmonics and they will limit the applicability of linear models (i.e. BELL, 1975,
CUSHMAN-ROISIN and SVENDSEN, 1982) close to the generation area of strongly forced sill flows.

BLACKFORD (1978) described a non-linear mechanism for generating internal tide oscillations in a pycnocline that lies deeper than the sill depth. The analysis is instructive though the model is restricted to a rather limited class of inlets. He points out that Bernoulli effects close to the sill will generate an even harmonic of the forcing frequency and a d.c. signal, and friction will generate odd harmonics (see also the discussion by STIGEBRANDT, 1980). He considers only the response to a single forcing frequency, presumably the same mechanism would predict a rich crop of exotic tidal lines if forcing at several tidal lines was allowed. In Knight Inlet, current observations at 170 m depth seaward of the sill do indeed exhibit a rich tidal spectrum with large amplitudes at non-linear tidal lines (i.e. MSf, MSN2 and many ter-, tetra- and penta-diurnal lines). A large Froude Number flow over the sill generates jets, lee waves and hydraulic jumps. No matter what the detailed mechanism of the sill flow in these cases it would seem inevitable that in the farfield, internal tides including some harmonic components, will always be found.

Internal tides can also be generated by tidal motions over the sloping shore near the head of a fjord. An analogous problem was first solved by RATTRAY (1960) and FARMER and OSBORN (1976) show evidence of such generation in Alberni Inlet. Thus in calculating energy fluxes in fjords we cannot always assume that the tide originates from only one source.

It was pointed out by FREELAND and FARMER (1980) that more energy will be extracted from the barotropic tide at spring tides than at neap tides. The hydraulic transitions that enable large amounts of energy to be extracted do not occur on every tide, but are more common near springs than near neaps. Hence, more turbulence is available for mixing near springs than neaps and we expect the general circulation to be modulated at the beat period of $M_2$ and $S_2$ which is the MSf tide of period 14.7d. Unequivocal observations of large, purely internal, MSf tides are shown and we would suggest that a large MSf internal tide is a signature of any inlet whose turbulence is derived from the tide. Preliminary observations in Observatory Inlet indicate that a large fortnightly tide occurs there also. These observations demonstrate how easy it appears to be to generate an internal tide. The highly non-linear events observed around the sills in Knight and Observatory Inlets force the large scale stratification at the dominant tidal frequencies and must inevitably produce a wide range of internal signals by non-linear interaction between the astronomical frequencies. At higher frequencies these become observable in the surface elevation signal and thus we see, in Knight Inlet for example, a rich tidal spectrum.

4.2. Deep water renewal

Major replacement of deep water in a fjord, by which we refer to the water below sill depth (Fig. 2), will occur whenever the outside water at or above sill depth exceeds the density of water within the basin. GADE and EDWARDS (1980) provide an excellent review of this topic, so we limit our discussion to a brief sketch of some of the dominant mechanisms.

Consider a deep fjord basin connected to a continental shelf via a shallow sill. In the absence of mixing the deep basin water will have the density of the densest water ever delivered by outside processes to the sill. The chances of exceeding that density by natural means are small so that water will rarely be replaced. Any biological oxygen demand will deplete the water of whatever oxygen it contains and bacterial action on biological material will produce hydrogen sulphide. This situation is nearly achieved in a number of locations; for example, Nitinat Lake.
a B.C. fjord undergoes infrequent renewals (OZRETICH, 1975). The renewals when they occur are partial and so water can remain stagnant for many years.

In general, however, some mixing does occur. The turbulence that drives the mixing processes may originate from tides, or the action of the wind on the free surface, or shear instabilities in the fjord near-surface circulation. Given even a low level of turbulence the density of the deep water will steadily decrease as near surface water is mixed downwards. Hence the deep water masses become steadily more predisposed towards undergoing a renewal.

The exchange process is generally unsteady and can embrace a wide range of time scales, from tidal periods to many years. The rate of exchange can be modulated by many factors. At the upper limit of exchange, internal hydraulic control [cf. equation (2.1)] will inhibit two-way flows over the sill unless a barotropic flow is impressed upon it. Large scale examples of such overmixed conditions include the Baltic and the Black Seas. STIGEBRANDT (1977) discusses the case of tidally induced forcing which can greatly increase the rate of exchange in the overmixed condition by periodically changing the flow to a single layer, thus overriding the internal hydraulic constraint. At lower Froude Numbers tides and meteorological effects can also modulate the exchange. GADE and EDWARDS (1980) comment that spring tides enhance renewal and this is likely to be true provided intense mixing does not occur in such a way that it inhibits the baroclinic flow. This latter point is discussed later in connection with GEYER and CANNON’s (1982) results (Section 5.2.2).

Fresh water runoff can also modulate the exchange, both through mixing over the sill, but also by accelerating the baroclinic currents that supply the replacement water. In extreme cases, fresh water discharge can cause blocking, so that the fresh water layer extends down to or beneath sill depth. Fluid accelerations that can bring water from some depth up to sill level are a manifestation of selective withdrawal. An interesting application of this concept on a much larger scale was made by STOMMEL, BRYDEN and MANGELSDORF (1973) who examined the upward movement of water exchanged over the sill in the Straits of Gibraltar. The analysis is based on Bernoulli’s law applied along successive streamlines and the assumption of hydrostatic balance. The depth from which water can be withdrawn is constrained by the density stratification, but in the case of the Mediterranean it was shown that even the deepest water can be exchanged in this way.

Once over the sill crest the replacement water enters the fjord as a gravity current. Since the flow is expected to be turbulent we anticipate entrainment into the flow as the density current descends; EDWARDS and EDELSTEN (1977) obtained consistent results using ELLISON and TURNER’s (1959) laboratory derived entrainment rates.

As the denser water descends, entrainment of surrounding fluid decreases its density. But the water of the fjord basin is itself stratified. Thus it is quite possible for the current to reach a depth at which it is neutrally buoyant, whereupon it separates from the sloping boundary and spreads out into the basin as an intrusion, possibly accompanied by internal waves (HAMBLIN, 1977). The behaviour of gravity currents has recently been the subject of active study in the laboratory, but it is beyond the scope of this review; we refer to the comprehensive review by PEDERSEN (1980) as well as recent studies by DENTON, FAUST and PLATE (1981) and FUKUOKA and FUKUSHIMA (1980). The latter discuss the formation of multiple interleaving layers that can form when the ambient fluid is appropriately stratified.

The problem of mixing in deep water is deferred to Section 5, but it is clear that the frequency of renewal is critically dependent both on the availability of dense water and on the mixing within the basin. These competing effects have motivated simple modelling approaches including the deterministic one-dimensional exchange model of WELANDER (1974) and
FIG. 13. Variations of temperature and dissolved oxygen in the Gulf of Cariaco, Venezuela, after OKUDA, 1981. This tropical example of periodic exchange is an analogue of the deep water exchange which occurs in fjords and serves as an example of the way in which fjord processes are representative of similar processes occurring in other oceanographic settings. Note the 'sawtooth' shape of the deep water properties in which a sudden jump is followed by a gradual decay towards equilibrium conditions.

A stochastic model in which external density conditions vary according to some known probability, developed by GADE (1973).

Unfortunately it is difficult to predict in advance what vertical exchange rates will prevail in a fjord. On the other hand the variability of coastal waters on a large scale is often much better known. GADE and EDWARDS (1980) have shown that the depth of the sill may be important in controlling the time of the renewal. If the annual cycle of density anomaly exhibits significant phase changes with respect to depth, then peak densities will occur at different times of year for different sill depths. It was found that in general, fjords with sills shallower than the depth of minimum density variation tend to renew in winter, whereas deeper silled fjords renew in summer.

We conclude this section with an example of deep water renewal reported by OKUDA (1981). The example is taken not from observations in a fjord, but from the Gulf of Cariaco in Venezuela, at 10°N. As such it serves to illustrate the way in which deep water modification and renewal, like so many other features of fjord oceanography, is representative of processes occurring elsewhere in the world’s oceans. The gulf has a sill and deep basin, and the water in the deep basin undergoes periodic stagnations and renewals. Okuda’s results (Fig. 13) clearly show the slow evolution of deep water properties between renewal events and the saw-tooth behaviour of deep oxygen values so characteristic of fjords.

Clearly the time period between successive renewals is largely determined by the rate at which mixing can be accomplished, and so any attempt to understand the renewal characteristics of a fjord must focus on the turbulence itself. Unfortunately it is not possible to predict in advance how large even so simple a number as $K_v$, the vertical salt diffusivity, will be. Estimates from Narrows Inlet, SMETHIE (1981), yield values ranging over two orders of magnitude. Each case has to be examined individually.
4.3. Shelf processes

It has recently become apparent that processes on the continental shelf may have a major impact on the flows and general ventilation of fjords. A number of papers have been published describing the response of the Juan de Fuca Strait to forcing from the Pacific Ocean, for example CANNON and HOLBROOK (1981), HOLBROOK, MUENCH and CANNON (1980) and others. Indeed some of the earliest observations of time-dependent density changes in fjords pointed clearly to meteorologically induced effects on the continental shelf (SANDSTROM, 1904 and PETTERSSON, 1920). We have already discussed the significance of density changes at the sill to exchange of water in the deep basins. However, if the sill is much deeper than the brackish layer we also expect exchange of the intermediate water (see Fig. 2). In this case the exchange is not inhibited by the sill and so occurs more readily than is the case for the deeper water.

A recent example studied in British Columbia will serve to show the potential of shelf processes for influencing intermediate as well as deep water properties in a fjord. As was apparent earlier in this section a fjord usually has stagnant water behind a sill that is depleted of oxygen until the occasional deep water renewals occur. Alberni Inlet, on the west coast of Vancouver Island, behaves differently. Off the west coast the interaction of geostrophic currents with a submarine canyon pumps very dense water onto the continental shelf during the summer months (FREELAND and DENMAN, 1982). This dense low-oxygen water is brought, by advection, near to the sill in Alberni Inlet and displaces the inner basin displacing both the deep and intermediate water. As the renewal takes place the net oxygen content of the inlet decreases. The overflow into the inlet is very strong and sufficient water comes in each year to replace the entire volume of the inlet several times over (STUCCHI, 1983).

The interaction between a fjord and the coastal oceanic environment is going to be a complex one. However, a start at understanding this interaction has been made by KLINCK, O'BRIEN and SVENDSEN (1982). In this paper a numerical model of a continental shelf environment driven by wind forcing is used to drive a fjord model. Some interesting results are presented, in particular the difference in the response of the fjord to alongshore or offshore winds is noted. An offshore wind causes a tilt in the free surface of the fjord and a corresponding response in the pycnocline. However, there is no net change in the volume of water in the fjord. Of more importance to fjord/shelf exchange is the alongshore wind. An alongshore wind over the continental shelf drives water on or offshore in the surface Ekman layer with a compensating deep return flow. Coupled with this is a large set in the free surface near the coast. In order to couple this behaviour to the fjord we need to change the mean sea surface elevation within the fjord and so change the total volume of water in the inlet.

KLINCK et al. (1982), while recognizing the possible importance of nonlinear mechanisms especially near the sill, propose matching fjord flows to the shelf flow field as a method of controlling fjord circulation, in contrast to the hydraulic control proposed by STOMMEL and FARMER (1952a,b) and others described earlier. However it is important to distinguish between the gravitational dynamics associated with the seaward moving, brackish surface layer on one hand and the exchange of intermediate water on the other. The dominance of one exchange mechanism over another will depend on the amount of river discharge, the dimensions of the fjord mouth, the magnitude of external forcing and perhaps other factors. KLINCK et al. (1982) considered a very special case, in which the fjord was represented strictly as a passive embayment with no river discharge and no mixing. Thus the estuarine circulation, in particular the movement of a surface brackish layer out towards the ocean which is the
physical process of interest in the theoretical models described in Section 3.2, is specifically excluded. Thus even if nonlinear terms are included in their model and if, as we suspect, transient exchanges of intermediate water over a sill often turn out to be limited by hydraulic constraints, the problem under examination will still differ in a fundamental way from the layered estuarine circulation models.

This does not mean that the intermediate exchange is unimportant, since as indicated earlier, it may dominate the movement of water into and out of a fjord. If river discharge is low and if there is little or no constriction at the fjord mouth, the density structure inside the fjord will be similar to that outside and Klinck et al.’s model is likely to be a fair representation of the dynamics. On the other hand if there is significant river discharge the hydraulic control at the mouth is required precisely because of the need to match surface layer conditions at the mouth of an essentially one-dimensional inlet to flows in a semi-infinite plane. In a deep silled fjord the intermediate water exchange will continue beneath the brackish outflow with only weak coupling between the two; shallower sills will impose stronger coupling and the combined dynamics of estuarine circulation and intermediate exchange will become more complicated and less predictable.

A related difficulty with the outer boundary condition arises in a numerical model described by Hodgins (1979). Here the numerical grid is extended one inlet length beyond the mouth and sets the upper layer thickness to some arbitrary value. We see no inherent advantage in this manipulation. It does not provide a physical basis for choosing a particular upper layer thickness at a given place and time, although in this case, as in the theoretical modelling of Farmer (1976), the motivation was the analysis of the time-dependent response of the density structure to variable forcing.

The choice of critical conditions [equation (2.1) or (2.2)] at the fjord mouth is mathematically satisfying, but we cannot say that it is unambiguously observed. From the theoretical modeller’s point of view it very conveniently decouples the estuarine circulation from the details of shelf processes, but we suspect that in many situations this condition, if it occurs at all, is masked by time dependent effects. In Knight Inlet we estimated maximum values at the mouth of $F_1 \approx 0.3$, based on steady state assumptions, but the reason for this may be quite different from that considered in any of the numerical models.

In a narrow fjord with strong stratification rotation can be relatively unimportant because the inlet width is substantially less than the internal Rossby radius. But when the brackish layer debouches over the continental shelf it finds itself, rather abruptly, in a region with characteristic length scale much greater than the internal Rossby radius. We would not therefore expect the upper layer to spread outwards in a two-dimensional plume which thins rapidly as it moves away from the fjord mouth. Rather, the behaviour might be more akin to that described by Stern (1980) resulting from laboratory experiments. In this case the fluid issuing from the model inlet is deflected to form a coastal current which has a width of about the Rossby radius of deformation. Variations in volume flux out of the inlet propagate as bores along the coastal current. By this means it may be possible to delay the matching condition to some distance along the coast. However the outer boundary of a fjord is often poorly defined, since the fjord itself may be separated from the ocean by a complex series of interconnected channels, see Fig. 1, for example.

5. MIXING PROCESSES IN FJORDS

One of the most interesting aspects of fjord oceanography is the study of turbulence and mixing. Most of what we know or can guess about these processes in fjords derives from studies
in other geophysical settings and from laboratory experiments; but from the broader perspective of the study of geophysical fluid dynamics, fjord oceanography is also of potential interest since the experimental conditions in fjords are much more favourable than in many other natural environments. In this respect, fjords offer the opportunity for observing mixing processes on scales comparable to those encountered in lakes, but with the added complexity of salt stratification and tidal forcing. They therefore offer opportunities for studying natural flows that have particular relevance to other coastal areas and also to the deeper ocean, but yet retain the relatively benign experimental conditions of inland waters.

This is not the place to summarize current knowledge of mixing that might be relevant to fjords. An excellent general review has recently been presented by TURNER (1981), much of which is directly applicable to fjord studies. SHERMAN, IMBERGER and CORCOS (1978) have also reviewed several aspects relevant to mixing in coastal waters and GARDNER, NOWELL and SMITH (1980) review several field experiments in estuaries. Here we shall mention a few of the key processes that seem especially important, emphasizing some of the differences between mixing in fjords and in the ocean and expanding on these and describing relevant observations in the subsequent sections.

The way in which turbulence is generated is often inextricably associated with several competing mechanisms; however we can identify certain principal sources. The differences between fjords, or between conditions in the same fjord at different times of the year, will be reflected in the dominance of one or more such mechanisms over others. A distinction between oceanographic conditions in fjords and in the deep ocean is that being an estuarine system, the stratification in a fjord is constantly replenished by fresh water. This strong, near surface stratification tends to inhibit convectively generated turbulence in the surface layer, except during periods of very low run-off or possibly in Arctic fjords where a convective layer may be driven by salt exclusion from a growing ice-sheet. Thus, most of the turbulence generated in fjords arises from mechanical input, by wind acting on the surface layer, by tidal action, by turbulent gravity currents at depth and by shear flows at the surface arising from river discharge. Only in winter, with low surface temperatures and low discharge, can thermal convection be important. Under the right conditions, however, the subtle effects of double diffusion may be important in the deep basins of a fjord.

Mixing is central to the problem of gravitational circulation in estuaries since it is responsible for altering the density gradients that drive the flow. These density gradients exist both in the vertical and horizontal dimensions, but we are principally concerned with vertical mixing. Horizontal fluxes, at least along the main axis of a fjord in the stratified zone, appear to be principally advective; horizontal mixing and diffusion is important in the transverse sense, however, and serves to reduce the transverse property gradients, thus supporting the two-dimensional approach adopted in theoretical studies in fjords.

Vertical mixing is dominated by the effects of vertical density gradients in the water. In fjords the density gradient typically has a maximum at a depth of a few metres just beneath the surface brackish layer, where the buoyancy period which contains the dynamic effect of the density gradient, defined as $2\pi/N$, with

$$N = \left(\frac{-g \partial \rho}{\rho_0 \partial z}\right)^{1/2},$$

(5.1)

can drop to about 10 s, rising to about 1 hour in the deep water (see Fig. 3). In contrast, typical values in the main North Pacific Ocean pycnoclines are around 400 s at depths of around 100 m rising to many hours in deeper water. It is the proximity of this highly stratified layer close to
the surface, constantly replenished in high run-off fjords, that isolates the deeper water from
the effects of wind-stress and thus serves to distinguish wind mixing in fjords from the ocean.
The run-off provides the surface buoyancy flux that opposes wind-mixing allowing, in principle,
steady conditions to be attained. This occurs when the vertical flux of momentum balances
the buoyancy flux $B$, thus defining the scale $L$ at which buoyancy forces become dominant:

$$ L = -u^2/kB, \quad (5.2) $$

where $u^2 = -u\overline{ww}$ and $k$ is a constant. This is just the Monin-Obukhov length (by convention
$k$ is the Von Karman constant) identified in the theoretical models of Section 3, equations
(3.15) and (3.16). It differs from the more typical oceanic situation where the buoyancy flux
arises from surface heat exchange which in the steady state leads to the depth scale first pro-
posed for the ocean thermocline by KITAIGORODSKII (1960), becoming negative for unstable
conditions.

Although the surface mixed layer depth might be proportional to $L$ in certain highly simpli-
fied situations, wind mixing usually involves many other competing factors. This has led to the
development of numerous one-dimensional vertically integrated models based on the original
version of KRAUS and TURNER (1967), (see, for example, NIILER and KRAUS, 1977).
Current models include terms proportional to the storage rate of turbulent energy in the mixed
layer, the rate of increase of potential energy due to entrainment from below, due to surface
buoyancy fluxes including penetrating solar radiation, the rate of working of the wind-stress
and of radiation stress from internal waves and the rate of energy loss to dissipation. At present,
the complexity of even these one-dimensional models is hardly justified on the basis of our
meagre understanding of the processes actually occurring. The problem of wind mixing in fjords
is further complicated by the influence of boundaries.

We have already indicated the different interpretation of the buoyancy flux, normally
applicable in fjords. Other differences may arise due to the narrowness of most fjords. For
example in the ocean, rotation starts to control the mixed layer deepening after about half a
pendulum day, at which point $h \approx u_\star/(N^2)^{1/2}$, (POLLARD, RHINES and THOMPSON, 1973),
whereas in a high run-off fjord the rotation effects should be small for a channel width signi-
ficantly less than the internal Rossby radius. Other effects such as seiching can start to domi-
nate the response within a few hours.

While wind mixing may be limited to the near surface, tidal effects can be felt throughout
the basin depth. Mixing can occur either at the boundaries or through instabilities generated
by internal waves or jets in the interior of the basin. We know of no bottom boundary layer
measurements in fjords. Well mixed boundary layers are observed in the ocean; for example
ARMI and MILLARD (1976) found benthic boundary layers which had a thickness $h$ depen-
dent on the Froude Number relationship

$$ F = u/(g' h)^{1/2} \approx 1.7. \quad (5.3) $$

In fjords with large tides, mixing can occur through breaking internal tides or higher frequency
internal waves or jets generated by tidal flow over constrictions, which are discussed in detail
in Section 5.2.2. and 5.2.3 following. The input of energy into these processes may be inferred
by direct measurement of energy loss from the surface tide, or by theoretical arguments. With
few exceptions the energy actually used in mixing has so far only been inferred from water
property changes in fjords although the field is clearly open to microstructure measurements
of the type carried out in the open ocean. The ratio of the rate of removal of energy by buoy-
ancy forces to its production by shear is the flux Richardson Number:
Since this parameter defines the efficiency of the mixing process there is some interest in knowing its values for particular mixing conditions in fjords. PEDERSEN (1980) has studied two-layer flows from this point of view and uses an extensive set of historical data in support of a new bulk flux Richardson Number that takes into account the gain in turbulent kinetic energy of the entrained fluid. The resulting parameter is found to have one of two values, depending upon the value of the densimetric Froude number. In applying this concept to real data, Pedersen analyzes a number of problems of direct relevance to fjord circulation.

In low run-off conditions, the stabilizing effect of fresh water discharge is reduced and each of the mixing mechanisms we have described can almost eliminate vertical stratification. Thermal convection may then become important, with penetration of the convective layer well below the depth normally achieved through wind-mixing. Winter cooling can have another effect by producing a vertical temperature gradient having the same sign as the salinity gradient and thus providing the conditions necessary for double diffusion. This topic has hardly been explored in fjords, but it seems probable that fjords provide ideal natural sites for study of the diffusive layered type of instability.

5.1. Winds in fjords

The wind can be expected to influence oceanographic conditions in inlets in several different ways. The frictional stress on the surface can retard or accelerate the mean surface layer flow; turbulence generated near the surface can drive mixing at the shallow pycnocline increasing the potential energy of water columns and hence driving the deep gravitational circulation.

The wind direction is usually along the length of the inlet following bends and is most often directed up-inlet in summer. At longer time scales the modulation of the wind is governed by large scale weather systems, but a prominent feature of fjord meteorology is the sea breeze effect. Solar heating in the daytime produces rising air over land, a compensating inflow from the sea and return flow aloft. It is a curious feature of the resulting wind field that the daytime sea breeze often dominates the weak land breeze at night. Wind speed resolved along the fjord axis often approximates a half wave rectified sinusoidal signal so that 24 hour average wind speeds are directed up-inlet, unless overridden by a strong opposing pressure gradient in the larger scale weather pattern. The lack of reciprocity arises partly from the relative thickness of the circulating air mass in each case: during a sea breeze, the air movement occurs over considerable depth and the predominant dynamic balance is between the pressure gradient and inertial effects, whereas in a land breeze the pressure gradient in the relatively thin nocturnal boundary layer is balanced by friction. In addition, the dew point provides a low limit to the air temperature over land at night, thus reducing the net horizontal gradient relative to daytime.

Fjords guide this inflowing air inland, so that the direction is merely a consequence of the channeling action of the steep sides of most fjords. Wind speed will vary along the channel, increasing with decreases in channel width but also in general showing a steady decrease towards the interior. This arises through the feeding of some of the inflowing air into tributary channels and river valleys, but also because heating on the north slope of a fjord will cause air to rise out of the lower inflowing mass. These effects are all observable in Knight Inlet. In August 1978 three anemometer sites were maintained along the inlet at distances of 70, 34, and 12 km from the head. The flow was landward and strongly diurnal in each case, with root
mean square wind speeds of 5.1, 4.0, and 3.8 m/s respectively, demonstrating the progressive decrease in wind speed with increasing distance from the mouth.

It is not known how well wind mixing in fjords follows the models generally used to describe the effect in the ocean. In its simplest form the 1-dimensional mixed layer representation, first analyzed by KRAUS and TURNER (1967), models the balance between the kinetic energy input of the wind and the transformation of a portion of it into potential energy by entrainment, the difference being lost to dissipation. The success of this type of modelling depends on the parameterization of entrainment and dissipation. As discussed in the introduction to this section, many refinements to the early model have been suggested, and we have already described the special models of LONG (1975a) and STIGEBRANDT (1981) applicable to wide fjords. The difficulty in modelling realistic conditions in fjords is partly due to the great range of spatial and temporal variability in wind forcing.

In the winter, outbreaks of polar air together with further cooling over the continental ice fields can produce strong katabatic winds. Winds created in such circumstances fed into narrow fjords can be very strong and destructive, as for instance the strong winds that blow down fjords off the Greenland ice-cap. In British Columbia the winds are known as Squamishes and have well known and predictable drainage patterns. Wind speeds of 130 km per hour have been observed in a Squamish in Dean Channel in the vicinity of Ocean Falls. In March 1950 an experienced coastal pilot encountered an intense wind blowing out of Dean Channel. We quote from a report by TYNER (1950), “Descending from ten thousand feet in smooth air he first encountered moderate turbulence at five thousand feet which increased to severe below two thousand feet. Surface wind speeds in this gale were estimated to exceed eighty miles per hour”.

Squamishes are comparatively rare in the sense that fjords are probably under the direct influence of a Squamish for only, at most, a few percent of the time each year. However the wind stress on the surface of the ocean is proportional to the square of the wind speed and the rate at which wind does work on the sea surface is proportional to stress times wind speed or wind speed cubed. Thus an occasional burst of high wind can promote a disproportionate amount of mixing. This will be especially true in winter when river discharge is low and the restratification that takes place between successive storms is inhibited.

While quasi-steady conditions might occur in some areas it is more often true that wind-conditions are strongly time dependent. Moreover in contrast to wind driven flows in the deep ocean which result in inertial motions, the narrowness of most fjords limits rotation effects, and a more important factor is the interaction of wind driven flows with boundaries. In this respect, wind effects in fjords, especially when the stratification is reduced, are similar to effects of strong winds on long, narrow lakes. Wind drift of the surface layer can lead to changes in the depth of the pycnocline which may occur either in the form of standing waves (i.e. seiches) or as frictionally damped progressive waves. Observations in Alberni Inlet and elsewhere demonstrate that fluctuations in up-inlet winds which are a typical feature of the spring and summer wind regimes, result in rapid changes in pycnocline depth near the head of the inlet that propagate back along the channel for several tens of kilometres. The pycnocline depth may double in less than six hours and the theory of long internal waves appears to provide a good description of its subsequent behaviour (FARMER and OSBORN, 1976; FARMER, 1976, SVENDSEN and THOMPSON, 1978; HODGINS, 1979).

If significant tilting of the pycnocline occurs the deeper water can be directly exposed, leading to rapid wind mixing accompanied in winter by strong heat exchange. THOMPSON and IMBERGER (1980) have demonstrated this effect with a two-dimensional numerical model
The physical oceanography of fjords

FIG. 14. Temperature profiles near the head of Knight Inlet (Station KN8) through the winter of 1977/78 (month of observation indicated by number). The figure shows the effects of a large surface heat loss accompanied by wind mixing in January which results in a temperature minimum at around 100 m after the surface layer warms in the spring. The higher fresh water discharge in spring and summer tend to inhibit wind mixing and thus serve to preserve the effects of the distributed heat loss that occurred in winter.

of a lake. A key parameter determining the wind-driven response of such a system is given by the balance between the surface-layer Richardson number $R_i = \frac{g' h}{u_*^2}$ and the aspect ratio of the lake $l'/h$, where $u_*$ is the surface friction velocity and $l'$ a horizontal length-scale for the basin. In a fjord the relevant length scale might be that of a long straight section or perhaps the total fjord length. For large values of the ratio $R_i/(l'/h)$, which Thompson and Imberger refer to as the Wedderburn Number (Wb), a sharp interface is maintained and wind effects lead to seiching. For smaller values, i.e. $Wb \lesssim 6$ the upwind interface approaches the surface causing local breakdown of the pycnocline and horizontal density gradients.

At still smaller values, typical of reduced stratification in fjords during low run-off periods, strong vertical mixing can occur during a single storm leaving a longitudinally stratified basin. This process can generate a temperature minimum near the head of a fjord at depths of 50–175 m (MCNEILL, 1974), an example of which is shown in Fig. 14. During winter it is quite possible for fjords subject to strong out-flowing winds to have higher surface salinities at the head than at the mouth. In such cases it is obvious that the classical estuarine circulation of the type discussed in Section 3 is completely masked by transient effects.

5.2. Tidal mixing

5.2.1. Mixing by internal tides. As was discussed in 4.1.2, internal tides may also lose significant energy in a fjord; some of this energy is then available for mixing. Laboratory experiments of CACCHIONE and WUNSCH (1974) demonstrate the breaking mechanism on a sloping boundary for internal waves in continuously stratified fluid. Rather than the modal development of Section 4.1.2 it is more convenient here to consider the complementary description in terms of rays (cf. TURNER, 1973). In a stratified fluid oscillations can occur at an angle $\phi$ between the horizontal and the resultant wavenumber; the natural frequency in this case is $\omega = \left\{f^2 + (N^2 - f^2) \cos^2 \phi \right\}^{1/2}$, so that external imposition of oscillations of given frequency leads to particle motions having a direction of oscillation permitting a matching to the forcing motion. The wave energy thus propagates along a ray having the characteristic slope $\theta = \frac{1}{2} \pi - \phi$. CACCHIONE and WUNSCH (1974) showed the way in which the breaking mechanism depends on the ratio of the bottom slope $\alpha_B$ to the wave characteristic slope $\theta$. For $\alpha_B > \theta$ waves are
reflected back into the interior, but for $\alpha_B < \theta$ the internal wave energy is trapped in the beach region and ultimately dissipated in the corner of the fluid wedge bounded by the surface and bottom. In the critical case ($\alpha_B = \theta$) vortices are generated along the boundary with subsequent mixing and layering spreading back into the interior of the fluid.

For semidiurnal tides the wave characteristic slope below the pycnocline in fjords is almost horizontal; for example, in Knight Inlet even in the deepest water $\theta = 2^\circ$. Thus we might expect that in general, internal tides occurring in the deep stratified water of a fjord and encountering a boundary will not be trapped and dissipated in the beach region unless the sea floor has a very gentle slope. On the other hand STIGEBRANDT (1976) carried out an analysis and laboratory experiment on the generation and dissipation of interfacial waves. As he points out, there is a difficulty in scaling his laboratory results to natural flows, but his shadowgraphs provide a striking demonstration of the way in which such waves can break on a sloping boundary.

Stigebrandt also presents a mixing model in which mixing is assumed to occur along a boundary layer below the pycnocline, caused by breaking effects, presumably along the lines indicated by CACCHIONE and WUNSCH (1974). Horizontal exchange occurs with fluid in the interior, through density current exchanges. If all the energy propagating into the fjord is in the form of internal waves and it is all converted to turbulent energy in the lower layer, the flux Richardson number can be written as

$$R_f = \int N^2 \frac{dV}{\Sigma \omega \epsilon_l(\omega)}$$

where $K(z)$ is the vertical exchange coefficient, $N$ is the buoyancy frequency and $\Sigma \omega \epsilon_l(\omega)$ is the energy flux summed over each harmonic component. An estimate of $K(z)$ can be made from the slow changes in salinity in the deeper water using the one-dimensional diffusion equation (GADE, 1970), allowing calculation of $R_f$ for given values of $\Sigma \omega \epsilon_l(\omega)$. Stigebrandt used his two-layer internal tidal model to calculate energy fluxes into the Oslofjord. Together with GADE's (1970) estimates of $K(z)$ he found $R_f \approx 0.05$. Unambiguous measurements of the breaking of internal waves, especially internal tides, have not yet been obtained. But it seems likely that in fjords in which internal tides are a prominent oceanographic feature, the mechanism described in Stigebrandt could be an important one for vertical mixing in the deep basins. STIGEBRANDT (1980) has interpreted results of dye studies that indicate much lower vertical diffusivities than implied by long term salt flux observations, as evidence of localized and perhaps intermittent mixing. The breaking of internal tides on sloping boundaries is one candidate for this process.

STIGEBRANDT (1976) derived expressions for the energy flux of a purely progressive long internal wave propagating into a fjord due to tidal forcing over a sill. This flux is proportional to the frequency squared, so that for a given barotropic tidal amplitude the energy flux would be four times as great for a semi-diurnal as for a diurnal tide. PERKIN and LEWIS (1978) studied mixing problems in an Arctic fjord forced at a period of 1.6 hours and directly measured velocity fluctuations near the sill thought to be associated with breaking of the waves. Although the amplitude of the 1.6 hour surface forcing was much smaller than that for the M2 tide, calculations showed that its contribution to the energy budget was at least as great. They report mixing down the slopes of the bay to the bottom, although the maximum depth at which temperature and salinity changes were detected in this case was only about 50 m.

A quite different example of mixing by internal tides, this time from a fjord in Greenland, has been presented by LEWIS and PERKIN (1982). During the winter, salt excluded from the
The physical oceanography of fjords

The growing ice sheet produces a convective layer that slowly thickens. The gradual change in stratification produces a corresponding change in the natural frequency of the basin. At a certain point this frequency matches that of the $M_2$ forcing and resonance occurs, with a rapid increase in the amplitude of the internal tide, presumably accompanied by increased turbulence and mixing. Figure 15 illustrates this effect on measured currents and salinity near the sill; the condition only lasts for about 7 days but apparently is responsible for active mixing over a large portion of the water column. We know of no other reported example of internal resonant forcing of this kind, but it would seem a likely condition in short fjords subject to a wide range of stratification. Analysis of seasonal data could include a simple check for resonance by calculation of the internal seiche frequency as a function of the time of year. Because of the large signal associated with resonant effects, they should be easy to identify.

We have already discussed (Section 4.1.2) the way in which differential mixing between spring and neap tides leads to a purely internal MSf tide. Unfortunately there exists at present no theory to relate the amplitude of the MSf tide to the amount of mixing, but it is clear that the detection of this tide can serve as a sensitive probe for the existence of such mixing.

5.2.2. Sill processes. The periodic forcing of a fjord by tidal flows provides a source of kinetic energy for mixing. However, since it is a characteristic of fjords that they are deep, the tidal currents are normally quite small except near constrictions. The kinetic energy associated with
FIG. 16. Above: Depth of Knight Inlet as a function of distance from the head. Below: Depth mean velocity squared, proportional to kinetic energy due to tidal forcing, subject to zero phase change along the channel, calculated according to equation (5.6). The calculation takes account of the cross-sectional area rather than the depth alone, but the figure serves to emphasize the importance of shallow sills, when they are well removed from the head, as locations for strong tidal mixing.

the tide is of course proportional to the square of the velocity. As an example we plot in Fig. 16 the depth mean velocity squared $u^2$ as a function of distance $x$ from the head of Knight Inlet, defined by

$$u^2(x) = \left( A^{-1} \frac{d}{dt} \int_0^x B \, dx' \right)^2$$

where $A(x)$ and $B(x)$ are the local cross-sectional area and channel breadth respectively and $\xi$ the tidal elevation. The figure clearly emphasizes the importance of flows in the vicinity of constrictions as a source of mixing energy.

The kind of interaction that results depends upon the length and shape of the constriction and the ratio of kinetic to potential energy production, represented by the Froude Number squared $F^2$. Over sills that are long relative to the tidal excursion, we expect mixing to be much more extensive than over short sills. On the other hand short sills may be effective at redistributing tidal energy over a large area through the generation of internal waves. In order to discuss the significance of tidal interaction in different topographic situations, it is necessary to explore specific examples. For this purpose we define an internal Froude Number

$$F_i = \frac{u}{c_i}$$

where $c_i$ is the phase speed of a long internal wave of mode $i$ at the sill crest. This choice is somewhat arbitrary, since $c_i$ might also have been calculated for the total fluid depth. The motivation for the choice is related to the potential of a sill to block flow in deeper water, (cf. FARMER and SMITH, 1980a.) Supercritical conditions, for which $F_i \gg 1$, imply that
The physical oceanography of fjords

Fig. 17. Schematic diagram of strong tidal mixing over a sill that is long relative to the tidal excursion and which separates denser oceanic water from less dense water in the fjord. The water is thoroughly mixed over the sill, producing a density current that periodically runs down the inner face of the sill into the deep basin of the fjord.

The kinetic energy exceeds potential energy of the baroclinic field, thus inhibiting the development of wave-like behaviour of mode 1. No wave-like behaviour of any mode is possible for $F_1 > 1$ and the density field behaves like a passive contaminant, but as discussed below, intermediate cases where $F_{i+1} > 1 > F_i$ can also occur. Even over sills for which the external Froude Number is very small, as in the case of Knight Inlet, internal flows that are critical or supercritical may readily occur. At lower Froude Numbers ($F_1 \leq 0.3$) we expect internal tides to be generated as discussed in Section 4.1.2 and 5.2.1, while for high Froude Numbers ($F_1 > 1$) the special barotropic effects were covered in 4.1.1.

Mixing is a consequence of turbulence, the generation of which is an expression of vorticity production. In the case of tidal flow through a long channel, vorticity will arise from boundary shear along the sides and bottom. Topographic variability can cause a redistribution of the vorticity into the interior of the fluid and bottom roughness can enhance the vertical extent over which it occurs. Free vortices can also be generated over bottom irregularities if not suppressed by stratification, through separation of the boundary layer.

The processes dominating over short and over long sills differ considerably. GEYER and CANNON (1982) describe observations over the sill in Puget Sound. In this case the sill length is 30 km, so that the tidal prism will not displace the water over the sill completely and there is mixing of estuarine and ocean water at all phases. During strong tidal flow the Froude Number lies in the range $2 \leq F_1 \leq 4$ so that internal waves are not generated over the sill, but there is intense mixing. The mixing is so strong that although denser water exists outside the sill, it cannot pass unmixed into the sound. Vertical salinity differences within Puget Sound amount to only $1\%$. However the interaction of tidally induced mixing with the mean flow results in a 14.7 day modulation of the estuarine circulation; in effect, strong mixing during spring tides inhibits gravitational flow. This is one example of the MSF forcing described in Section 4.1.2. Estimates of the vertical diffusivity of momentum show an increase by a factor of two between neap and spring tides and there are also corresponding decreases in the bulk Richardson Number. This is an extreme example which serves to emphasize the effectiveness with which tidal mixing over a long sill can influence the oceanography.

Strong mixing over long sills may produce water of density between that found at the surface and that found at depth some distance away from the sill, Fig. 17. MAXWORTHY (1979) pointed out that such pulses of mixed water could evolve into internal undular bores. However in many practical situations, including the case analyzed by GEYER and CANNON (1982), the mixed water can be of higher density than the deep water within the fjord, so that tidal pulses of well mixed water run down the inside slope of the sill as gravity currents which may be detectable for some distance into the fjord.
Over much shorter sills in which the tidal excursion far exceeds the sill length, mixing will be more sporadic. A satisfactory description must include the time dependent behaviour, which imposes considerable demands on both the experimental and theoretical approach. Since the existence of strong tidal currents over short sills appears to be an important feature of many fjords subject to large tides, especially along the NW coast of North America, we will discuss several features of the resulting flow together with some recent field observations. We also refer here to an excellent review of topographic effects in stratified fluids by Huppert (1980).

Such flows have an important analogy with the flow of the atmosphere over mountains. Early work with the steady linearized equations, subject to the assumption that the flow does not separate, demonstrate the existence of lee waves with wavelength $2\pi u/N$ as part of the response in an infinitely deep fluid with constant stratification $N$. The problem of tidally driven stratified flow over a fjord sill differs in several ways from the mountain lee wave analogy, although lee waves are certainly one of the features of such flows. First, the flow being tidally forced cannot be considered stationary, although under certain conditions 'quasi-steady' conditions may prevail near the sill. Second, the fluid is not infinitely deep, nor in many cases is it even deep relative to the sill height, so that the upper boundary reflects energy propagating upwards from the sill crest. Third, in most cases the shape and relative size of the sill is such as to violate the linearity assumption: finite amplitude theory must be invoked. Observations of such flows in fjords, to be described later, exhibit a broad range of responses that appear sensitively dependent upon the sill shape, which is often asymmetric, the vertical density stratification and the strength of the tidal forcing. Boundary layer separation also appears to be a prominent feature of observed sill flows.

The linearized problem of lee waves generated by periodic or transient forcing has been studied by Bell (1975) and Lee (1972); Baines (1979) extensively tested the linear theory in the laboratory. Two limiting cases of Bell's (1975) theory are the quasi-static limit, for which $\omega/N \ll 1$ and the 'acoustic' limit, for which $\omega \gg U_T/L_\eta$, where $\omega$ is the tidal frequency and $L_\eta$ is a scale length for the sill. While the linearization limits the applicability of this theory to real flows in fjords, it can provide insight into several aspects of lee wave response. Bell (1975) showed that many different harmonics could be generated, the amplitude of each component depending upon the corresponding spatial derivatives defining the obstacle shape. Lee's (1972) analysis of the quasi-static limit in a fluid of finite depth demonstrated the movement of the wave energy peak back and forth across the sill within the range of the tidal excursion.

The existence of an upper boundary limits the conditions under which lee waves may develop. Specifically, waves can only maintain their position with respect to the sill if the flow is subcritical. For constant stratification $N$ and speed $U$, the requirement for lee waves of mode $i$ to exist is

$$U^2 < \frac{N^2 H^2}{L^2 \pi^2}.$$  

Thus the flow may be subcritical with respect to one or more modes, but critical or supercritical with respect to others. A change in stratification or the transition between spring and neap tides can alter the modal structure of the lee wave response. For supercritical flows over small obstacles Baines (1979) showed that the linear theory worked well and that the predicted flows were reproduced in the laboratory model. For subcritical flows there is a parameter range for which long columnar disturbances are found which are not predicted by linear theory. As critical conditions are approached, the time dependent linear theory predicts that the time taken for the flow to reach steady state near the obstacle increases without limit.
The wavelength $\lambda$ of the lee waves is also dependent on the fluid depth, tidal velocity and stratification. For the simple example of constant stratification $N$,

$$\lambda = 2HF_i(1 - F_i^2)^{-1/2}$$  \hspace{1cm} (5.9)

Thus lee waves of given mode have a wavelength that increases with flow speed: the wavelength is modulated by the slowly varying tidal current. This feature appears to play a central role in the process of boundary layer separation over sills, discussed later. Moreover, since lee waves are stationary with respect to the bottom, wave energy propagates downstream at a speed $C_E$ equal to the difference between the phase speed $C_p$ and the group velocity $C_g$, which is always less than $C_p$. For constant stratification $N$,

$$C_E = C_p - C_g = U(1 - F_i^2).$$  \hspace{1cm} (5.10)

With increasing flow speed, the wave energy propagates away from the sill more slowly, resulting in an accumulation of energy close to the area of generation. As the wave amplitude increases, the linear theory breaks down, but the relationship (5.10) suggests a natural transition between a train of small amplitude lee waves at low flow speed and a large breaking lee wave that occurs as critical conditions ($F_i \rightarrow 1$) are approached and the wave energy is concentrated close to the sill.

The assumption used in deriving the linearized equations are violated for flow over large obstacles. Two quite different approaches have been used to investigate such flows. For the case of layered stratification, the flow has been treated within the framework of internal hydraulics for which the hydrostatic approximation is assumed. Continuously stratified flows over finite obstacles have been studied through a transformation of the two-dimensional nonlinear equations which under certain conditions can be reduced to the linear Helmholtz equation. Each of these approaches sheds some light on observed flows over fjord sills. The hydrostatic approximation will only be valid over sills that are long so that vertical accelerations can be neglected.

The theory of steady 2-layer flow yields solutions for several different types of response which depend on the numbers $F_1$, $F_2$ for each layer defined in Section 2 (see LONG 1954, 1970, 1972, 1974; HOUGHTON and KASAHARA 1968; ARMI 1974). If a transition between super-critical and sub-critical conditions occurs a hydraulic jump is expected downstream of the obstacle and evidence of such jumps has been observed near the sills of tidally forced fjords as discussed below. In the steady solution for which critical conditions occur over the sill crest, there must be a bore that travels upstream away from the crest; thus the existence of the obstacle in the flow controls the upstream conditions required for steady flow over the crest. In fjords the problem is somewhat different in that the flow is modulated by the ebb and flood of the tide and the slow increase in velocity may not result in an upstream jump. However FARMER and SMITH (1980a) observed an undular bore propagating upstream from the sill in Knight Inlet which would be consistent with this interpretation. The bore was associated with a net increase in surface layer thickness and it was observed to travel upstream from the sill just before maximum ebb. A similar effect is seen in Fig. 6 of FARMER and SMITH (1980b).

Within the jump itself turbulence is intense and entrainment and mixing occurs. As yet there have been few attempts to measure the amount of mixing that takes place, but it is clear from laboratory measurements (WILKINSON and WOOD, 1971; MACAGNO and MACAGNO, 1975) that hydraulic jumps are associated with high entrainment rates.

The theory of hydraulic flows and transitions has been extended to account for shear at the interface and more general upstream conditions (ARMI 1974). SU (1976) and LEE and SU
considered the problem of multiple layers as an approach to representing internal hydraulic jumps in continuously stratified fluids and showed that for an exponential density stratification jumps could only occur over an extremely narrow range of flow speeds. However, as pointed out by YIH (1980), it is questionable whether such multiple layer models which specifically exclude turbulence and mixing, can adequately represent hydraulic jumps in real flows.

Attempts to compare solutions of LEE and SU's (1977) model with flows observed in nature have proved useful under certain restricted conditions (GARDNER and SMITH, 1980). Experience shows however (J. SMITH, personal communication) that while the nonlinear model reproduces the initial distortion of the flow quite well, the inviscid assumption prevents the modelling of realistic flow speeds on the lee face of the obstacle. In effect, failure to account for drag on the bottom, as well as mass and momentum exchange between layers, leads to unrealistically high velocities. As with most of the flows in this section, the dynamics of the inviscid solutions are understood fairly well; the next step, namely the incorporation of a satisfactory representation of mixing processes into these simpler models, guided by appropriate field observations, has yet to be taken.

The theory of continuously stratified flow over obstacles of finite amplitude is largely based on an exact first integral of the equations of motion derived by DUBREIL-JACOTIN (1937) and LONG (1953) which becomes linear for special choices of stratification and velocity. An unfortunate limitation of this approach is that the flow is only specified along streamlines that originate upstream and there are also difficulties associated with the problem of upstream influence (BAINES, 1977). Nevertheless the theory appears to describe many features of observed flows including the formation of jets and eddies, which LONG (1955) demonstrated using a series of laboratory and theoretical comparisons. Although the form of the differential equation is determined upstream and thus the flow is not defined within closed streamlines, Long's solutions still bear remarkable fidelity to the laboratory observations of rotors and eddies. YIH (1980) questions whether ruling out the validity of solutions with closed streamlines may not entail the sacrifice of solutions for more interesting flows. An alternative approach to modelling stratified flow around finite amplitude obstacles, which avoids the problem of upstream influence encountered in Long's model, is obtained through use of the inviscid unsteady Oseen equations (JANOWITZ, 1968, 1981).

An important series of experiments together with a linear analysis, carried out by BRIGHTON (1977), demonstrate the role played by lee waves in controlling the separation of the boundary layer from the obstacle. This work has been complemented by SYKES' (1978) nonlinear calculations of flow over a ridge, which accurately predict the suppression and onset of separation for a range of Froude Numbers, and experiments carried out on three dimensional hills both by BRIGHTON (1977) and HUNT and SNYDER (1980).

The lee waves can either promote or suppress separation depending upon the ratio of wavelength to the half length of the sill. If \( \lambda/2 \) is comparable to the length of the obstacle's lee slope, the downslope acceleration will suppress separation while promoting it downstream. However, shorter wavelengths lead to a retreat of the separation point so that it can occur at some mid-point on the lee slope. For large Froude Numbers, separation is controlled by the boundary layer flow; the wave length of the lee waves, if they occur at all, is too large to suppress the separation. These ideas have been extended to the study of boundary layer separation in hydraulic flow (HUPPERT, 1980).

Observations of the complex processes of tidal interaction with topography is made difficult by the range of time and space scales at which they occur. For example a steep sill may produce
significant distortion of the flow at horizontal scales of a few hundred metres or less, while the full length of the sill may be several km and the far field manifestation of the interaction several tens of km. Moreover the time scale for tidal interaction is at most a half tidal period (~ 6 hours). This means that a lot of territory must be covered in a relatively short period if the processes are to be properly resolved. While the periodic nature of the tide might be expected to allow repeated measurements of the same process at the same place but at different phases for different tidal cycles, experience has shown that the interaction can be sensitively dependent upon tidal amplitude and stratification so that periodic repeatability is not assured.

All of these considerations point to the necessity of using a remote sensing technique, in which the flow field can be rapidly scanned, in combination with more traditional measurements. We have recently exploited acoustic techniques for this purpose. Although the potential for this type of remote sensing remains to be fully developed, early results have proved encouraging and we shall use some of these to illustrate the kind of processes described above. The observations were made in two fjords, Knight Inlet and Observatory Inlet on the British Columbia mainland. Both have short sills and are subject to large tides but otherwise have many oceanographic differences. For example in Knight Inlet the fresh water enters the fjord at the head while in Observatory it enters from the side, down-inlet of the sill. As might be expected there are some differences in the response to tidal forcing in each inlet, but there is also a striking similarity in the general structure of the interaction as well as in the range of different types of response. This tends to support our contention that the processes we describe are quite general for fjords with short sills subject to significant tidal action.

The simplest and one of the most effective techniques for remotely sensing perturbations to the flow is high frequency echo-sounding. An echo-sounder operating at 50–200 kHz detects sound reflected mainly from biological sources, such as zooplankton and also from microstructure in the sound speed profile due to temperature and salinity gradients. Since the ocean tends to be vertically stratified, microstructure and also biological scatterers tend to be aligned in nearly flat horizontal planes. When the flow is distorted, for example by internal waves or by flow over an obstacle, these distortions appear on the acoustic trace. Disorganized motion or turbulence also has a characteristic acoustic signature, giving rise to irregular cloud-like features. Figure 18(a) shows an acoustic trace obtained from a ship travelling over the sill in Observatory Inlet. The tidal flow is from left to right in the image and a train of lee waves is seen just down-inlet of the sill crest. The waves are visible not only because of gross variations in acoustic target strength, but also as a result of the slope of traces derived from individual targets. Moreover the length of time an individual target remains detectable on the sounder is dependent on its speed relative to the vessel, which provides a qualitative indication of flow speed.

The lee waves visible in Fig. 18(a) are a relatively tranquil response to flow over the sill during an early part of a tidal cycle. Calculations of the wavelength of lee waves using (5.9) are consistent with wavelengths estimated from the acoustic images. As the flow speed increases however, the response becomes more energetic [Figure 18(b)] and bears little similarity to the solutions of linear theory. The train of lee waves has now been replaced by a single large lee wave and there is evidence of a shear zone, presumably associated with boundary layer separation. Within the limitations of linear theory we interpret this transition as that implied by (5.10) as $F_l \rightarrow 1$. Acoustic images of this sort cannot in themselves provide a measurement of turbulence, but the distorted structure of the acoustic signal above the wave trough is certainly suggestive of unstable flow. The third image in Fig. 18(c) is made over the same sill during a flood tide, when the different shape of the lee face appears to induce a quite different response.
FIG. 18(a). Echo sounding made by a ship travelling seaward over the sill in Observatory Inlet during the early stages of an ebb tide. The sound is scattered by microstructure, bubbles and zooplankton, clearly indicating the wave-like character of the internal response.

FIG. 18(b). Image of lee wave response during maximum ebb, made approximately one hour later than Fig. 18(a). The lee wave train has coalesced to form a single breaking lee wave.

FIG. 18(c). Supercritical flow during a flood tide. Separation of the flow does not occur within the range of observation (110 m), due to the gentler slope on the landward face of the sill. Note the numerous instabilities at the interface, generated by the strongly sheared flow.
The sill is less steep on the landward side and separation of the flow is apparently suppressed. The different shape of the sill is partly a consequence of higher ship speed and thus a more compressed horizontal scale and partly due to the measurement being taken along a different section of the sill (slightly to the north of the other sections). In this example the water driven over the crest runs down the lee face of the sill as a super-critical flow. Instabilities on the interface between the super-critical and sub-critical flows are clearly visible. These instabilities are one manifestation of the mass and momentum transfers between layers that limit the applicability of inviscid nonlinear models to sill flows.

As the flow separates from the bottom just beneath the trough of a lee wave it spreads vertically as a rapidly growing mixing layer that entrains fluid from above and beneath. Thus mixing takes place in two quite different ways: through slow overturning motions in the trough of the lee wave and through entrainment in the shear layer associated with flow separation. We can probe the structure of these different zones by lowering an instrument to the appropriate depth and drifting slowly over the lee wave. Figure 19 shows the result of such an experiment using a CTD and a 3-axis ultra-sonic current meter. The current meter data has been corrected for ship drift using microwave positioning data. This example is taken from measurements made in Knight Inlet during an ebb tide. The simultaneous acoustic image aids in the interpretation of the current meter and CTD traces. Since the acoustic reflection of the instrument can also be detected, the measurements can be precisely related to the acoustic image of the flow field.

It is clear from both the acoustic image and the salinity trace in Fig. 19 that the instrument passes through at least two lee waves. Between the first two wave troughs it also briefly passes through the upper portion of the deep mixing layer that originates from the flow separation. Downstream of the second lee wave the mixing layer and the disorganized structure within the lee wave merge. The deformation of the flow is emphasized both by the salinity excursions and velocity changes. When the instrument passes through the trough of the first wave the salinity observed at 51 m is equal to that at 11 m just upstream of the wave. Moreover the salinity signal does not fluctuate much as the probe drifts through this portion of the wave, implying that the water is well mixed. Vertical velocities of up to 38 cm s⁻¹ occur. Both within the disorganized flow in the core of the lee wave and also in the deeper shear layer, velocities are lower than in the acoustically 'clear' jet that separates these two layers. It appears that this high speed jet acts as a source of kinetic energy for the production of turbulence above and beneath. As the two mixing layers merge there is a noticeable increase in variance of the observed salinity and velocity signal.

Figure 19 also illustrates the way in which the salinity profile changes across the sill. Both the downcast, made above the sill crest and the upcast, made at the end of the pass are plotted together with the axial component of the velocity measured during the downcast. The surface layer is appreciably less saline (above 11 m) and the deeper water is fresher downstream of the sill, consistent with our interpretation of mixing within the lee waves. Such measurements do not allow direct calculation of the amount of mixing that is taking place, but they do provide a basis for carefully scaled laboratory models of the flow which will permit quantitative estimates. Such modelling efforts are now underway (G. LAWRENCE, U. of California, personal communication).

We have already seen how flows over sills can take the form of lee waves or hydraulic jumps. Subtle differences in density structure can also determine whether the response is in the first or second mode. In Knight Inlet a sharp pycnocline in summer inhibits development of mode 1 responses except during very strong tides. In effect the surface layer acts as a flexible lid.
Distance along Knight Inlet in metres, arbitrary origin.

FIG. 19. Salinity and velocity time series obtained simultaneously with an echo-sounding of a lee wave on the seaward side of the Knight Inlet sill during an ebb tide on July 25, 1979, 0800h. The overlay is positioned such that the echo-sounding and the salinity and velocity data have a common time base. The echo-sounding shows the path of the instrument that was lowered and maintained at fixed depth (51 m) during passage over the sill. Only along-inlet \((u)\) and vertical \((w)\) components of velocity are shown although significant cross-channel components were observed. Microwave positioning data was used to correct the speed measurements for ship motion, which varied considerably during the period of drift, and to provide an appropriate horizontal scale. Vertical profiles of along-inlet speed and of salinity obtained at the start of the pass, are shown at upper left, together with the upcast salinity obtained at the end of the pass. The symbol \(\circ\) identifies the salinity subsequently observed at a depth of 51 m as the instrument passed through the trough of the first lee wave. The upcast salinity profile shows an elevated salinity near the surface and a depressed salinity at depth relative to the earlier profile, indicative of vertical mixing behind the sill.
beneath which most of the streamline deformation and mixing occurs. Moreover the pycnocline affects the way mixed water spreads away from the sill and thus alters the kind of circulation that can occur. FARMER and SMITH (1980) show how the model response is related to an appropriate internal Froude Number. The range of different flows over sills appears to be limited only by the range of stratification, of tidal currents and of sill shapes that occur, but in Fig. 20 we present a highly idealized sketch of a few of these processes. In Fig. 20(A), the current has generated a train of mode 1 lee waves. Upstream of the sill there is some blocking (b) of the deeper flow (i.e. $U^2 < N^2 A^2$ where $A$ is the sill height). The boundary layer also separates at S. As the tidal current slackens the lee waves move back upstream (ii), (iii) against the current, evolving into a train of nonlinear progressive internal waves.

In Fig. 20(B), we represent a slightly different version of the preceding events that might obtain at higher stratification and higher tidal current speeds. In this case the flow is marginally subcritical with respect to mode 1, but critical or super-critical with respect to mode 2. Initially (i) the flow separates at the crest, the shear layer spreading vertically and ultimately collapsing along the lines modelled in the laboratory by KOOP and BROWAND (1979). In frame (ii), mode 2 lee waves have evolved and the point of boundary layer suppression (S) has moved down the lee face of the sill. However a small mode 1 internal disturbance or a train of nonlinear internal waves is advancing upstream (a). This is a manifestation of upstream influence discussed earlier and clearly identifiable in Fig. 13 of FARMER and SMITH (1980a) and in Fig. 6 of FARMER and SMITH (1980b). The theory of time dependent flow of stratified fluid over finite obstacles has yet to be developed, but in the absence of such a theory it appears reasonable that upstream adjustment for a mode 2 response can occur by way of a mode 1 disturbance. The mode 1 adjustment can readily travel back through the critical flow over the sill crest by virtue of its higher phase speed. Frame (iii) shows the energy concentrated in a large breaking lee wave or internal hydraulic jump in which intense mixing occurs. The disturbance collapses (iv) and spreads out as an undular bore of mode 2 in frame (v) which travels away from the sill behind the mode 1 disturbance generated earlier. A small wave (c) moves off in the opposite direction, as observed in the laboratory experiment of MAXWORTHY (1979).

Figure 20(C) represents the evolution of an internal bore from flow over a somewhat longer sill, where mixing produces a large mass of water having density lying between that of the two layers in the surrounding area. As the tide slackens (iii) the mixed fluid collapses, forming an undular bore that spreads out mainly to the left in the sketch as the tide turns, but also including a secondary disturbance (c) similar to that observed in Fig. 20(B) frame (v). These sketches are not intended to be all inclusive, but they are included here to provide an indication of the range of phenomena that can be expected due to tidal forcing over a sill.

The broad range of phenomena that can occur due to tidal flows over sills is a consequence of the range of different scales that are relevant, both in the geometry of the channel and in the structure of the flow. The processes isolated above, including lee waves and jumps, blocking, boundary layer separation as well as the spread and collapse of turbulent mixing layers shed from the sill crest, have all been observed in Knight Inlet. However it seems likely that accurate parameterization of the flow and of the mixing processes that result, will most readily arise from well controlled laboratory studies. Figure 21 illustrates two such laboratory runs designed to shed light on the details of tidal interaction with a sill.

5.2.3. Nonlinear internal wave trains. We have described some of the quasi-steady responses to high Froude Number tidal forcing over a sill and have shown how the slowly changing background flow can give rise to trains of finite amplitude internal waves that travel away from
FIG. 20(A). Schematic diagram of a mode 1 lee wave response to tidally forced flow over a sill. Upstream of the sill there is blocking (b) of the deep flow and separation (s) of the boundary layer beneath the trough of the lee wave downstream of the sill. As the tidal current slackens the lee waves are released successively to form a nonlinear wavetrain that travels over the sill crest and back along the fjord.

FIG. 20(B). Schematic representation of flow that is subcritical with respect to mode 1, but critical with respect to mode 2. In the early stages of the tidal flow the boundary layer separates from the sill crest, but subsequently separation is suppressed as mode 2 lee waves evolve. The upstream flow is modified by a small mode 1 internal wave that travels up-inlet away from the sill. The growing lee wave or jump eventually collapses to spread out as an undular bore as the flow slackens. A small wave (c) travels downstream. All of these features have been observed in Knight Inlet, B.C.

FIG. 20(C). Sketch of tidal mixing over a longer sill leading to collapse of water of intermediate density. The mixed water spreads outwards as an undular bore between the two layers from which it was formed. A small disturbance (c) travels in the downstream direction.
The physical oceanography of fjords

FIG. 21(a). Photographs taken at successive intervals during a laboratory tank study of harmonically forced stratified flow over an obstacle. Although the obstacle is moved relative to the tank, the photographs are positioned so that their frame of reference is fixed with respect to the obstacle as the flow moves through 270° of a 'tidal' cycle. The surface layer is fresh and dyed black, the deep layer is saline and clear. Shortly after starting from rest the flow over the sill, indicated by a horizontal arrow, induces a lee wave (W1). By frame 5 a second trough (W2) has evolved, though less regular than the first. As the flow slackens the waves move over the crest and travel at a steady speed up the channel. With the change in flow direction a new lee wave (W3) forms on the left hand side of the sill. Not directly visible in this series is the separation of the boundary layer from the sharp crest of the sill. The separated flow evolves as a spreading sequence of vortices that interacts with the surface layer and accounts for the irregular structure of the interface at W2 in frames 5–8. (Laboratory experiment by D. FARMER and J. ZELT.)
FIG. 21(b). Shadowgraph images of steady two-layer stratified flow over a sill in a towing tank at two different Froude Numbers. In these examples, the obstacle is less steep than in the experiment of Fig. 20(A), and the flow does not separate from the crest. The vertical scale is in cm, the reduced gravity is 14 cm s$^{-2}$ in each case and the two speeds 3.8 and 6.3 cm s$^{-1}$ respectively for the upper and lower figures. These dimensionless Froude Numbers correspond to densimetric Froude Numbers calculated $upstream$ of the obstacle using the undisturbed interfacial ($h_2$) and total fluid ($h_1 + h_2$) depths, i.e. $F_r = \frac{\bar{u}}{[g'\rho_1 h_1/(\rho_1 - \rho_2)]^{1/2}}$, of 0.48 and 0.89 respectively. Turbulence and mixing occur in a roller region downstream of the crest. At the higher Froude Number, mixing below the interface occurs earlier, just downstream of the crest and larger instabilities appear on the interface below the roller region. The density structure can be determined at fixed depth as a function of position relative to the sill, or by vertical profiling using the conductivity probe to the left of the obstacle, in a way quite similar to that used in field experiments as shown in Fig. 17. (Laboratory experiment by R. DENTON, D. FARMER and G. LAWRENCE.)
the sill at each slack tide. The evolution of cnoidal or solitary wave trains from given initial disturbances has recently received much attention both as a theoretical entity and because of observations in the atmosphere and ocean. For this reason their generation in fjords is interesting not only from the point of view of fjord oceanography but also because solitary waves in fjords are well suited to detailed observational study.

Comparisons suggest that the internal waves exhibit several of the features predicted by the nonlinear theories. An essential feature of these theories, which combine linear dispersion with a nonlinear convection operator, is the prediction of waves of permanent form. Subtle differences in their shape and in their dependence of speed on amplitude, arise from the relative scales of stratification and fluid depth. JOSEPH (1977) has shown that the streamline displacement in fluid of finite depth $h$ has a horizontal dependence of the form $[\cosh^2(x/h) + (h/\lambda)^2 \sinh^2(x/h)]^{-1}$. In the fairly typical case of solitary internal waves in fjords that have a thin, highly stratified layer close to the surface, waves can occur for which the scale length $\lambda$ is long relative to the surface layer, but short relative to the depth of water below. In this limiting case, BENJAMIN (1967) found the wave shape to be $f(x) = \lambda^2(x^2 + \lambda^2)^{-1}$.

Comparisons with observed wave shapes (FARMER and SMITH 1978) show that the streamline displacement predicted by the theory leads to a somewhat narrower wave for a given amplitude. However, there are many features of the waves that serve to limit the validity of the theory. The waves typically possess an amplitude comparable to the stratification depth so that it is remarkable that the "small" parameter expansion of the theory works as well as it does. Recent theoretical developments (R. GRIMSHAW, personal communication) can account for some of the discrepancy between observed and theoretical shapes using higher order expansions, but there are other features of the observations that defy analytical description including the occurrence of turbulence and mixing. Attempts to measure directly the turbulence within the waves have been made using a small submersible, a preliminary account of which is given by GARGETT (1980). Figure 22 shows one set of measurements made by Gargett as the vessel approached the waves from behind at a depth of approximately 20 m. Bursts of signal in the plots of temperature and velocity fluctuations within the troughs of the waves indicate energy containing scales within the dissipation range, in sharp contrast to the quiescent flow ahead of the first wave. Data from the pitch sensor provides additional evidence of larger scale motions affecting the vessel including a particularly noticeable downwelling observed at the leading edge of the first wave.

The waves can also be observed using the acoustic technique described earlier (cf. FARMER and SMITH, 1980a). Simultaneous measurements with an acoustic velocity sensor and CTD, demonstrate the large downwards component of mean motion, about 0.5 m s$^{-1}$, near the leading edge of the wave, consistent with Gargett’s observations from the submersible. Since the turbulent core persists as far as the waves can be detected, the turbulence must draw its energy from the wave itself, presumably from shear flow instabilities associated with the deformation of streamlines near the leading edge of the wave and perhaps also from convective instabilities in the turbulent core.

High frequency non-linear internal waves trapped on the pycnocline overlying weakly stratified water may also radiate energy down into the water column. In discussing these waves PEREIRA and REDEKOPP (1980) point out that the waves will only be ducted along the thermocline if the ratio of the amplitude to the pycnocline thickness $(a/h)$ exceeds the ratio of the local and deep buoyancy frequencies $(N_0/N_\infty)$:

$$a/h \geq 0(N_\infty/N_0).$$  \hspace{1cm} (5.11)
This ducting, which is clearly observed in fjords, is thus a finite amplitude effect, since although $N$ becomes small at depth (Fig. 3) it does not vanish. It is a consequence of this effect that energy is radiated downwards into the weakly stratified water column in the form of internal waves; thus we must consider the possibility of this energy also being made available for mixing.

In contrast to the example of internal tides discussed in Section 5.2.1, the characteristic slope $\theta = \sin^{-1} \omega/N$ along which energy is radiated will now be very much greater and must be vertical at depths for which $\omega = N$. The detailed mechanism of radiation under these conditions, which covers a range of wavenumbers with a low frequency cut-off related to (5.11), is discussed by MASLOWE a. REDEKOPP (1980), but it is clear that for the internal solitary waves commonly observed in strongly stratified fjords such as Knight Inlet, which in summer pass a fixed point in 2–3 minutes and are spaced at intervals of 6–10 minutes, total internal reflection of the energy can be expected within a few tens of metres depth below the pycnocline. Intense boundary mixing is possible, as indicated in Section 5.2.1, for critical bottom slopes identical to the local value of $\theta$. Thus especially near the sides of the fjord or in shallower areas, this mechanism may allow mixing well below the pycnocline, in addition to the mixing already noted in the turbulent core of the waves.

While estimates of the amount of mixing caused by the waves are still uncertain, there is little question that they can strongly influence the water structure in fjords. Density profiles before and after passage of a wave, typically show a substantial increase in potential energy of
the water column, only part of which can be attributed to advection. The mixing is dependent on the modal structure of the internal wave. In the case of mode 2 waves, most mixing occurs just beneath the pycnocline. For mode 1 waves which include the example presented by Gargett (Fig. 22), the turbulence occurs around and above the pycnocline. Thus we anticipate quite different dynamical consequences of this mixing, depending upon the mode of the waves. It seems probable that mode 1 waves enhance the entrainment of salt water into the surface layer, much as wind mixing does. However the bore-like structure of these waves also implies a net deepening of the surface layer following passage of the waves. An acoustic image of the wave train that has departed from the sill in Observatory Inlet (Fig. 23) shows this effect as a long depression of the pycnocline for a mode 1 response. In the mode 2 response, such as typically observed in Knight Inlet during summer, the water mixed by the ebb tide over the sill, advances back up the inlet just beneath the pycnocline, the leading edge of this intrusion appearing as a sequence of internal solitary waves. This suggests that the resulting circulation is much more complicated than the simple 2-layered prototype used by some of the theoretical models. An attempt to model some of the features of sill-induced mixing was made by LEE and HSUEH (1978), both theoretically and experimentally. The detailed structure of mixing over the sill was not specified, but the spread of mixed fluid and resulting recirculation may have some features in common with the corresponding fjord circulation problem.

5.3. Lateral variability; mixing in river plumes

In many fjords significant mixing takes place between recently discharged river water and the surrounding brackish layer. The river water spreads out as a plume that may mix laterally through incorporation of brackish water into unstable tongues or vortices and is often bounded by well defined density fronts. A distinctive feature of these fronts is the large velocity shear across them. MCCLIMANS (1979) reports entrainment through a river front equivalent to about 16% volume increase per unit river width along the plume. The weak return flow of mixed water below the plume shields it from direct contact with the fjord water beneath. The energy used in the mixing may be derived from kinetic energy associated with the river discharge itself or by tidal mixing over shoals.

Beyond the immediate vicinity of the river mouth fronts can also form under the influence of various mixing processes and rotation effects which, in the northern hemisphere tend to make the river plume hug the righthand shore. In calculating the effect of rotation MCCLIMANS (1979) argues that the relevant internal Rossby radius of deformation $R_i$ is based on the density difference between the river water and surrounding brackish layer rather than the density jump from fresh to sea-water. In narrow fjords transverse gradients are small but may be measurable (CAMERON, 1951). In a fjord that is wide with respect to $R_i$ a longitudinal front can form which may be subject to various forms of unstable flow (STERN, 1980). The extent to which mixing is induced along fronts can however be controlled by a constriction (MCCLIMANS, 1979).

In sinuous fjords where the radius of curvature is comparable to or less than the internal Rossby radius of deformation, lateral variations in the flow field can also be influenced by the centrifugal force (i.e. term $(x)$ in equation (3.1); see STEWART, 1957). These effects are dramatically visible in air photographs showing colour changes associated with river water. BUCKLEY (1977) found that in the upper reaches of Howe Sound the river was the cause of considerable spatial inhomogeneity in the flow although not of temporal variations. Except
Echo sounding image obtained in Observatory Inlet showing train of nonlinear internal waves travelling up-inlet away from the sill following their release at the end of an ebb tide. Deformation of the flow during the subsequent flood tide is apparent. The bore-like character of the waves is apparent from density profiles which show a net deepening of the surface layer after their passage.
for wide fjords however it appears that lateral mixing, whether by wind, tide or instabilities associated with the plume, is sufficiently vigorous to eliminate most transverse gradients further than about 10 km from the river mouth. The interaction of tide and topography may play a role in this mixing. Streams entering the fjord can sometimes be seen to form complex patterns as the tidally induced vortices shed from irregular shoreline features incorporate the fresh water into the brackish layer (Fig. 24).

5.4. **Thermal convection**

The effects of wind, tide and frontal processes are so energetic that it might appear unlikely that convective instability resulting from surface cooling or from double diffusion would be a significant mixing mechanism in fjords. However their importance in the deep ocean demands a closer look at their potential for modifying the properties of fjord waters.

Thermal convection must be expected whenever the surface heat flux is directed positively upwards. This balance includes the effects of latent and sensible heat flux and of radiant heat exchange. Downwelling radiant energy is of course a distributed source of heat, a fact of great biological importance, but probably of minor consequence to the present discussion. A strong net loss of heat can occur in winter; extreme examples will occur during the polar outbreaks with associated katabatic winds discussed earlier (see Section 5.1), but large surface heat exchange is also possible in calm winter conditions, especially at night under clear skies. The significance of convection depends upon the stratification and the existence of other mixing mechanisms such as wind stress.

Consider first the case of convective mixing in the absence of wind. This problem has been widely discussed with reference to the formation of a surface mixed layer in the ocean and
atmosphere (cf. NILLER and KRAUS, 1977; and TURNER, 1973). Convective mixing occurs over a progressively deepening mixed layer that penetrates into the stratified water beneath. The rate of advance of a convective layer subject to a surface buoyancy flux per unit area $B_0$, into water of uniform stratification $N$ is

$$\frac{dh}{dt} = \frac{1}{2}(rB_0)^{1/2}N^{-1}t^{-1/2},$$  \hspace{1cm} (5.12)$$

where $r$ has a value between 2 and 6 depending on the importance of dissipation. We take the dissipative case in what follows (i.e. $r = 2$). During periods of high run-off, the pycnocline effectively blocks the convective deepening, but the situation may be quite different in winter, with low run-off and intense cooling. For example in Knight Inlet it is not uncommon in winter to observe near surface density profiles having values of $N \approx 1 \times 10^{-2}$ s$^{-1}$ corresponding to a buoyancy period of about 10 minutes. For a typical winter heat flux of 150 Wm$^{-2}$, corresponding to a buoyancy flux of $1.61 \times 10^4$ Jm$^{-1}$ s$^{-1}$, the rate of advance after 1 day would be 5 m d$^{-1}$. Under these conditions convection could play a significant role in determining the water characteristics.

More often strong cooling is accompanied by wind mixing. Again the relative significance of thermal convection will be negligible during high run-off when the fresh-water supply inhibits the depth of wind mixing, but for deep mixing layers such as can occur during low run-off conditions, convection may dominate. The relative significance can be seen from the appropriate energy scaling [cf. equation (5.2)]. Using the same surface heat loss of 150 Wm$^{-2}$ and taking a typical cube-root-mean-cube-wind-speed of 5.3 m s$^{-1}$ for Knight Inlet (cf. FREELAND and FARMER, 1980), the resulting length scale is about 60 m. This value far exceeds the pycnocline depth during high run-off thus confirming the negligible role of heat exchange compared to wind effects during high run-off, but it is also significantly less than the total depth to which mixing is observed during periods of low run-off. For example, winter observations both in Alice Arm and in Knight Inlet, B.C., have shown mixing to within 0.02 ø from the surface down to a depth of 225 m. Under such thoroughly mixed conditions, convection may dominate. Once such a deep mixed layer has formed, even though convection may not have been responsible for its formation, it seems likely that thermal convection would serve to maintain the layer. This may also be true of tidal mixing (see Section 6).

5.5. Double-diffusive mixing

When the vertical gradient of both temperature and salinity have the same sign, one of the properties is unstably distributed in the sense of providing a component of the density profile that would be hydrostatically unstable in the absence of an opposing effect of the other property gradient. Differences in the molecular diffusion rates of salt and heat allow release of potential energy from the component that is unstably distributed, even though the mean distribution is hydrostatically stable. The kind of instability that results depends upon the sign of the gradients. In fjords the salinity almost always increases with depth, so that double-diffusive effects can be expected when the temperature increases with depth; this is a common condition in winter and spring and may exist at all times in deeper basins. The result of the different diffusion rates in this case is the formation of the ‘layered’ type of instability, which is distinguished from the salt-finger instability that can occur when temperature and salinity decrease with depth.

This topic has been extensively reviewed (cf. SHERMAN et al., 1978; HUPPERT and
The physical oceanography of fjords

Turner, 1981) and is the focus of current research both in the laboratory and in the ocean; we shall limit this discussion to a brief description of its relevance to fjord oceanography. In the layered type of convective instability a series of discrete layers of well mixed fluid will form, separated by sharp density gradients. The convection is driven by the larger vertical flux of heat versus salt through the sharp interfaces. The thickness of the convecting layers is determined by the viscosity, thermal diffusivities, buoyancy flux, initial density gradient and a critical Rayleigh number for the layer at which instability sets in (Turner, 1968; Huppert and Linden, 1979). Double-diffusive layers can also merge (Linden, 1976) and of course can be broken up and distorted by internal waves and other disturbances. Thus it is not always possible to interpret observed layering scales in terms of the mechanism of their initial formation. Moreover, the absence of well-defined layers does not in itself imply the absence of double-diffusive instabilities. Double diffusion can also occur as a result of horizontal property gradients. Since horizontal gradients of salinity and temperature without a corresponding density gradient can easily occur in fjords, for example after a partial exchange of deep basin water, intrusive effects can be expected with individual flows bounded above and below by the appropriate type of double-diffusive instability (Turner, 1978).

The flux of heat and salt through double-diffusive interfaces depends on the relative contribution of each component to the density gradient. For example, the heat flux $\alpha F_H$, defined in density units, can be expressed as

$$\alpha F_H = A_1 (\alpha \Delta T)^{4/3},$$

(5.13)

where $\alpha$ is the coefficient of thermal expansion and $\Delta T$ the temperature jump across the interface (see, for example, the discussion in Turner, 1981). $A_1$ depends on the ratio $R_\rho$ defined as

$$R_\rho = \beta \Delta S / \alpha \Delta T$$

(5.14)

where $\beta$ is the corresponding factor relating salinity to density. Empirical forms have been found to relate $A_1$ to $R_\rho$, in terms of exchange coefficients for solid boundaries (Huppert, 1971). Moreover, the ratio of salt flux expressed in density units, to the corresponding heat flux $\beta F_s/\alpha F_H$ is also a function of $R_\rho$ alone, falling from unity at $R_\rho = 1$ to 0.15 for $2 < R_\rho < 7$. Thus the large-scale or "eddy" coefficients of salt $K_s$ and of heat $K_H$ must be different for the two properties with the eddy diffusivity of the driving component ($K_H$ in the usual fjord example) always larger than the driven. For a diffusive interface

$$\frac{K_s}{K_H} = \frac{F_s \Delta T}{F_H \Delta S} = R_\rho^{-1} \left( \frac{\kappa_s}{\kappa_T} \right)^{1/2}$$

(5.15)

with $\kappa_s$ and $\kappa_T$ the molecular diffusion coefficients, $1.55 \times 10^{-9}$ m$^2$s$^{-1}$ and $1.5 \times 10^{-7}$ m$^2$s$^{-1}$ respectively yielding $(\kappa_s/\kappa_T)^{1/2} = 10$.

It is clear from the foregoing that a simple interpretation of "eddy diffusion" in the deep basin of a fjord may not be appropriate. Both heat and salt are transported down their respective gradients, although at quite different rates. But the net density flux is transported against the gradient, since the potential energy is decreasing and the density difference tends to increase. Of course double-diffusion may not be the only, or even the dominant vertical exchange mechanism in a fjord, but its contribution needs to be considered in energy arguments related to mixing in deep basins.

A first check on the potential for double-diffusion effects is provided by a calculation of $R_\rho$ for temperature and salinity profiles of interest. Figure 25 shows a segment of the $R_\rho$ profile.
from Knight Inlet during February 1978, at a station well removed from the area of active tidal mixing. Calculation of density shows the profile to be hydrostatically stable, but the potential for the layering type of instability exists over much of the water column and salt fingering may be occurring at one depth (140 m). We use the convention of assigning negative values to $R_\phi$ for the layering instability, positive for the salt fingering instability. For this profile, which is not atypical in a fjord, we would therefore expect double-diffusive effects to lead to an eddy diffusivity of heat greater than that of salt.

There appear to be very few examples for which eddy diffusivities have been calculated separately for both heat and salt in the deep basins of fjords and to our knowledge none in which the results have been analyzed in terms of the recently developed understanding of double-diffusive processes. In Table 1 we list ratios of $K_H/K_s$ for the Oslofjord given by GADE (1970) and from Loch Etive given by EDWARDS and EDELSTEN (1977). In the Oslofjord

<table>
<thead>
<tr>
<th>Location</th>
<th>$K_H/K_s$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oslofjord:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vestfjord basin, St.Fl  (1964)</td>
<td>5.0</td>
<td>Gade, 1970</td>
</tr>
<tr>
<td>St.DK</td>
<td>5.5</td>
<td>Gade, 1970</td>
</tr>
<tr>
<td>Bunnefjord basin</td>
<td>&gt; 3.0</td>
<td>Gade, 1970</td>
</tr>
<tr>
<td></td>
<td>1.3-4.0</td>
<td>Gade, 1970</td>
</tr>
<tr>
<td>Loch Etive</td>
<td>2.5</td>
<td>Edwards and Edelsten, 1975</td>
</tr>
</tbody>
</table>
below 40 m both $S$ and $T$ increase with depth throughout the year, thus providing conditions that could lead to the layering type of instability. Detailed salinity and temperature profiles are not given for Loch Etive, but apparently both $S$ and $T$ increase with depth for at least part of the year. In each of these examples $K_H/K_s$ is significantly greater than unity, suggesting that double-diffusion plays a role in modification of the deep waters of these fjords. Much higher exchange rates occur in the Vestfjord basin of the Oslofjord than in the Bunnefjord basin, which has been attributed to breaking internal tides (see Section 5.2.1), but this process alone cannot explain the proportionately greater heat diffusivity.

Calculation of eddy diffusivities using slow changes in salinity and temperature profiles is difficult to accomplish accurately and is subject to several sources of error. Nevertheless the consistent bias towards higher values for $K_H$, especially in the Oslofjord, appears hard to explain by other means. Double-diffusion in the deep water of fjords has important implications for the calculation of vertical exchange of both active and passive components. For example estimates of biological oxygen demand, or of other chemical changes (i.e. OZRETICH, 1975), which utilize computed exchange coefficients, should also allow for the possibility of convective instability. The molecular diffusivity of different salts (and presumably gases) influences the turbulent transfer by multiple-diffusion (GRIFFITHS, 1979). Calculation of energy sources and sinks, such as the significance of internal waves to mixing in deep basins, discussed in Section 5.2, and of flux Richardson Numbers (i.e. equation 5.4) based on vertical diffusivity estimates, must also allow for the effects of double-diffusion. There is a need for careful analysis of double-diffusion processes in fjords, in which the relatively isolated conditions of deep basins permit indirect estimates of the bulk transport coefficients of different properties (heat, salt, dye, silicates, oxygen) for comparison with direct measurement of diffusive interfaces and layering over an extended period.

6. CONCLUDING REMARKS

6.1. Energy constraints on mixing

Although the motivations for the study of fjords may be scientific curiosity or concern over some practical aspects such as the environmental effects of mine discharge or oxygen depletion in the deep water, it is useful to consider a coherent approach to the problem of describing processes in fjords. Such an approach also serves as a focus for summarizing several earlier parts of this review and we hope will provide guidance in the design of future studies. Fjords vary so much from one place to another that a consistent approach must begin with an assessment of the magnitude of various factors influencing the circulation and of the various scales of temporal and spatial variability. This is required not only so that measurement programs can be designed to minimize problems of aliasing or inappropriate instrument deployment, but also to determine the processes which dominate at different times of year and in different parts of the fjord.

To some extent the proposals for fjord classification (Section 2) constitute just such an attempt; for example the Hansen-Rattray scheme and Stigebrandt’s hydraulic scheme are expressions of distinct dynamical models. But these models are of necessity extreme simplifications. Even if they represent correctly the balance of competing effects in some averaged sense, they still leave out the time dependence associated with exchange and mixing processes (Sections 4 and 5) that constitute much of what is important and interesting to the physical oceanographer. In this section we sketch an approach based on energy considerations, again
illustrating with our observations in Knight Inlet, so as to demonstrate how some of the pro-
cesses described earlier contribute to the larger picture of circulation in a fjord.

SVENSSON (1980) attempted to assess the relative significance of the various terms in
the equations of motion through a detailed scale analysis. However, such an approach has a
strong tendency to ignore physical processes in favour of gross statistical estimates. For
example, a term like $u\xi$ is critically important in the evaluation of tidal energy fluxes. An
estimate of the size of this term by scale analysis would suggest a very large energy flux;
however, consideration of the basic physical processes indicates that $u$ and $\xi$ must almost be
in quadrature leading to an estimate of their covariance that is perhaps two orders of magnitude
smaller. We prefer to attempt a more physical, process-oriented approach.

Consider first the various factors that influence mixing and circulation. One way of doing
this is to identify various sources of buoyancy that might drive the gravitational circulation
and identify the various sources of turbulent kinetic energy that can redistribute the buoyancy.
Dimensionally the buoyancy sources are expressed in $J\, m^{-1}\, s^{-1}$ (SI units) and the turbulent
kinetic energy sources as $J\, s^{-1}$. The ratio of these defines a length scale related to the depth
over which the kinetic energy might be expected to redistribute buoyancy. This length scale $L$,
is just the Monin–Obukhov length [equation (5.2)] discussed earlier. If we define a control
volume bounded by the bottom, surface, sides, head and mouth of a fjord then an inwards
buoyancy flux is positive; the length scale will be positive when the buoyancy flux is a source
and negative when it is a sink. This concept allows comparison of the various different factors
that serve to stabilize or to mix water in a fjord. The uncertainty in such assessments lies in
determining the efficiency of the mixing process, specifically the flux Richardson Number
[equation (5.4)]. In the absence of direct measurements of turbulence we must rely entirely on
previous work; coherent approaches such as that presented by PEDERSEN (1980) are of
particular value for this purpose.

Table 2 lists several of these sources along with some estimates for Knight Inlet. The table
also indicates the spatial distribution of each buoyancy or turbulent kinetic energy source. This
list could be extended, but we believe it includes most of the processes likely to dominate in
fjords. Other mechanisms (i.e., salt exclusion beneath growing ice sheets, double diffusion
effects associated with icebergs, suspended sediment movement, turbulent density currents
associated with sporadic renewals, effects due to human intervention, etc.) can be included as
appropriate in particular examples. In assessing each process we need estimates of the magni-
tude of different factors such as wind stress, tidal components and so on as well as estimates
of various scales of variability, including temporal variability to be discussed later. Of course
this information may allow us to follow the equivalent approach of estimating the size of
different terms in the momentum and conservation equations, but it is more useful at this
stage and more consistent with the process-oriented approach we have taken in this review, to
identify the dominant mechanisms directly. Such an approach provides insight into the
dynamics of circulation, but does not in itself lead to a circulation model, since the local
dominance of certain processes must depend on the boundary conditions (see Section 3.2).
Our approach to the measurement problem in fjords must therefore take account of these
too.

**Bl. River discharge.** Among buoyancy sources, relatively straightforward measurement
provides information on the fresh-water discharge. If the flow is not dominated by a single
large river, estimates must be made of the contribution from the different sources based on
watershed area, precipitation and snowmelt information. Let us assume the dominant source
is a single river at the inlet head ($x = 0$), then the buoyancy flux is
The physical oceanography of fjords

<table>
<thead>
<tr>
<th>Process</th>
<th>Distribution</th>
<th>Estimate for Knight Inlet ($J , m^{-1} , s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$ River discharge</td>
<td>horizontally localized,</td>
<td>winter $\leq 1.2 \times 10^4$</td>
</tr>
<tr>
<td></td>
<td>confined to surface</td>
<td>summer $20 \times 10^4$</td>
</tr>
<tr>
<td>$B_2$ Gravitational circulation</td>
<td>distributed vertically</td>
<td>winter $-1.2 \times 10^4$</td>
</tr>
<tr>
<td></td>
<td>mainly near-surface</td>
<td>summer $-20 \times 10^4$</td>
</tr>
<tr>
<td>$B_3$ Exchange of water with outside:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Intermediate water</td>
<td>localized from pycnocline</td>
<td>$\pm 2 \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>to sill depth</td>
<td></td>
</tr>
<tr>
<td>(b) Deep water</td>
<td>vertically distributed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>below sill depth</td>
<td></td>
</tr>
<tr>
<td>$B_4$ Surface heat exchange</td>
<td>horizontally distributed</td>
<td>$\pm 0.5 \times 10^4$</td>
</tr>
<tr>
<td>(a) Summer heating</td>
<td>at the surface</td>
<td></td>
</tr>
<tr>
<td>(b) Winter cooling</td>
<td></td>
<td>$-1.6 \times 10^4$</td>
</tr>
</tbody>
</table>

Turbulent kinetic energy sources

<table>
<thead>
<tr>
<th>Process</th>
<th>Distribution</th>
<th>Estimate for Knight Inlet ($J , s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$ Wind stress</td>
<td>horizontally distributed</td>
<td>(summer) $1.4 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>confined to surface</td>
<td></td>
</tr>
<tr>
<td>$K_2$ Tidal interaction with</td>
<td>horizontally localized,</td>
<td></td>
</tr>
<tr>
<td>variable topography</td>
<td>vertically distributed</td>
<td></td>
</tr>
<tr>
<td>$K_3$ Diffusive instabilities in deep water</td>
<td>horizontally and vertically</td>
<td></td>
</tr>
<tr>
<td></td>
<td>distributed in deep water</td>
<td></td>
</tr>
<tr>
<td>$K_4$ Convective instability by surface</td>
<td>horizontally distributed</td>
<td>(winter) $1.6 \times 10^5$</td>
</tr>
<tr>
<td>cooling</td>
<td>at surface</td>
<td></td>
</tr>
<tr>
<td>$K_5$ Mixing due to kinetic energy of river</td>
<td>localized vertically and</td>
<td>$&lt; 8.9 \times 10^4$</td>
</tr>
<tr>
<td>discharge</td>
<td>horizontally, near surface</td>
<td></td>
</tr>
</tbody>
</table>

$$B_f = \int_0^B \int_0^H g[\rho(z) - \bar{\rho}] u(z) \cdot n \cdot d z \cdot dy,$$

(6.1)

where $\bar{\rho}$ is the reference density and $n$ is the unit outward normal. If the river discharge takes some time to mix laterally (Section 5.3) then (6.1) must be based on measurements at an appropriate distance from the head at which point frontal mixing ($K_5$) may have raised the surface salinity significantly. If the reference density $\bar{\rho}$ is taken to be some deep value near the head then most of the contribution to the integral comes from the near surface. For a homogeneous, fresh surface layer, which is not atypical near the head of a fjord during intermediate and high run-off, (6.1) reduces to

$$B_f \approx g(\rho - \bar{\rho}) Q_f.$$

(6.2)

Values for Knight Inlet range from a winter minimum of $\leq 1.2 \times 10^4 \, J \, m^{-1} \, s^{-1}$ to typical
spring values of $5.5 \times 10^4$ J m$^{-1}$ s$^{-1}$ and summer maxima of about $20 \times 10^4$ J m$^{-1}$ s$^{-1}$. The annual mean is $9.7 \times 10^4$ J m$^{-1}$ s$^{-1}$.

**B2. Gravitational circulation at the mouth.** This is the buoyancy sink for the influx at the head and can be calculated using (6.1) at the fjord mouth. If there are no other significant buoyancy inputs and if the circulation is steady then $B_2$ must be equal and opposite to $B_1$, a result which is implicit in the various theoretical models (Section 3) that use the Knudsen relationships. We have entered the value accordingly in Table 2. However this equality will not exist instantaneously since exchanges at the mouth are highly time dependent; thus care is required in determining a suitable integration period if steady state calculations are to be made on the basis of an assumed equality between $|B_1|$ and $|B_2|$. Since we expect that in narrow fjords (Section 2) the stratification will be both deeper and weaker at the mouth than at the head, contributions to the integral (6.1) will come from greater depths for $B_2$ compared with $B_1$. Practical measurement of $B_2$ will be difficult due to the spatial and temporal variability of flows near the mouth. In principle, automatically profiling Salinity-Temperature-Velocity instruments that have variable buoyancy might be used for this purpose, although experience indicates difficulty in obtaining profiles close enough to the surface, owing to the steep density gradients there. Unfortunately these gradients exist just because of the high near-surface buoyancy flux, which must be measured in order to evaluate (6.1). Moreover the flow is unlikely to be strictly two dimensional near the mouth, further complicating the measurement problem. Thus we will usually have to settle for the time averaged equality $B_1 = -B_2$ without experimental verification.

**B3. Exchange of intermediate and deep water.** In this case we consider the possibility that the density of water beneath the brackish surface layer has changed as a result of advective exchange with the water outside the fjord (Sections 4.2 and 5.3). These exchanges can be evaluated indirectly from hydrographic surveys or directly from automatically profiling S–T–V instruments, using equation (6.1). The contributions are restricted to exchanges beneath the pycnocline and, at least for Knight Inlet, appear to be relatively small for intermediate water exchanges $(10 < z < 100$ m) compared to the buoyancy fluxes associated with the surface layer. Our estimates for Knight Inlet are based on equation (6.1) using the observed horizontal density gradient along the fjord, the vertical gradient at the mouth, over a depth range from just below the pycnocline down to 100 m (slightly below sill depth) and horizontal transports calculated from Knudsen's relations for a two layer model. The data are those presented by FREELAND and FARMER (1980).

The buoyancy flux associated with deep $(z > 100$ m) water exchanges are highly episodic but might be estimated from hydrographic and current meter data. Changes between hydrographic data on successive cruises can be used to determine the influx of new water at different depths (GADE and EDWARDS, 1980) and current meters located on the inside of the sill, close to the bottom, can provide information on the timing of the exchanges. No estimates are available for Knight Inlet.

**B4. Surface heat exchange.** The buoyancy flux due to heat gained or lost through the surface is

$$B_r = gA\alpha H_f \rho c_p,$$

(6.3)

where $A$ is the surface area of the inlet, $\alpha$ is the coefficient of thermal expansion and $H_f$ the surface heat flux per unit area. Estimates of $H_f$ can be made on the basis of local climate information or by direct measurement. For Knight Inlet we use approximate values of $+50$ J m$^{-2}$ s$^{-1}$ for summer and a minimum of $-150$ J m$^{-2}$ s$^{-1}$ for severe winter cooling. It
is clear from Table 2 that the buoyancy flux due to heat exchange is of small or marginal consequence in summer, but definitely of consequence in winter. This conclusion was arrived at by different means in the beginning of Section 5. During the winter the buoyancy flux due to cooling is negative and thus serves to diminish the flux due to fresh water discharge. However, it is probable that wind induced transients would mask the gravitational flow during conditions of low run-off.

$K_1$. **Kinetic energy due to wind stress.** The rate of working of the wind on the surface of the fjord may be represented as

$$K_1 = \int_A \left( V_d \tau / \rho_w \right) \mathrm{d}A$$

(6.4)

where $\tau = \rho_a C_a U_a^2$ is the wind stress and $V_d$ is a surface drift speed. The form used by FREELAND and FARMER (1980) is essentially that used by NIILER and KRAUS (1977) viz.

$$K_1 = \int_A mU_a^2 \mathrm{d}A$$

where $U_a$ is the friction velocity defined as $(\tau / \rho_w)^{1/2}$. The latter form implicitly includes an efficiency factor and is the rate at which work is done by the wind to increase the potential energy of water columns. For the present purposes equation (6.4) will be used as we will want to make comparisons with rates of working by other processes for which efficiency factors are unknown. Unfortunately the relationship between the surface drift velocity and the air speed $U_a$ is poorly known. KIRWAN et al. (1979) in an analysis of satellite tracked drifting buoys suggest $V_d \approx 1.6\% U_a$. Using that figure in (6.4) and a drag coefficient $C_a = 0.002$ (possibly a little high) and the cube-root-mean-cube wind speed of 5.3 m/sec given by FREELAND and FARMER (1980) we derive the estimate for $K_1$ entered in Table 2.

$K_2$. **Kinetic energy due to tidal interaction with topography.** As discussed in detail earlier (Sections 4.1.1, 5.2, 5.3, 5.4), energy lost from the tide can lead to mixing in a variety of different ways. The energy can be distributed locally, for example due to Kelvin-Helmholtz instabilities on shear zones bounding internal jets or hydraulic jumps, or it can be distributed far from the place of generation by breaking internal waves and the collapse of fluid mixed near the sill. The most intense turbulence due to tidal action will be in constrictions and typically distributed vertically from near the surface to well below sill depth. Each of the various mechanisms contributing to mixing must be separately assessed but an overall energy loss from the tide can be found from accurate tidal measurements at different points along the fjord. As shown in Section 4.1.1, the energy loss from the tide between two points in the inlet depends on the phase difference $\epsilon$

$$K_2 = \frac{1}{2} \rho_s g^2 \omega \theta_0 \sin(\epsilon).$$

(6.5)

For Knight Inlet $\epsilon$ is greater in summer than in winter. The annual average of 0.5° for the $M_2$ tide has been used in our estimate.

$K_3$. **Kinetic energy from doubly diffusive instabilities.** The measurement problem in this case is confused by difficulty in direct observation of double diffusion. An upper bound on the contribution can be assessed from successive hydrographic profiles over a period during which there is no influx of water from outside the fjord and no deep mixing of surface water. A check can be made on the redistribution of heat and salt in the deep water and the changes in potential energy can be calculated; significantly different redistribution rates ($K_H \neq K_3$) indicate a contribution due to different molecular diffusivities (see Section 5.5). No such estimates are available from Knight Inlet, but when such instabilities occur they are likely to
be a very small contribution to the energetics of the inlet compared to other sources. The possible importance of these processes lies in their distribution; they may be the dominant mixing mechanism in the deep water of some fjords.

\( K_4 \). Kinetic energy due to surface cooling in winter. This is just the energy transfer rate corresponding to \( B_4 \):

\[
K_4 = \frac{B_4}{h}.
\]  

(6.6)

In winter Knight Inlet has a much more variable stratification and can be well mixed down to over 200 m (see Section 5.4). For the purpose of our estimate in Table 2 we use a more typical value for Knight Inlet of 10 m.

\( K_5 \). Kinetic energy associated with fronts. An upper bound on the kinetic energy input associated with turbulence generated by river discharge is simply the net energy flux out of the river:

\[
\int_0^h \int_0^B \frac{1}{2} \rho U^3 \, dz \, dy
\]

(6.7)

where the integration is carried out at the river mouth. This requires information on the dimensions and discharge of fresh water at the river mouth. Estimates for Knight Inlet in the summer show that this represents a substantial energy source. However, the efficiency with which this energy is used to redistribute buoyancy must be very low, since the surface layer in summer is completely fresh in the upper reaches of the fjord. Since the mean river depth (3 m) is substantially less than the surface layer thickness (~10 m) it is probable that most of the kinetic energy is dissipated within the fresh water; this example contrasts with the situation in a salt wedge estuary, or in fjords with higher surface salinities.

The relative importance of each of the sources described above depends critically on their distribution in space and time and on the efficiency with which the kinetic energy is recovered as potential energy through mixing. A few examples will illustrate.

Consider first the balances of buoyancy flux due to fresh water discharge and gravitational circulation and the effects of mixing by wind stress. The distribution of both buoyancy and kinetic energy is horizontally distributed but vertically confined to the surface layer. If they are locally balanced and horizontally unconfined, the ratio

\[
\frac{K_4}{B_2} = \frac{\gamma U_\infty^3 A}{Q \beta S_2}
\]

(6.8)

which corresponds to the surface layer depth \( h \) in equation (3.14) and also to the first term on the right in equation (3.15), of LONG's (1975a) and STIGEBRANDT's (1975, 1981) model of wide fjords. The coefficient \( \gamma \) is related to the efficiency of the entrainment process; for this example we take an accepted value of 1.2, but it can hardly be considered a constant since the deepening mechanism changes with time (see Section 5.1). For Knight Inlet, which has horizontal surface layer gradients and thus is not a 'wide' fjord, as well as having gradients in the wind-stress, the calculation is strictly meaningful only locally. However an idea of the depth over which mixing can be sustained may be gained by using the values given in Table 2. The mixing depth (6.8) varies from 1.4 m in spring to 0.35 m during peak discharge. Since these values are much less than actually observed it is apparent that other factors must play an important role in determining the mean layer depth; these might include the hydraulic control as implied by the layered models described in Section 3.2.

In winter during strong winds, for which records are not available, we anticipate that wind-mixing will be dominant over much greater depths. For example, even if we take a wind speed of 10 m s\(^{-1}\), which is probably conservative for winter, the mixing depth during low discharge (~50 m\(^3\) s\(^{-1}\)) would be 40 m.
It is evident that surface heating is a small buoyancy source compared to river discharge, so that surface heating is unlikely to be dynamically important. The ratio

$$K_1/B_4 = \frac{\gamma U_0^3}{H c_p}$$

for Knight Inlet in summer leads to a scale depth of order 10 m, so that we expect wind stress to redistribute the surface heat flux over the surface layer, provided that there is no opposing salinity gradient. However, observation often shows that considerable vertical temperature structure can occur in the surface layer, stabilized by the salinity structure. We have already discussed the inverse problem of cooling in winter (Section 5.4), for which case the surface buoyancy flux acts as a source of turbulent kinetic energy through gravitational instability.

The effects of wind stress, solar heating and fresh water discharge tend to be concentrated near the surface. This need not be true of tidal effects. In our example of Knight Inlet, tidal interaction is an important kinetic energy source. Less than 4% of the turbulent kinetic energy liberated by tidal interaction with the sill is required to account for the divergence of total energy flux over the whole inlet \((1.5 \times 10^5 \text{ J s}^{-1})\) so that even with modest efficiencies, the tidal input appears sufficient to account for observed mixing. On the other hand wind-stress, even during the summer, can also account for the divergence of potential energy flux. However the wind always acts on the surface, whereas mixing events of tidal origin are initiated at or near the pycnocline. As discussed in Section 5.2, during periods of high run-off at neap tides, interaction with the surface layer is suppressed and most of the mixing proceeds just beneath the pycnocline; at these times the surface layer can remain almost unmixed right across the sill. At lower discharges the surface layer is actively mixed near the sill. These highly variable interactions can be readily detected using high frequency echo sounders as illustrated earlier.

Each of the factors determining the circulation in a fjord vary with time. It is necessary to learn the nature of this variability if oceanographic observations are to be properly planned. The dominant time scales can be found by suitable measurement programs. More than one scale may be applicable for a particular process; for example winds in fjords typically have a strong 24 hour sea-breeze component, a 3–5 day component due to the passage of weather systems and a strong annual component.

The effect of fluctuations in the forcing must be interpreted in terms of the time scales of the response of the inlet. For example, short glacially fed rivers have a strong 24 hour fluctuation, but the response time of the estuarine system will be of order \(BLh/Q_t\) which is normally much more than 1 day. Conversely the baroclinic response to 24 hour fluctuations in the wind stress will be increased if the internal resonant period of the fjord is about 24 hour. These time scales also provide a guide to the length of time that a field program must continue in order to assess the full range of oceanographic phenomena likely to be encountered. Undoubtedly the longest time scales are associated with diffusion effects in the deep water, which typically have a time scale of 1 year, but often substantially more. In contrast, the replacement of deep water can occur very rapidly, in just a few days. Unfortunately many oceanographic surveys designed for practical applications, allow insufficient time for adequate measurement of deep water modification and exchange processes which have long time scales. In such cases use of theoretical models, such as GADE's (1973), may provide a suitable approach if sufficient data are available on local variability of the coastal waters.

6.2. **Summary**

The study of fjord oceanography has reached the stage at which much of the world's fjord coastline has received at least a preliminary synoptic survey, along with quite detailed physical
studies of a few Scandinavian and N.W. American fjords. There have also been a number of laboratory models of fjord processes as well as several attempts to derive steady state theoretical models of the gravitational circulation. As attempts are made to arrive at progressively more precise descriptions of fjord circulation, the major difficulty becomes that of adequate representation of turbulence and of properly accounting for the time dependent nature of real flows. A satisfactory description of the scales and intensities of turbulent flows is essential to the proper description of fjords, as it is of all estuaries.

We have seen how almost all processes in fjords are strongly time dependent. This is particularly true of wind effects and of course tidal processes although perhaps less so of processes associated with river plumes. Any modelling approach that averages over these effects, with all their nonlinearities, inevitably masks much of the underlying physics. Field measurements such as those described earlier for Knight Inlet emphasize that strong mixing processes tend to occur in relatively brief periods. Fjords that are wind mixed and thus forced by the highly skewed parameter $U_a^3$ must be similarly episodic in character.

These considerations suggest that future research on fjords concentrate on a thorough understanding of processes, in particular the scales and intensities of turbulent flows and the mechanisms by which energy is fed into such flows and ultimately distributed. In near surface waters the dominance of the buoyancy effects ensures that density flows, internal wave and hydraulic phenomena will play a prominent role in the redistribution of energy. In weakly stratified water the mechanism may also include the subtle effects of double diffusion.

A deeper understanding of fjords must come through new observational approaches. In this paper we have presented a few examples of remote sensing applications to the study of fjords. Echo sounding has proven particularly effective in probing the small to medium scale (1–50 m vertical, 1 m–several km horizontal) processes associated with tidal mixing and advection. Other techniques now proving useful include range-gated acoustic Doppler velocity sensing which can also be used from a moving vessel. The new generation of microstructure instruments should prove especially useful for direct estimates of turbulence intensity. For measuring the slow processes of diffusion and advection in the deep water of fjords new classes of chemical tracers may prove valuable, such as exotic fluorocarbons (Freons), which have excellent stability and can be detected in concentrations much lower than commonly used dye tracers.

Careful laboratory experiments designed to explore specific processes under idealized conditions will complement the observations and help to establish good parametric representation of mixing processes. With good descriptions of turbulence and mixing the way will be prepared for development of the next generation of mathematical models. This complementary approach of observation, laboratory and mathematical modelling will undoubtedly yield scientific dividends beyond the better description of fjords. Indeed, one of the fascinations for oceanographers studying these deep, strongly stratified estuaries is the enormous range of fluid processes, common to many other geophysical settings, that can be studied in their relatively benign and accessible environments. Fjords have already proven to be ideal places to study internal waves (Fjelstad's classical analysis is a prime example), internal hydraulics, problems of density currents and shear flow instabilities, wind mixing under a wide variety of density stratification, the slow processes of diffusion in deep basins, several problems associated with the formation and melting of ice, surface wave modification by internal waves, frontal mixing and many other processes. In concluding this review we emphasize that a fjord coastline should attract oceanographers not only because of practical problems associated with man's interaction with his environment, but also because it provides an excellent laboratory for studying natural flows.
Acknowledgements – The measurement of flows in B.C. fjords would not have been possible without the skilled assistance of the Coastal Oceanography staff at I.O.S. and in particular D. Stucchi, A. Stickland, R. Bigham, L. Spearling, J. Meikle, D. Sieberg, N. Delacretaz and the computing skills of G. Kamitakahara-King and A. Lee; we are also indebted to the Master and crew of the C.S.S. VECTOR. We wish to acknowledge the contributions of several of our colleagues and a referee who provided valuable comments on the manuscript. The first author also wishes to express his indebtedness to Professor J. D. Smith of the University of Washington, with whom many of the field projects were jointly undertaken over several years and who greatly contributed to the development of our knowledge of fjord oceanography and to the improvement of our experimental approach.

REFERENCES


EKMAN, F. L. (1875) Om de strømninjer som uppstå i närheten af flodmynningar etc. Ofvers. K. Vetensk. – Akad. Förhandlingar. Mr. 7.


FJELSTAD, J. E. (1933) *Interne Wellen*, Geofysiske publikasjoner, 10(6), 1–35.


