Abstract

A two-dimensional, time-dependent solution to the transport equation is formulated to account for advection and diffusion of sediment suspended in the bottom boundary layer of continental shelves. This model utilizes a semi-implicit, upwind-differencing scheme to solve the advection-diffusion equation across a two-dimensional transect that is configured so that one dimension is the vertical, and the other is a horizontal dimension usually aligned perpendicular to shelf bathymetry. The model calculates suspended sediment concentration and flux; and requires as input wave properties, current velocities, sediment size distributions, and hydrodynamic sediment properties. From the calculated two-dimensional suspended sediment fluxes, we quantify the redistribution of shelf sediment, bed erosion, and deposition for several sediment sizes during resuspension events. The two-dimensional, time-dependent approach directly accounts for cross-shelf gradients in bed shear stress and sediment properties, as well as transport that occurs before steady-state suspended sediment concentrations have been attained. By including the vertical dimension in the calculations, we avoid depth-averaging suspended sediment concentrations and fluxes, and directly account for differences in transport rates and directions for fine and coarse sediment in the bottom boundary layer. A flux condition is used as the bottom boundary condition for the transport equation in order to capture time-dependence of the suspended sediment field. Model calculations demonstrate the significance of both time-dependent and spatial terms on transport and depositional patterns on continental shelves.

1. Introduction

On many continental shelves, including those off California and the mid-Atlantic coast of the United States, resuspension by energetic waves and currents is the dominant mechanism for sediment transport (see, e.g., Butman et al., 1979; Drake and Cacchione, 1985; Wiberg et al., 1994; Wright et al., 1994; Li et al., 1997; Cacchione et al., 1999). Once sediment is suspended, it is transported by bottom boundary layer flows that can redistribute sediment along and across the shelf. Net erosion or deposition occur when sediment entrained and advected away from one location is not exactly replaced from an upstream source. On continental shelves, such flux divergence in the sediment transport field can result from spatial gradients in wave energy, current velocity, or sediment properties. Because the effect of these gradients on sediment transport and bed evolution cannot be captured with one-dimensional models (e.g., Wiberg et al., 1994), we have developed a two-dimensional, time-dependent model of shelf sediment transport. This paper describes the model and an example of its application.

Our model has three components that are solved sequentially at each time step. At the beginning of a time step, the flow field in the bottom boundary layer is determined for specified wave and current conditions...
using a standard wave–current interaction model applied to each location along the model transect (e.g., Smith, 1977; Grant and Madsen, 1979). Next, the two-dimensional, time-dependent advection–diffusion equation is solved for the given flow field to obtain suspended sediment concentration for each sediment size class present in the bed. The calculated velocity and concentration fields are used to determine suspended sediment flux and the volume of sediment in suspension at each location. Finally, divergences in suspended sediment flux and temporal variations in the volume of suspended sediment are related to changes in bed elevation and grain size distribution using the erosion equation.

The model domain is a two-dimensional slice of the bottom boundary layer of the continental shelf, which includes the bottom Ekman layer (tens of meters thick) and a thin wave boundary layer adjacent to the bed (~10 cm thick). One dimension, z, is taken to be the vertical dimension (positive upward) and the other, x, is any horizontal direction. For many shelves, the spatial gradients in both wave energy and sediment size distributions are much larger in the cross-shelf direction than in the along-shelf direction, so that the most significant sources of flux divergence are included when the horizontal dimension of the model is oriented in the cross-shelf direction. In areas that also display significant along-shelf variation in sediment size or current circulation patterns, a fully three-dimensional model would be advantageous.

Our model is an extension of the one-dimensional models that previously have been developed to compute suspended sediment concentration and flux on continental shelves (e.g., Smith, 1977; Glenn and Grant, 1987). The one-dimensional approach has successfully reproduced field measurements of velocity and suspended sediment concentration at several sites (e.g., Wiberg and Smith, 1983; Glenn and Grant, 1987; Wiberg et al., 1994; Cacchione et al., 1999; Wiberg et al., 2001). It has also been linked to calculations of bed grain size evolution and the formation of fine-scale stratigraphy due to wave–current resuspension (Kachel and Smith, 1986; Niedoroda et al., 1989; Wiberg and Butman, 1991; Harris and Wiberg, 1997). A one-dimensional approach may be inaccurate, however, in shelf areas that display significant spatial variability in flow or sediment properties. In addition, patterns of net deposition and erosion cannot be obtained from one-dimensional calculations of sediment transport.

One approach to estimating the effect of spatial gradients in flow or sediment properties on transport and depositional patterns is to complete the one-dimensional calculations at several locations and compare the fluxes calculated for each site (e.g., Kachel and Smith, 1989; Sherwood, 1995). This approach assumes that the suspended sediment field is equilibrated with local conditions and neglects the contribution of advection to the suspended sediment field. The vertically integrated flux at each of these grid points can also be input into an advection–diffusion model and used to adjust the bed sediment size distribution for each grid cell to account for erosion and deposition during each time-step (Sherwood, 1995). This approach requires separate frameworks for the vertical structure of the suspended sediment field and horizontal advection, however, and does not account for non-equilibrium transport. The steady-state assumption is most likely to be accurate during the peak of a resuspension event, but not when sediment is being diffused upward or settling out of suspension. It will also be violated when the time-scale associated with winnowing of fine-grained sediment from the bed or advection from different source areas is shorter than the duration of a resuspension event. Therefore, a complete treatment of the spatial terms requires that the time-dependent terms be included as well. Some one-dimensional shelf transport models have retained the time-dependent terms (Kachel and Smith, 1989; Wiberg and Butman, 1991), but they lack a spatial component.

Advection can be accounted for directly using a transport equation that retains multiple spatial dimensions and the time-dependent terms. The time-dependent terms are necessary to account for changes to the suspended sediment field from lateral advection or settling. Inclusion of lateral spatial terms facilitates direct calculations of flux divergence that allow us to estimate net sediment erosion and deposition. Combining the time-dependent and multi-dimensional transport equation with a model of bed reworking also avoids many of the difficulties and inconsistencies that were confronted when models that assumed local and temporal equilibrium were used to predict modifications to the bed due to transport processes (see Sherwood, 1995; Harris and Wiberg, 1997).

During the time that we developed our model, another two-dimensional model that solves the same fundamental equations was developed for sediment transport on continental shelves (Zhang et al., 1999). Our model extends a well-tested one-dimensional transport model, but requires several grid cells to represent the bottom 10 cm of the flow. The Zhang et al. (1999) model parameterizes wave boundary layer processes so that they can be represented within one grid cell. Their approach has the advantage of requiring fewer grid cells in the water column, however, until the parameterizations are tested against a variety of field conditions, it is difficult to evaluate their applicability.

2. Two-dimensional, time-dependent model

The two-dimensional, time-dependent transport model calculates suspended sediment concentrations and...
fluxes for given flow conditions using the advection-diffusion equation. This conserves mass of sediment that is advected with ocean currents, settles due to gravity, and diffuses due to turbulence:

\[ \frac{\partial C_s}{\partial t} = -u \frac{\partial C_s}{\partial x} + \frac{\partial}{\partial x} \left( K_x \frac{\partial C_s}{\partial x} \right) + w_s \frac{\partial C_s}{\partial z} + \frac{\partial}{\partial z} \left( K_z \frac{\partial C_s}{\partial z} \right). \]  

(1)

Here, \( C_s \) is the volumetric concentration of suspended sediment (cm\(^3\)/cm\(^3\)), \( u \) is flow velocity, \( w_s \) is sediment settling velocity (for \( w_s > 0 \) by convention), and \( K_x \) and \( K_z \) are the horizontal and vertical turbulent diffusion coefficients, respectively. Eddy diffusivity, \( K_z \), is a function of height above the bed and shear stress, and is often assumed to be equal to the eddy viscosity of the fluid. In Eq. (1), the time-averaged vertical flow velocity is assumed to be negligible so that the vertical sediment velocity is equal to the settling velocity (\( w_s \)). This is reasonable provided there is no significant mean vertical flow and that the time scales of the calculations are longer than the period of wave oscillations.

Modifications to the sediment bed caused by resuspension and redistribution are calculated using the erosion equation, which conserves sediment mass between the water column and sediment bed:

\[ \frac{\partial \eta}{\partial t} = -\frac{1}{C_b} \left( \frac{\partial q_s}{\partial x} + \frac{\partial V_s}{\partial t} \right), \]  

(2)

where \( \eta \) is the location of the bed surface, \( C_b \) is bed sediment concentration (1-porosity), \( q_s \) is the depth-integrated volume flux of suspended sediment (\( q_s = \int_0^h C_s u \, dz \)), \( V_s \) is the depth-integrated volume of suspended sediment (\( V_s = \int_0^h C_s \, dz \)), and \( h \) is the thickness of the flow being considered. Eqs. (1) and (2) can be applied to each sediment size in a mixed grain size bed to calculate suspended sediment concentrations, fluxes and rates of deposition or erosion for each size class. A right-hand coordinate system is used for both the water column and sea-bed, with \( \eta \) and \( z \) increasing upward.

Eqs. (1) and (2) are solved using a finite-difference approximation on a two-dimensional spatial grid. The necessary velocities and diffusivities are calculated from the wave and current conditions specified at each time step. The grain size distribution of the bed is updated each time step based on the net erosion and deposition determined for each grain size across the model grid. The general procedure used to solve the two-dimensional transport and bed reworking equations is as follows. Details of the theories and methods are provided after this overview.

1. Set up model grid to be uniformly spaced in the horizontal (\( x \)-direction), but logarithmically spaced in the vertical (\( z \)-direction) to resolve the high gradients in the velocity and suspended sediment concentrations near the bed. Read the input model transect, which includes water depth and initial grain size distribution for each \( x \) location. For each grain size class read the sediment data including nominal grain size, critical shear stress, and settling velocity.

2. Initialize variables so that there is initially no suspended sediment, and the sediment size distribution is constant with depth in the bed at each \( x \) location.

3. For each time step:
   
   (a) Read input wave conditions (either \( H_{sig} \) and \( T_{dom} \), or wave spectra), and current velocity at the top of the model grid.

   (b) Calculate velocity profile at each \( x \) location:
      
      (i) Calculate near-bed orbital velocity (\( u_b \)) and wave period from either surface wave characterisitics or wave spectra.

      (ii) Complete flow-field calculations to determine velocity (\( u, v \)), eddy viscosity (\( K_z \)), shear stress (\( \tau \)) and bed roughness (\( z_0 \)). The bed roughness estimate depends on bedform dimensions, which are also estimated in this step.

      (iii) Calculate depth of the active layer, \( \delta_{mix} \), as a function of mean sediment size and bed shear stress. Update grain size distribution of the active layer if it deepens, by incorporating sediment from below.

   (c) For each sediment size class:
      
      (i) Solve transport equation (Eq. (1)) to obtain suspended sediment field (\( C_s \)).

      (ii) Calculate net erosion and deposition using the erosion equation (Eq. (2)).

      (iii) If net erosion exceeds the volume of sediment available for any grain size at any \( x \) location, apply bed armoring by decreasing the entrainment of that sediment size.

      (iv) If bed armoring limits have been exceeded, repeat steps 3(e)(i)–(iii) using the adjusted entrainment rates until erosion does not exceed available sediment at any \( x \) location.

   (d) Update bed grain size distribution and stratigraphy to account for net erosion (or deposition) of each grain size at each grid point.

2.1. Velocity calculations

At the beginning of each time step, geostrophic current velocity (\( u_{geo} \) and \( v_{geo} \)) at the top of the bottom boundary layer and surface wave conditions are read
from an input file. Either surface wave characteristics \( (H_{d0g} \text{ and } T_{d0m}) \) or wave spectra can be used to specify the wave component of the flow. If surface wave height and period are used, the calculations use linear wave theory to calculate near-bed wave orbital velocity \( (u_b) \) and wave period is taken to be equal to \( T_{d0m} \). If surface wave spectra are available, near-bed wave orbital velocity \( (u_b) \) and near-bed wave period \( (T_{mb}) \) are calculated following Sherwood (1995) and Madsen (1994). For each \( x \) location, \( u_b \) is calculated from a surface spectrum that contains \( N \) frequency bins, described by the wave amplitude \( (a_n) \) within each bin of angular frequency \( (\omega_n) \):

\[
u_b = \left( \frac{2}{N} \sum_{n=1}^{N} \omega_n^2 a_n^2 \right)^{1/2} = \left( \frac{2}{N} \sum_{n=1}^{N} \frac{\omega_n^4 a_n^2}{[\sinh(k_nh)]^2} \right)^{1/2},
\]

where \( h \) is water depth and \( k_n \) is wave number for frequency bin \( n \). The near-bed average wave period is calculated as

\[ T_b = 2\pi u_b \left( \frac{1}{N} \sum_{n=1}^{N} \omega_n u_b n^2 \right)^{-1}. \]

Wave boundary shear stress, \( \tau_{bw} \), at each \( x \) location is related to orbital velocity through a wave friction factor, \( f_w \):

\[ \tau_{bw} = \rho u_w^2 = 0.5 f_w \rho u_b^2, \]

where \( u_w \) is the wave shear velocity and \( \rho \) is fluid density. The wave friction factor can be approximated as \( f_w = 0.04(u_b T_{mb})^{-0.25} \) (Fredsoe, 1993). The physical roughness length scale of the sediment bed, \( k_b \), is described in a later section.

Current velocity in the bottom boundary layer is calculated using the momentum equations for an unstratified, steady uniform planetary boundary layer beneath an interior region in which the flow is in geostrophic balance:

\[
\begin{align*}
\frac{\partial}{\partial z} \left( K_z \frac{\partial u}{\partial z} \right) - f (u_{geo} - v) &= 0, \\
\frac{\partial}{\partial z} \left( K_z \frac{\partial v}{\partial z} \right) + f (u_{geo} - u) &= 0,
\end{align*}
\]

where \( u \) and \( v \) are the \( x \) - and \( y \) -components of velocity, \( K_z \) is the vertical eddy viscosity, \( f \) is the Coriolis frequency, and \( u_{geo} \) and \( v_{geo} \) are the velocity components at the top of the boundary layer, which is assumed to lie in the geostrophically balanced region (Smith, 1977). Eq. (6) is solved by finite differences, subject to the boundary conditions that \( u = u_{geo} \) and \( v = v_{geo} \) at the top of the model grid and \( u = 0 \) and \( v = 0 \) at a roughness height, \( z_0 \), close to the bed; determination of \( z_0 \) is described below. Time dependence, horizontal momentum advection and density stratification in the currents are neglected in these calculations. This simplification of the flow field reduces model complexity and computation time by decoupling the velocity and suspended sediment calculations.

The eddy viscosity profile which is required to solve Eq. (6) depends on current and wave shear velocities:

\[ K_z = \kappa \left[ \left( u_{cw} e^{-z/\delta_c} \right)^2 + \left( u_{cw} e^{-z/\delta_w} \right)^2 \right]^{1/2} \]

(Wiberg and Smith, 1983), where \( \kappa = 0.408 \) is von Karman’s constant. The length scales of the wave and bottom boundary layers are given by \( \delta_w = u_{cw} / 3 \omega \) and \( \delta_c = u_{cw} / 6 f \), respectively; where \( f \) is the Coriolis frequency, \( \omega \) is the near-bed wave frequency \( (\omega = 2 \pi / T_{mb}) \), and the wave–current shear velocity, \( u_{cw} \), is defined below. To ensure that the bottom boundary layer neither becomes unreasonably small or large, the value of \( \delta \) is constrained to be within 250 cm or \( h/6 \), where \( h \) is the total depth of the flow.

Once the eddy viscosity and current velocity profiles have been calculated, they are used to find the current shear stress profile:

\[ \tau_{cx} = \rho K_z \frac{\partial u}{\partial z}, \]

\[ \tau_{cy} = \rho K_z \frac{\partial v}{\partial z}. \]

The boundary shear stress components due to the currents are estimated by evaluating Eq. (8) at \( z = z_0 \):

\[ \tau_{cx} = \tau_{cx}|_{z=z_0} \quad \text{and} \quad \tau_{cy} = \tau_{cy}|_{z=z_0}. \]

The resulting values are added to the wave shear stress (Eq. (5)) to obtain the total wave–current boundary shear stress \( (\tau_{cw}) \) and shear velocity \( (u_{cw}) \):

\[ \tau_{cw} = \sqrt{\left( |\tau_{cx}| + |\tau_{cy}| \right)^2} + \tau_{cy}^2 \]

(9) (Smith, 1977; Wiberg and Smith, 1983). This implementation assumes that the wave approach is always parallel to the \( x \)-direction, but it can be generalized to account for waves approaching from any direction by using the appropriate vector addition in place of Eq. (9).

Interdependencies between roughness height, eddy viscosity and boundary shear stress require that an iterative solution be used to solve for the current velocity profile, \( u \). Using a first guess for current shear velocity \( (u_c) \), the total boundary shear stress is initially estimated by \( \tau_{cw} = \rho u_c^2 + \rho u_{cw}^2 \). This shear stress is used to reevaluate bed roughness \( (k_b \text{ and } z_0) \), eddy viscosity \( (K_z) \), and current shear stress \( (\tau_c) \). The resulting current boundary shear stress \( (\tau_{cb}) \) is combined with the wave shear stress that was derived in Eq. (5) to revise the estimate of total bed shear stress \( (\tau_{cw}) \). This process (Eqs. (6)-(9)) is repeated until consecutive iterations yield estimates of total boundary shear stress \( \tau_{cw} \) that are within a specified tolerance level.
Different formulations are used to estimate bed roughness on silty and sandy beds because ripples and bedload usually dominate roughness on sandy beds, whereas bioturbated mounds typically dominate the roughness of silty beds. Bedform and bedload roughness heights often vary by an order of magnitude. For this reason, we take the roughness of sandy beds to be the larger of the bedform and bedload (saltation) roughness heights rather than their sum as is used in some other roughness formulations (e.g., Grant and Madsen, 1982). On many shelves, bed ripples are dominantly wave-formed, symmetric features, termed anorlital ripples by Clifton and Dingler (1984). Estimates of anorlital ripple height, \( \eta_{\text{rip}} \), and spacing, \( \lambda_{\text{rip}} \), are calculated based on sediment size and wave orbital velocities following Wiberg and Harris (1994). Their relationships approximate ripple spacing as \( \lambda_{\text{rip}} = 535D_{50} \) and ripple steepness as

\[
\eta_{\text{rip}} / \lambda_{\text{rip}} = \exp \left[ -0.095 \left( \frac{d_{0}}{\eta_{\text{rip}}} \right)^2 + 0.442 \ln \left( \frac{d_{0}}{\eta_{\text{rip}}} \right) - 2.28 \right],
\]

where \( D_{50} \) is mean grain size and \( d_{0} = (u_{*}T_{W}/\pi) \) is wave orbital diameter. Using these relationships, ripple height will decrease under energetic waves with large orbital velocities. The physical roughness length of bedforms \( (k_{b}) \) is estimated as \( k_{b} = 27.7\eta_{\text{rip}}^{2} / \lambda_{\text{rip}} \) (Grant and Madsen, 1982). Flow over the ripples is assumed to be hydraulically rough, with \( z_{0} = k_{b}/30 \) (Nikuradse, 1933).

Roughness due to saltating sediment tends to dominate bed roughness under energetic conditions when ripple crests are eroded, and is estimated following Wiberg and Rubin (1989):

\[
z_{0} = \frac{2D_{50}a_{1}T_{*}}{1 + a_{2}T_{*}},
\]

where \( a_{2} = 0.0204 \ln D_{50}^2 + 0.0220 \ln D_{50} + 0.0709 \)

\[
a_{1} = -0.056 \text{ and } a_{1} = -0.68 \text{ are coefficients set by Wiberg and Rubin (1989), and } T_{*} = \tau_{cw}/\tau_{sf}.
\]

For silty beds, the spacing of biogenic roughness elements \( (\lambda_{\text{bio}}) \) is estimated in the same manner as for ripples: \( k_{b} = 27.7\eta_{\text{bio}}^{2} / \lambda_{\text{bio}} \) and \( z_{0} = k_{b}/30 \).

For mixed sand/silt beds a weighted average of the silty-bed roughness scale and the sandy-bed roughness scale is used, weighted by the sand percent of the bed, \( \eta = \eta_{\text{rip}}fr_{s} + \eta_{\text{bio}}(1.0 - fr_{s}) \), and \( \lambda = \lambda_{\text{rip}}fr_{s} + \lambda_{\text{bio}}(1.0 - fr_{s}) \), where \( fr_{s} \) is the fraction of the bed that is sand. A minimum value of \( z_{0} = 0.005 \text{ cm} \) is specified so that roughness estimates do not become unreasonably small given the small-scale bed variations generally present on the sea floor.

The presence of bedforms reduces the shear stress acting on the bed surface, termed the skin friction shear stress \( (\tau_{sf}) \), relative to the total shear stress, \( \tau_{cw} \). It is the skin friction component of shear stress that is important for sediment transport calculations. The total shear stress is divided into its bedform and skin friction components using the estimated bedform dimensions:

\[
\tau_{sf} = \tau_{cw} \left[ 1.0 + 0.5C_{d} \frac{1}{k} \left( \frac{\eta}{\eta_{\text{sf}}} - 1 \right)^{2} \right],
\]

(Smith and McLean, 1977; Wiberg and Nelson, 1992), where \( C_{d} \) is the bedform drag coefficient and \( \eta_{\text{sf}} \) is the hydraulic roughness of the bed surface. Shear stress varies spatially over a bedform, with the highest stresses near the crest. As a result, it is possible for the spatially averaged shear stress over a ripple to be below the threshold of motion while the shear stress near the crest exceeds the threshold. To account for this, the spatially averaged skin-friction shear stress (Eq. (13)) is adjusted to provide an estimate of skin-friction shear stress near the ripple crest: \( \tau_{\text{sfm}} = \tau_{sf}(1.0 + 8.0\eta/\lambda) \). This shear stress is used to calculate entrainment rates in the suspended sediment calculations that are described in the following section.

2.2. Suspended sediment calculations

Once the flow field is calculated, the suspended sediment concentration field for each grain size is found by solving Eq. (1) using finite differences. This requires that boundary conditions be specified at the lateral boundaries and the top and bottom of the model grid. The top of the model grid is in the middle of the water column \( (h/2) \), because these calculations are intended to capture sediment transport that occurs in the bottom boundary layer, and the velocity profile used (Eq. (6)) is not valid above the layer that is in geostrophic balance. Because sediment concentration and the vertical diffusion coefficient are both low at this level, loss of sediment through the top of the model domain should be minimal. Therefore, this boundary is considered to be closed in the model. The cross-shelf boundaries are usually placed far from the region of interest or in locations where transport of sediment is limited by either the lack of available fine
sediment (inshore boundaries), or low wave energy (offshore boundaries). Because an upwind differencing scheme is used, only upstream boundary conditions will affect the finite difference solution. On the continental shelf, however, either of the cross-shelf boundaries can act as an “upstream” site, depending on whether flows are on- or off-shelf at that location. The upstream model boundary can be made open or closed by specifying the input sediment concentrations at this \( x \) location. For the cases described here, these boundaries were considered closed, and no sediment was input from upstream.

The horizontal diffusion coefficient \( (K_x) \) should account for sub-grid-scale mixing processes so that the horizontal mixing estimated by the model is similar to the observations from continental shelves. We have used \( K_x = 0 \text{ m}^2/\text{s} \) in our calculations, because the model introduces horizontal mixing in other ways. It directly accounts for some mixing processes, such as shear dispersion, that are not differentiated from other sources of mixing by many field observations (see Okubo, 1971).

In addition to this, the upwind differencing method we used contributes numerical diffusion, as is discussed in Section 3.

The bottom boundary condition has long been problematic in suspended sediment transport calculations. A flux boundary condition is needed to represent transport during times when sediment is settling out of suspension (Parker, 1978), but no clear choice exists for a flux boundary condition valid for both shelf sands and muds. We use a procedure based on the reference concentration formulation of Smith and McLean (1977) to specify the flux boundary condition. This is described below. We also considered the entrainment function proposed by Garcia and Parker (1991, 1993) which uses settling velocity in the calculation of the entrainment function, \( E \), which has units of concentration. It is not clear how to properly calculate the Garcia and Parker entrainment function, however, for fine-grained shelf environments where flocculation is likely to enhance settling velocity.

Following the formulation of Parker (1978), the upward flux of sediment from the bed is defined to be the product of settling velocity and an entrainment function \( (E) \) that has units of concentration. At steady state the rate of entrainment of sediment, \( E_w \), must equal the downward sediment flux near the bed, \( C_a w_a \), where \( C_a \) is the volumetric equilibrium concentration at a reference level close to the bed. Smith and McLean (1977) proposed an expression for \( C_a \) at the near-bed elevation \( z_a \),

\[
S_{sfm} > 0 \quad \text{and} \quad \frac{w_s}{\kappa u_{*sfm}} < 2.5, \quad C_a = \frac{fr C_{b,0} S_{sfm}}{1 + \gamma_0 S_{sfm}},
\]

\[
C_a = 0, \quad S_{sfm} \leq 0 \quad \text{or} \quad \frac{w_s}{\kappa u_{*sfm}} \geq 2.5, \quad (14)
\]

where \( S_{sfm} = (\tau_{sfm} - \tau_{cr})/\tau_{cr} \) is the excess skin-friction shear stress, \( fr \) is the volumetric fraction of a size class in the bed, \( C_b \), is the bed concentration of sediment and \( \gamma_0 \) is the resuspension coefficient. The conditions on the excess skin-friction shear stress and Rouse parameter \( (w_s/\kappa u_{*sfm}) \) ensure that the value calculated for \( C_a \) will be zero during times of low energy. Values cited for \( \gamma_0 \) range from \( 10^{-3} \) to \( 10^{-5} \) (Drake and Cacchione, 1989).

For this implementation, a value of \( \gamma_0 = 10^{-3} \) was used. This value is smaller than that used in similar one-dimensional calculations (i.e., \( \gamma_0 = 2 \times 10^{-3} \); Wiberg et al., 1994; Cacchione et al., 1999) to offset the fact that our calculations neglect density stratification of the water column by suspended sediments. We take \( z_a \) to be the top of the bedload layer, approximately \( 3D_{50} \).

To apply this boundary condition to unsteady conditions, we assume that the entrainment function, \( E \), equals the reference concentration estimated by Eq. (14), and equate entrainment with upward mixing due to turbulence at the level \( z_a' \),

\[
-K_z \frac{\partial C_a}{\partial z} \bigg|_{z_a} = E w_s = C_a w_a. \quad (15)
\]

Eq. (15) is applied separately for each sediment size in a mixed grain size distribution. When coupled with our approach for tracking the evolution of the grain size distribution of the bed and the development of bed grading, as described below, this formulation partially accounts for armoring of the bed by winnowing. Resuspension of fine sediment reduces the bed fraction of these sizes, thereby decreasing their entrainment rates.

In summary, the suspended sediment concentration field is found by solving Eq. (1) after specifying the velocity field, the horizontal eddy viscosity, sediment settling velocities, appropriate boundary conditions, and the horizontal diffusion coefficient. The transport equation (Eq. (1)) is discretized using a semi-implicit (implicit in \( z \)), upwind-differencing scheme with the specified boundary conditions (see Appendix A). This provides a tri-diagonal linear system of equations that is solved for \( C_z \). The suspended sediment concentration field is used to estimate net erosion/deposition across the transect by depth-integrating the suspended sediment concentration and flux for each grain size at each \( x \) location, and applying the erosion equation (Eq. (2)).

Modifications to bed sediment size and fine-scale stratigraphy by resuspension and transport are calculated in a manner similar to that used for the steady, one-dimensional version of the model (see Harris and Wiberg, 1997). At each \( x \) location, the bed is modeled as two layers. The uppermost layer is the surficial active, or mixed, layer that is available for suspension. It has a thickness \( (\delta_{mix}) \) that depends on grain size and shear stress. Sediment below the active layer is unavailable for resuspension until the active layer moves downward.
either because erosion has occurred, or it has thickened due to increased shear stresses. At each time step, the model updates the grain size distribution in the active layer and several underlying layers to account for changes to the thickness of the active layer and for erosion and deposition of sediment in each size class.

For each size class, the volume per unit bed area of sediment removed from the bed during any time step is limited by the amount available in the active layer. The thickness of the active layer is calculated differently for sandy and silty beds. For sandy beds, the depth of the active layer is related to the depth of the bed that is mixed by migrating ripples or sheet flow (Wiberg et al., 1994). The depth of sediment transport on sandy beds is estimated using an expression based on the migration of a bedform over the time scale of a half-wave period:

\[ \delta_{\text{mix}}^{(\text{rip})} = \frac{Q_b T_{\text{run}}}{2 C_{h} f_{\text{rip}}} + 6 D_{50}, \]

where \( Q_b \) is volumetric bedload transport rate estimated by:

\[ Q_b = \sum f_{\text{rip}} g (\rho_s - \rho) g^{-1} (\tau_{\text{sfm}} - \tau_{\text{cr}}(t))^{1.5}, \]

where \( f_{\text{rip}} \) is the fraction of the active layer in size class \( l \), and \( \rho_s \) is the density of the sediment grains (Meyer-Peter and Muller, 1948). The depth of the active layer is reduced to one-half the ripple height if the value estimated by Eq. (16) exceeds this value. The \( 6D_{50} \) term represents the grain-scale irregularities of the bed surface that results in a non-zero thickness of the active layer even when there is no transport (\( \tau_{\text{sfm}} < \tau_{\text{cr}} \)). For silty beds, the depth of the active layer is assumed to be proportional to excess shear stress relative to the critical shear stress (\( \tau_{\text{cr}}(s) \)) of the median grain size (\( D_{50} \)).

\[ \delta_{\text{mix}}^{(\text{silt})} = 8 (\tau_{\text{sfm}} - \tau_{\text{cr}}(s)) + 6 D_{50} \]

(Harris and Wiberg, 1997). For locations that contain both sands and silts, the depth of the active layer is estimated using an average of the two estimates of \( \delta_{\text{mix}} \) that is weighted by the fraction of the bed \( f_{\text{rs}} \) that is sand; \( \delta_{\text{mix}} = \delta_{\text{mix}}^{(\text{rip})} f_{\text{rs}} + \delta_{\text{mix}}^{(\text{silt})} (1 - f_{\text{rs}}) \). Using this procedure, active layer depths of the order of a few centimeters for energetic, sandy sites, to a few millimeters typical for less-energetic, muddy sites have been estimated (Harris, 1999). The volume of sediment available for suspension in a size class \( l \) per unit area of the bed is \( f_{\text{rip}} C_{\text{h}} \delta_{\text{mix}} \).

An additional step is necessary to explicitly account for bed armoring of the sediment bed. If the erosion rate calculated by solving Eq. (2) removes more sediment than is available in the active layer at a grid point, the entrainment rate, \( E \), for that bed location and sediment size is decreased from the value calculated in Eq. (15). When this occurs, the advection–diffusion equation (Eq. (1)) must be solved again using the modified entrainment rate(s), and the process is repeated until sediment availability is not exceeded.

3. Stability constraints

The tendency of the two-dimensional transport equation to exhibit numerical instability and numerical dispersion was investigated using three non-dimensional parameters. The values of these parameters depend on the length of the time step (\( \Delta t \)), horizontal and vertical length scales (\( \Delta x \) and \( \Delta z \)), diffusivity (\( K_x, K_z \)), and velocity scales (\( u, w_z \)). The settling velocity of sediment (\( w_s \)) is used as the vertical velocity scale, following the assumption that it is much larger than the time-averaged vertical velocity of the flow.

The Courant numbers indicate the number of grid points that would be traversed during a single time step. The vertical and horizontal Courant numbers can be, respectively, defined as \( Cr_z = w_s \Delta t/\Delta z \) and \( Cr_x = u \Delta t/\Delta x \), where \( w_s \) and \( u \) are vertical and horizontal velocities and \( \Delta z \) and \( \Delta x \) are the vertical and horizontal grid spacings. Courant numbers that exceed 1 indicate that an explicit solution will be unstable. For a two-dimensional system, an implicit formulation should be used if the sum of the Courant numbers \( (|Cr_x| + |Cr_z|) \) exceeds 1. For the system studied here, the vertical Courant number is much larger than 1 throughout the model domain, but the horizontal Courant number does not exceed 1 for velocities up to \( u_{\text{geo}} = 100 \text{ cm/s} \). Much of the difference between the scales of the horizontal and vertical Courant numbers stems from the fact that the vertical grid is very dense in order to accurately represent high gradients in shear stress and suspended sediment concentration near the sea floor.

The diffusion in the system is non-dimensionalized by length and time-scales of the finite difference solution; \( \delta_x = K_x \Delta t/\Delta x^2; \delta_z = K_z \Delta t/\Delta z^2 \). An explicit finite difference solution will not be stable if \( \delta_x \) or \( \delta_z \) exceed 1/2. For typical choices of eddy viscosity, \( \delta_z \) exceeds 1/2 throughout the water column. Stability should not be problematic in the horizontal direction, however, since \( \delta_x < 1/2 \) for values of \( K_x \sim 0.05 \text{ m}^2/\text{s} \), similar to values observed in the coastal zone.

Both the Courant and stability parameters indicate that the transport equation can be applied to continental shelves by using a semi-implicit finite difference approximation that is explicit in the horizontal direction (\( Cr_x < 1, \delta_x < 1/2 \)) and implicit in the vertical (\( Cr_z > 1, \delta_z > 1/2 \)). Time steps of the order of 5 min, and horizontal grid spacing of about \( \Delta x \sim 0.4 \text{ km} \) were used to represent resuspension events on the continental shelf. Because it is always stable, an upwind scheme has been used to discretize Eq. (1), (see Appendix A; Fletcher, 1991; Patankar, 1980; Peyret and Taylor, 1982). The
The Peclet number indicates the relative importance of advection and diffusion, \( Pe = Cr_x/\Delta x = u\Delta x/K_x \); \( Pe_v = Cr_v/\Delta x = u\Delta x/K_v \). Systems that have high Peclet numbers (\( > 2 \)) are dominantly advective. The Peclet numbers for typical velocity and diffusion scales indicate that the continental shelf is dominantly diffusive in the vertical direction. The spatial and temporal grid used can result in high Peclet numbers in the upper part of the model grid, indicating that the solution is subject to numerical dispersion at these heights. This is only true for coarse sediment, however, which is usually not suspended to these heights. In the horizontal direction, the solution is dominantly advective for diffusion coefficients that are close to those observed for length scales equal to the model grid spacing \( (K_x = 0.05 \text{m}^2/\text{s}; \text{see Okubo, 1971}) \). The upwind differencing scheme introduces numerical diffusivity, however, that result in an effective diffusivity equal to \( K_x^{\text{eff}} \approx u\Delta x/2 \). We typically use horizontal grid spacings of 400 m, and velocities on the order of 10 cm/s, so that \( K_x^{\text{eff}} = 20 \text{m}^2/\text{s} \), which is equivalent to the diffusivity observed over distances of 20 km (see Okubo, 1971). The Peclet number calculated using the effective diffusivity indicates that the calculations produce a diffusive system. A sensitivity test showed that numerical diffusion is not significant for the case presented in the following section. The advective flux, estimated by \( u\Delta V_x/\Delta x \), dominated the diffusive flux \( (K_x^{\text{eff}}\Delta V_x/\Delta x^2) \), by a factor of 8. Calculations were insensitive to a doubling of the horizontal grid resolution, indicating that the numerical diffusion was unimportant. Similar tests should be conducted whenever gradients that act over spatial scales smaller than those that correspond to the numerical diffusivity are of interest.

4. Example of model calculations

Application of the model to a relatively simple continental shelf example allows us to demonstrate model performance and provides a framework for discussing the effect of the nearshore boundary condition. A cross-shelf model transect that has an initially uniform and ungraded sediment bed is subjected to steady, energetic waves with significant heights of \( H_{\text{sig}} = 5 \text{m} \). Velocity calculations are driven by input currents with speeds of 10 cm/s directed 45° offshore and poleward, which were applied uniformly at the top of the model transect. A silty sand sediment bed was chosen for this example (Table 1). A minimum settling velocity of 0.1 cm/s was assumed, based on observations of flocculated sediment size and settling velocity in fine-grained marine settings (Sternberg et al., 1999; Hill and McCave, 1999). Cohesive behavior was accounted for by adopting a minimum critical shear stress for entrainment of \( \tau_{cr} = 1 \text{dy/cm}^2 \), following previous work (Wiberg et al., 1994; Harris and Wiberg, 1997).

The model covers an area that is 30 m deep on the inner-shelf boundary, and 150 m deep at the offshelf boundary (Fig. 1A). The top of the model grid is taken to be one-half the water depth at each location along the transect. Seaward of 30 m water depth, the transect initially has a uniform sediment size distribution equal to the one in Table 1. A boundary zone having constant water depth of 30 m was added to the model domain shoreward of the region of interest in an attempt to minimize boundary effects. The sediment size distribution was coarsened towards the nearshore edge of this boundary zone to minimize transport there (Figs. 1A and C).

Initially, wave energy was low and there was no suspended sediment. After a brief spin-up time, wave energy was increased until a steady peak wave height of \( H_{\text{sig}} = 5 \text{m} \) was reached (Fig. 1B), at which time bottom wave orbital velocities across the shelf ranged from 85 cm/s to less than 1 cm/s (Fig. 1D). Fine-grained sediment typically can be eroded when bottom orbital velocities exceed 10–15 cm/s, so that for this case sediment was initially entrained and suspended in water depths less than 80 m (Fig. 1D). Suspended sediment flux was given by the product of calculated velocity and suspended sediment concentration (Fig. 2).

At the beginning of the wave event, suspended sediment concentration and suspended sediment flux were highest in the shallow, energetic portion of the model grid (Fig. 3). As the system evolved, however, fine sediment was removed from the upstream portion of the transect, where wave shear stresses were highest, and advected offshore. This reduced the bed fraction of the fine-grained sediment (Fig. 3B) and the predicted entrainment rates decreased (see Eq. (15)). The suspended sediment flux thereby exhibited a time-dependent response to the steady forcing function (Fig. 3C). After three days of energetic waves, the model calculated a net import of \( 7.5 \times 10^4 \text{g} \) of sediment per unit width (cm) of the bed. Of this, 56% was deposited on the sea floor. Accounting for flux into and out of the model...
domain, suspended sediment, and net deposition, these calculations conserved mass to within 2%.

For the offshore-directed currents considered in this example, the cross-shelf-oriented model grid required a lateral boundary condition at the shoreward edge of the computational domain. The choice of boundary configuration for the shoreward edge affects transport calculations in the region of interest in two ways. First, fine-grained sediment entrained from the boundary zone can be advected into the region of interest, which may import an unrealistic amount of fine-grained sediment at the inner-shelf boundary. Secondly, once the supply of fine sediment in the boundary zone has been depleted, a region of flux divergence and erosion may migrate...
offshore and enhance erosion rates calculated for the region of interest.

To evaluate these effects, calculations were made using two boundary configurations for the shoreward portion of the model domain for the steady forcing shown in Fig. 1B. The boundary configuration used had a 3–4 km wide zone of grid points shoreward of $x = 0$ km (Figs. 1 and 4). Calculations made using this model grid were compared to a similarly constructed one that used a 10 km wide zone of boundary grid points shoreward of $x = 0$ km (Fig. 4). It should be noted that this represents an extreme test of the lateral boundary condition, since it shows the response of the system to a prolonged period (3 days) of energetic waves, with flows consistently directed towards offshore. More realistic conditions would contain periods when currents were directed towards shore, which could decrease the rate of removal of fine-grained sediment from the boundary zone.

Results using the boundary configurations in Fig. 4 and the steady forcing function shown in Fig. 1B illustrate the trade-off between supplying fine grained sediment from upstream and eroding sediment at the inshore boundary of the region of interest (Fig. 5). The 10 km wide boundary zone had more fine-grained sediment available for resuspension upstream of the model domain. Over the three day period modeled, that resulted in twice as much fine grained sediment input into the model domain than the case using a 4 km wide boundary zone. After three days of energetic waves, the 10 km boundary zone had imported $16 \times 10^4$ g/cm of sediment into the region of interest, about 73% of it from the $<45 \mu m$ size class. This amounts to an average flux of sediment into the region of interest of about 0.5 g/(cm s). The higher sediment input for the 10 km case reduced erosion depths calculated for the shoreward edge of the region of interest and increased bed deposition for the outer-shelf portion of the model domain (Fig. 5B). The wider boundary zone also resulted in smoother calculations of both flux and bed elevation (Fig. 5). Actual applications of this type of model must take care to treat the shoreward boundary in a manner that appropriately represents the real situation.

5. Discussion

This model has been used to predict bed reworking and suspended sediment transport along a cross-shelf transect for time scales of up to 128 days (Harris and Wiberg, 1998; Harris, 1999). The speed of the computations depends on the computer system, the number of grain sizes, the length of the time step, and the resolution of the vertical grid. Using an Ultra-Sparc workstation and modeling the response of three grain sizes, the 128 day simulations were completed overnight (Harris, 1999). The calculations described in Section 4 required less than 1 h. With the semi-implicit scheme, calculations are fairly insensitive to the length of the time step. Time steps greater than 10 min produced instabilities in the calculations, however, because the bed roughness estimates for one time step are based on the shear stress from the previous time step. For this reason, 5 min time steps were used.

The calculations of shear stress and suspended sediment concentration are more sensitive to the
resolution used for the vertical (z) dimension than the time step, because the upwind differencing scheme used results in a first-order dependence on the size of the vertical grid spacing. High resolution in the vertical grid near the bed is also required to represent accurately the high concentration and velocity gradients in this region, so the vertical grid is logarithmically spaced. Between 300 and 500 vertical grid points were required, with 50 grid points per decade, to achieve accuracy for these model runs. The grid spacing near the sea-bed are on the order of $10^{-4}$ cm. While preserving a high degree of vertical resolution, the model neglects some effects that other, more coarsely gridded models are able to include, such as density stratification, and non-steady-state conditions in the velocity field. Use of a second order van Leer (1977) style upwind differencing scheme could decrease the number of grid points required.

Spatial terms are required to model situations where flux divergence and advection significantly affect transport and deposition. One-dimensional models that assume local uniformity will not directly account for these terms. The significance of advection and flux divergence on cross-shelf transport is illustrated by comparing calculations made using the two-dimensional model for the transect in Fig. 1 to those calculated using a similarly structured version of the one-dimensional, time-dependent model applied at several water depths (Fig. 6).

Time-dependence in the one-dimensional calculations depends only on the time required for a site to reach equilibrium (Fig. 6A). The one-dimensional calculations therefore do not predict the behavior of the suspended sediment field seen in Fig. 3 that derives from advection and winnowing of the sediment bed. This has implications for calculations of net erosion and deposition that are based on a one-dimensional model. For example, the flux divergence pattern calculated using the two-dimensional model predicts that the area between 50 and 60 m water depths would be depositional until approximately 1.3 days into the model simulation (Fig. 3C). After this, the available fine sediment has been removed from shoreward of 50 m, thereby decreasing the delivery of sediment to the 50 m site. The system then begins to entrain sediment between 50 and 60 m water depths, and net erosion occurs between these water depths (see Fig. 3C). This time-dependence is not accounted for in one-dimensional models. Instead, the one-dimensional calculations predict net deposition between water depths of 50 and 60 m for the entire simulation (Fig. 6C).

The model calculations, particularly the erosion rates at the shoreward edge of the region of interest, are sensitive to sediment import from upstream. Because of the influence of flux at the upstream boundary on erosional patterns, it is important to have some confidence in the rate of sediment supplied to the inner shelf from inshore. This is a difficult value to constrain, however, in the absence of site-specific information. Ideally, the capacity of the innermost shelf and nearshore zone to supply fine-grained sediment to the landward edge of the model domain should be quantified. This could be accomplished either by estimating the volume of fine-grained sediment typically available in the innermost shelf or measuring sediment flux at the interface between the nearshore zone and shelf at the site of interest.

While this model makes some assumptions that limit its applicability, it can reproduce many of the important transport processes on continental shelves and can be modified to treat situations other than the one example shown. Because it uses linear wave theory to estimate

---

Fig. 6. Calculations made using gridded one-dimensional model of (A) bed elevation and (B) sediment flux for three water depths (see legend) as system responds to steady, energetic waves.
bottom orbital velocities it cannot be applied to nearshore areas where waves are highly nonlinear or breaking. The eddy viscosity formulation is averaged over the wave period and will therefore not account for processes that operate over a smaller time-scale than the wave period. It does not include the effects of density gradients on the mixing profiles, and should not be applied to areas in which stratification by suspended sediments, temperature, or salinity are dominant effects.

In many shelf environments, however, sediment transport occurs primarily during winter storms when the bottom-boundary layer is relatively well mixed with respect to temperature and salinity. The value of the resuspension coefficient, $\gamma_0$, was reduced to partially account for the effects of stratification due to the presence of suspended sediment. It is therefore assumed that this approach will adequately represent times of significant transport.

The particular methods we adopted for estimating parameters such as entrainment rate, bedform dimensions, and the bed roughness length may not be optimal for all shelf settings. Other formulations can be substituted as appropriate. For example, the particular bed roughness formulation presented here for sandy beds assumes that the bedforms are dominantly formed by wave oscillations. This would not be appropriate in areas where tides or currents dominate ripple formation. Furthermore, there are several alternate formulations for wave-formed ripple roughness that could be used (e.g. Nielsen, 1992; Grant and Madsen, 1982; Li and Amos, 1998).

The forcing wave and current velocities are not limited to the steady, uniform waves and currents used in the example presented here. This model has been run using wave and current fields that varied in both time and space. For example, it has been used to evaluate depositional patterns and cross-shelf flux divergences associated with variations in current across the shelf and with storm sequences (Harris, 1999; Harris and Wiberg, 1998).

This model can be applied to shelf systems where the important spatial variations are two-dimensional, and applications of this model are not limited to cross-shelf transects. The model grid could be aligned so that the x-direction parallels shelf bathymetry to represent situations where sediment characteristics, or flow velocity varies in the alongshelf direction. For example, an along-shelf transect would be useful to study sedimentation in areas near a discrete source of suspended material, such as in the vicinity of a mud patch, or downstream from the mouth of a river, or a sewage outfall.

While a two-dimensional approach may represent the important processes in many shelf environments, others will require a fully three-dimensional model to adequately represent transport and deposition. Three-dimensional coastal shelf circulation models are available that also include the effects of density and advection of momentum (see, e.g. Blumberg and Mellor, 1987). Some of these also include sediment transport, but they do not represent sediment transport with the level of detail included in our model. Furthermore, many coastal circulation models do not allow for a detailed representation of bottom boundary layer processes because their vertical resolution is too coarse to fully resolve the wave boundary layer (Blumberg and Mellor, 1987). While a typical circulation model uses about 10 vertical grid cells to represent the water column, our calculations required several hundred. Our resolution could be coarsened with use of a higher-order upwind differencing scheme (e.g., van Leer, 1977), but probably not to a degree that it could be applied to regional spatial scales. An alternative approach is to parameterize the bottom boundary layer, wave boundary layer, and suspended sediment concentration profile so that accuracy is achieved without high resolutions of grid cells (e.g. Keen and Glenn, 1994; Zhang et al., 1999). While this approach is promising, it has yet to be applied in a fully time-dependent manner as we have done with the two-dimensional model, and has not been fully tested against continental shelf field measurements.

6. Conclusions

A two-dimensional, time-dependent solution to the advection–diffusion equation has been developed to calculate the transport of suspended sediment in the bottom boundary layer of continental shelves. Spatial and temporal variation in the suspended sediment field are related to erosion and deposition of bed sediment through the erosion equation (Eq. (2)). The estimates of entrainment and deposition of sediment are used to predict changes to seabed stratigraphy. Model calculations demonstrate that advection of sediment can impact transport and bed reworking over timescales typical of energetic waves on the shelf. A semi-implicit formulation takes advantage of the disparate velocity scales in the horizontal and vertical directions. An upwind differencing scheme has been used to avoid stability problems. This solution is efficient enough to simulate seasonal timescales in a reasonable computation time.

The advective and time-dependent terms are needed in the suspended sediment transport equation to calculate net erosion and deposition. The two-dimensional, time-dependent approach represents shelf-wide transport processes in a more direct manner than was possible using the one-dimensional solutions on which it is based. The one-dimensional approach, however, has been tested favorably against measurements from several sites with very little adjustment to model parameters (e.g. Wiberg et al., 1994; Cacchione et al., 1999; Wiberg
et al., 2001). When the two-dimensional, time-dependent model is configured to run under uniform conditions and steady forcing functions, it provides results that are similar to the more widely tested one-dimensional, steady model.

The model can be used to study a variety of shelf processes. It is a promising tool for investigating general shelf sediment transport processes, as was illustrated by the example in this paper of cross-shelf transport during steady wave and current conditions. It can also be driven using realistic wave and velocity time series in order to simulate transport and bed reworking during a particular time period. Further investigations could include the importance of gradients in wave energy, current velocity, and sediment size relative to the time-dependent terms that derive from fluctuations in wave energy and current velocity. The model can also be modified to simulate other types of shelf systems than the example presented here, such as an along-shore transect downstream of a sediment source.

Acknowledgements

The development of this model benefited from discussions with Chris Sherwood, now at the U.S.G.S. in Menlo Park, Ca. and Rocky Geyer of Woods Hole Oceanographic Institute. Funding for this work came from the Office of Naval Research’s Marine Geology and Geophysics Program, Grant No, N00014-91-J-1349, under the guidance of Joe Kravitz. The authors also appreciate the input from Rich Signell (U.S.G.S., Woods Hole, MA), Chris Sherwood, and two anonymous reviewers on earlier drafts of this paper.

Appendix. A: Discretization of transport equation using semi-implicit, upwind-differencing

The transport equation (Eq. (1)) was discretized using a scheme that is implicit in the vertical (z) and explicit in the horizontal (x) directions. Upwind methods were used to estimate the spatial terms, \( \partial / \partial x, \partial / \partial z \). This results in a system of linear equations of the form

\[
AC_{z_{n+1}}^{k+1} = BC_{z_{n+1}}^{k} + D,
\]

\[
A_{n,n-1}C_{z_{n-1},j}^{k+1} + A_{n,n}C_{z_{n},j}^{k+1} + A_{n,n+1}C_{z_{n+1},j}^{k+1} = B_{n,n-1}C_{z_{n-1},j}^{k} + B_{n,n}C_{z_{n},j}^{k} + B_{n,n+1}C_{z_{n+1},j}^{k} + D_{n},
\]

where \( i \) and \( j \) are the \( x \)- and \( z \)-indices of the grid point \( k \) is the index of the time-step, \( J \) is the number of vertical gridpoints, and the two-dimensional space is vectorized using \( n = (i-1)J + j \). The matrices \( A \) and \( B \) contain the terms derived from vertical (A) and horizontal (B) mixing and advection. The vector \( D \) contains boundary terms. Because \( A \) is tridiagonal, Eq. (A.1) can be efficiently solved once the right-hand side is evaluated using the known values of \( C_{z_{n}}^{k} \). Following the upwind-differencing, when \( (w_{s} + K_{z}) < 0 \); the matrix \( A \) is specified as

\[
A_{n,n-1} = -\left(w_{s} + K_{z}\right) \frac{\Delta t}{\Delta z_{l}} + K_{z} \frac{\Delta t}{\Delta z_{l} \Delta z_{m}},
\]

\[
A_{n,n} = -1 + \left(w_{s} + K_{z}\right) \frac{\Delta t}{\Delta z_{u}} - 2K_{z} \frac{\Delta t}{\Delta z_{l} \Delta z_{u}},
\]

\[
A_{n,n+1} = K_{z} \frac{\Delta t}{\Delta z_{m} \Delta z_{l}},
\]

\[
A_{n,m} = 0 \text{ for all other } m's.
\]

When \( (w_{s} + K_{z}) \geq 0 \) the terms in \( A \) are specified as

\[
A_{n,n-1} = K_{z} \frac{\Delta t}{\Delta z_{l} \Delta z_{m}},
\]

\[
A_{n,n} = -1 - \left(w_{s} + K_{z}\right) \frac{\Delta t}{\Delta z_{u}} - 2K_{z} \frac{\Delta t}{\Delta z_{l} \Delta z_{u}},
\]

\[
A_{n,n+1} = \left(w_{s} + K_{z}\right) \frac{\Delta t}{\Delta z_{u}} + K_{z} \frac{\Delta t}{\Delta z_{l} \Delta z_{m}},
\]

\[
A_{n,m} = 0 \text{ for all other } m's.
\]

Here, \( K_{z} = \partial K_{z} / \partial z, \Delta z_{u} = z_{l+1,j} - z_{l,j}, \Delta z_{l} = z_{l,j} - z_{l-1,j}, \) and \( \Delta z_{m} = \left(\Delta z_{u} + \Delta z_{l}\right)/2 \). The elements in the matrix \( B \) account for horizontal mixing and advection, and when \( u \leq 0 \), are specified as

\[
B_{n,n-1} = \frac{K_{x} \Delta t}{\Delta x^{2}},
\]

\[
B_{n,n} = -1 + \frac{u \Delta t}{\Delta x} + 2 \frac{K_{x} \Delta t}{\Delta x^{2}},
\]

\[
B_{n,n+1} = \frac{u \Delta t}{\Delta x} - \frac{K_{x} \Delta t}{\Delta x^{2}},
\]

\[
B_{n,m} = 0 \text{ for all other } m's.
\]

The terms in \( B \) are specified as follows when \( u > 0 \);

\[
B_{n,n-1} = -\frac{u \Delta t}{\Delta x} - \frac{K_{x} \Delta t}{\Delta x^{2}},
\]

\[
B_{n,n} = -1 + \frac{u \Delta t}{\Delta x} + 2 \frac{K_{x} \Delta t}{\Delta x^{2}},
\]

\[
B_{n,n+1} = \frac{K_{x} \Delta t}{\Delta x^{2}},
\]

\[
B_{n,m} = 0 \text{ for all other } m's.
\]

The vector \( D \) in Eq. (A.1) is zero for all gridpoints except those on a boundary. The value used for \( D_{n} \)'s at boundary grid-points will depend on the particular
boundary condition specified, and whether that grid point is a top, bottom, upstream or downstream boundary. Eq. (A.1) is then solved by back substitution to obtain $C_{k+1}$.

### Appendix. B: Table of Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t$</td>
<td>length of time step (s)</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>grid spacing in horizontal direction (cm)</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>grid spacing in the vertical direction (cm)</td>
</tr>
<tr>
<td>$\Delta z_m$</td>
<td>average of $\Delta z_l$ and $\Delta z_u$ (cm)</td>
</tr>
<tr>
<td>$\Delta z_u$</td>
<td>spacing between a grid point and the point above it (cm)</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>non-dimensional constant, horizontal direction</td>
</tr>
<tr>
<td>$\alpha_z$</td>
<td>non-dimensional constant, vertical direction</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>resuspension coefficient</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>height of bottom boundary layer (cm)</td>
</tr>
<tr>
<td>$\delta_w$</td>
<td>height of wave boundary layer (cm)</td>
</tr>
<tr>
<td>$\delta_{mix}$</td>
<td>depth of the active, or mixed layer of surficial sediments (cm)</td>
</tr>
<tr>
<td>$\delta_{(rip)}_{mix}$</td>
<td>depth of the active, or mixed layer of sandy, rippled sediments (cm)</td>
</tr>
<tr>
<td>$\delta_{(silt)}_{mix}$</td>
<td>depth of the active, or mixed layer of silty, bioturbated sediments (cm)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(1) bed elevation relative to datum of $z = 0$; or (2) bedform height (cm)</td>
</tr>
<tr>
<td>$\eta_{bio}$</td>
<td>height of biogenically formed bedforms (cm)</td>
</tr>
<tr>
<td>$\eta_{rip}$</td>
<td>ripple height of sandy sediments (cm)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>von Karman’s constant</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength of bedforms (cm)</td>
</tr>
<tr>
<td>$\lambda_{rip}$</td>
<td>wavelength of ripples; sandy sediments (cm)</td>
</tr>
<tr>
<td>$\lambda_{bio}$</td>
<td>wavelength of biogenic bedforms (cm)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>3.1415</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of water (g/cm$^3$)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of sediment (g/cm$^3$)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress (dy/cm$^2$)</td>
</tr>
<tr>
<td>$\tau_{bc}$</td>
<td>bed shear stress due to currents (dy/cm$^2$)</td>
</tr>
<tr>
<td>$\tau_{bw}$</td>
<td>bed shear stress due to waves (dy/cm$^2$)</td>
</tr>
<tr>
<td>$\tau_{cr}$</td>
<td>critical shear stress for initial motion (dy/cm$^2$)</td>
</tr>
<tr>
<td>$\tau_{cr(50)}$</td>
<td>critical shear stress; mean sediment size (dy/cm$^2$)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency of wave (rad/s)</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>angular frequency of spectral bin $n$ (rad/s)</td>
</tr>
<tr>
<td>$A$</td>
<td>matrix used in solution of suspended sediment transport equation</td>
</tr>
<tr>
<td>$A_{n,m}$</td>
<td>elements of matrix $A$</td>
</tr>
<tr>
<td>$B$</td>
<td>matrix used in solution of suspended sediment transport equation</td>
</tr>
<tr>
<td>$B_{n,m}$</td>
<td>elements of matrix $B$</td>
</tr>
<tr>
<td>$C_{s}$</td>
<td>concentration of sediment near the bed at equilibrium (cm$^3$/cm$^3$)</td>
</tr>
<tr>
<td>$C_{s}$</td>
<td>concentration of sediment (1–porosity, cm$^3$/cm$^3$)</td>
</tr>
<tr>
<td>$C_d$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$C_{rx}$</td>
<td>Courant number in horizontal direction</td>
</tr>
<tr>
<td>$C_{rz}$</td>
<td>Courant number in vertical direction</td>
</tr>
<tr>
<td>$C_{s}$</td>
<td>suspended sediment concentration (cm$^3$/cm$^3$)</td>
</tr>
<tr>
<td>$D$</td>
<td>vector used in solution of suspended sediment transport equation</td>
</tr>
<tr>
<td>$D_{50}$</td>
<td>mean grain size of sediment bed (cm)</td>
</tr>
<tr>
<td>$D_f$</td>
<td>grain size of sediment size class $l$ (cm)</td>
</tr>
<tr>
<td>$E$</td>
<td>entrainment function of sediment, same units as sediment concentration (cm$^3$/cm$^3$)</td>
</tr>
<tr>
<td>$H_{sig}$</td>
<td>significant wave height (m)</td>
</tr>
<tr>
<td>$K_x$</td>
<td>horizontal diffusion coefficient (cm$^2$/s)</td>
</tr>
<tr>
<td>$K_{xeff}$</td>
<td>effective horizontal diffusivity (cm$^2$/s)</td>
</tr>
<tr>
<td>$K_z$</td>
<td>vertical diffusion coefficient (cm$^2$/s)</td>
</tr>
<tr>
<td>$K_z$</td>
<td>gradient of vertical diffusion coefficient ($\partial K_z/\partial z$); (cm/s)</td>
</tr>
<tr>
<td>$N$</td>
<td>number of frequency bins in wave spectra</td>
</tr>
<tr>
<td>$Pe_x$</td>
<td>horizontal Peclet number</td>
</tr>
<tr>
<td>$Pe_z$</td>
<td>vertical Peclet number</td>
</tr>
<tr>
<td>$Q_{b}$</td>
<td>bedload transport rate by volume: (cm$^3$/cm$^2$s))</td>
</tr>
<tr>
<td>$S_{sfm}$</td>
<td>excess skin-friction shear stress; maximum value over the crest of bedforms</td>
</tr>
<tr>
<td>$T_{amb}$</td>
<td>average period of wave orbital motion near seabed (s)</td>
</tr>
<tr>
<td>$T_{dom}$</td>
<td>dominant wave period (s)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>transport parameter; shear stress non-dimensionalized by critical shear stress</td>
</tr>
<tr>
<td>$T_{w}$</td>
<td>transport parameter of wave shear stress</td>
</tr>
<tr>
<td>$V_s$</td>
<td>depth-integrated volume of sediment in suspension (cm$^3$/cm$^3$)</td>
</tr>
<tr>
<td>$a_n$</td>
<td>amplitude of wave frequency bin $n$ (cm)</td>
</tr>
<tr>
<td>$a_t$</td>
<td>constant used in calculation of saltation roughness</td>
</tr>
<tr>
<td>$a_z$</td>
<td>constant used in calculation of saltation roughness</td>
</tr>
<tr>
<td>$d_o$</td>
<td>wave orbital diameter near the bed (cm)</td>
</tr>
<tr>
<td>$f$</td>
<td>Coriolis frequency (s$^{-1}$)</td>
</tr>
<tr>
<td>$f_{rf}$</td>
<td>fraction of sediment bed that is in size class $l$</td>
</tr>
<tr>
<td>$f_{rs}$</td>
<td>fraction of sediment bed that is sand</td>
</tr>
<tr>
<td>$f_w$</td>
<td>wave friction factor</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration (cm/s$^2$)</td>
</tr>
<tr>
<td>$h$</td>
<td>total depth of flow (cm)</td>
</tr>
<tr>
<td>$k_b$</td>
<td>physical roughness of bed (cm)</td>
</tr>
<tr>
<td>$k_n$</td>
<td>wave number of frequency bin $n$ (cm$^{-1}$)</td>
</tr>
</tbody>
</table>
References


