Abstract

In this study, numerical solution of advection–diffusion equation with third-order upwind scheme by using spreadsheet simulation (ADE-TUSS) is carried out. This is a user-friendly and a flexible solution algorithm for the numerical solution of the one dimensional advection–diffusion equation (ADE). The ADE-TUSS algorithm is based on the description of ADE by using the third-order upwind scheme (TU) for advection term and second-order central finite representation. For the solution of the governing equations, spreadsheet simulation (SS) technique is used instead of conventional solution techniques. The solution of ADE can be obtained for explicit, implicit and Crank–Nicolson schemes by only changing the values of temporal weighted parameter in the ADE-TUSS algorithm. It is clear in the literature that numerical diffusion causes great deviations in the model results when the first- or second-order upwind schemes are used. In order to decrease the numerical diffusion and to obtain oscillation free results, an artificial diffusion term is usually defined or sizes of the time step and/or grid sizes are set small values. Reduce of the grid sizes and/or time step increases the computational time and generally requires writing a fairly complex code when high order schemes are used. However, numerical solution of ADE by taking into account TU scheme has been carried out by using the iterative spreadsheet solution technique in the proposed solution algorithm. In order to simulate transient solution, a simple macro that carries out time cycle is defined with the help of VBA feature of spreadsheets. One of the most important advantages of ADE-TUSS algorithm is that it does not require the matrix algebra at each time step of the transient solutions. In order to test the ADE-TUSS model, two examples having numerical and analytical solutions are solved. Results showed that use of the high-order schemes in the spreadsheet simulation is very applicable for the numerical solution of ADE. Moreover, numerical diffusion problem is drastically prevented by using the ADE-TUSS model.

Keywords: Advection–diffusion; Third-order upwind schemes; High-order schemes; Spreadsheet; Numerical diffusion

1. Introduction

The mathematical model that describes the transport and diffusion processes is the advection–diffusion equation. Mathematical modeling of heat transport, pollutants and suspended matter involves the solution of advection–diffusion equation (ADE). In order to solve ADE analytical and numerical solution techniques are used. Analytical solution of ADE may be carried out when simple and idealized conditions are satisfied. However, if the solution parameters and transport processes change in time, use of the numerical solution techniques is necessary for the solution of ADE. One of the numerical solution techniques is the finite-difference method which may be easily applied to flow and transport modeling. The theory and solution techniques of the finite-difference method can be found in many textbooks [1–5].

In the standard finite-difference method, the interface concentration is evaluated by using either the upstream or the central in space weighting scheme. For the upwind weighting scheme, the interface concentration between two neighboring nodes in a particular direction, (e.g., \( x \)) is set equal to the concentration at the upstream node along
the same direction. Therefore, the upstream weighting schemes result in oscillation-free solutions. However, numerical solution of the advection term has first order accuracy and numerical solution of this scheme may lead to significant numerical dispersions in the advection dominated problems. In the central of the space weighting scheme, the solution of the advection term has the second order accuracy and the numerical solution does not lead to any numerical diffusion. However, it may lead to excessive artificial oscillation, which is typical of higher-order truncation errors [5], in the advection dominated transport problems. Due to the dual problems of numerical diffusion and artificial oscillation, use of the standard finite-difference solution techniques may be applicable only for solving ADE that is not dominated by advection. Several studies indicated that standard finite difference methods may be applicable when the grid Peclet number is smaller than four.

In order to solve one-dimensional ADE with the finite difference method, Noye and Tan [6] has used a weighted discretisation with the modified equivalent partial differential equation. Later, the authors extended this scheme to solve two-dimensional ADE [7]. However, solution of two- and three-dimensional problems by using these solution techniques is difficult since requirement of matrix inversions at each time step.

The upwind scheme of Spalding [8] and the flux-corrected scheme [9] are available for the solution of the depth-averaged form of the ADE. Another widely used approach is the split-operator approach [10,11], in which the advection and diffusion terms are solved by two different numerical methods.

Many studies show that the use of central differencing for the advection terms results in negative species concentration. Lam [12] pointed out that the central difference approximation will overestimate the advective flux so much that it often causes a negative concentration to appear in the neighboring cells. Some researchers have suggested flux corrected methods which take into account the mass flow rates and flow directions on the grid cell boundaries by interpolation [13–16]. The quadratic differencing algorithm proposed by Leonard [17] introduces an upstream interpolation method, namely QUICK (quadratic upstream interpolation convective kinematics) for one dimensional unsteady flow to prevent this situation. Several variations of the QUICK schemes retaining its desirable attributes while eliminating unphysical overshoots and oscillations have been developed. Among these are Sharpe and monotonic algorithm for realistic transport (SMART) [18]. Later, Leonard [19] improved this scheme by eliminating the wiggles completely by introducing exponential integration into regions with sharp fronts.

The upwind or donor cell method introduced by Gentry et al. [20] is generally used. Grima and Newman [21] have found that the “master equation discretization” (MED) has generally more accuracy than both “linear centered discretization” (LCD) and upwind schemes. Also, it allows the use of two and four times larger grid sizes than tradi-

tional schemes. The moment propagation method has been used to solve the advection–diffusion equation in the simulation of the transport of nutrients to a growing coral colony [22] and was further developed to solve electro-viscous transport problems [23]. Merks et al. [24] showed that a modification of the moment propagation method allows advection–diffusion simulations with higher Peclet numbers, in particular in the low Reynolds number limit.

Sommerijer and Kok [25] improved a finite differences model for the numerical solution of three dimensional ADE based on various time-integration techniques. This model was validated by comparing the results obtained by analytical solutions in the case of transport of a Gaussian pulse in unsteady flow.

Huang et al. [26] developed an accurate third-order numerical approximation of the solute transport equation. This approach deals with the corrections for both the diffusion coefficient and the convective velocity. The third-order algorithm is shown to yield very accurate solutions near sharp concentration fronts, thereby showing its ability to eliminate numerical diffusion. However, this scheme does suffer from numerical oscillations.

Zoppou et al. [27] demonstrated the use of shape-preserving exponential spline interpolation in a characteristic based numerical scheme for the solution of the ADE. This is an accurate scheme and it captures discontinuities, does not introduce spurious oscillations, and preserves the monotonicity and positivity properties of the exact solution.

In this study, a third-order upwind finite difference scheme is modified in respect to time dimension as given in Kowalik and Murty [28] for the advective terms of ADE. Earlier, the authors Shankar et al. [29] used this scheme for the advective terms of the shallow water momentum equations and Sankaranarayanan et al. [30] have used for the transport of conservative pollutants.

However, solution of ADE with previously mentioned techniques is difficult because of the necessity for extensive matrix algebra at each time step. The development of computer technology may ease the solution the partial differential equations (PDE) without solving matrix algebra. One of the best tools for solving the PDE is spreadsheet. There are many advantages of spreadsheets such as having numerical and visual feedback, fast calculating capabilities. One of the most important advantages of spreadsheets is its graphical interface. The solution obtained through the spreadsheet can easily be plotted at the same worksheet. Any changes in the input parameters of the solution domain will be directly reflected to the graphical representation of the solutions. Spreadsheets are user-friendly and easy to program.

Spreadsheets have an increasing popularity in engineering problems. Several studies have been carried out using spreadsheets for the last ten years. The application of them are carried out in different fields of engineering problems such as the solutions of partial differential equations [31], one-dimensional transient heat-conduction problems [32],
free-surface seepage problems [33], steady-state groundwater applications [34,35], transient groundwater applications [36,37], and groundwater parameter estimation [38].

The solution of ADE is quite difficult in terms of time dimension in the governing equations. Inclusion of time dimension in PDEs may lead to an increase in the CPU time if conventional methods are used. Therefore, advection–diffusion–filtration problems are solved by Karahan [39] based on iterative spreadsheet calculations with second-order implicit FDM schemes since spreadsheets eliminate the matrix algebra to vector form. Thus, it has been shown that spreadsheets can be used to solve transient ADE.

In this study, a user-friendly and a flexible solution algorithm is proposed for the numerical solution of ADE. The proposed solution algorithm is based on the description of ADE by using the third-order upwind scheme (TU) for advection term and second-order central scheme for diffusion term. For the solution of the obtained equations, spreadsheet simulation (SS) technique is used instead of computer code. Proposed ADE-TUSS model has been solved by using the iterative calculation feature of spreadsheets. The results of the model are validated with the analytical and numerical solutions in the literature.

In the ADE-TUSS model, the governing equation is solved and the results can be examined simultaneously on the spreadsheet by changing only the weighting parameter. Thus, the effects of the model parameters (such as \( u, \Delta t, \Delta x, D \)) on the results can easily be examined visually. In addition, model parameters (such as \( \Delta t, \Delta x \)) may be adjusted easily in order to overcome the problem of overshooting and negative concentrations.

2. Mathematical model

Problems of environmental pollution (for rivers, coasts, groundwater and the atmosphere) can be reduced to the solution of a mathematical model of diffusion–dispersion. The mathematical model describing the transport and diffusion processes is the one-dimensional advection–diffusion equation

\[
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2} \quad 0 < x < L \text{ and } 0 < t \leq T
\]

with initial conditions

\( c(x, 0) = f(x) \quad 0 \leq x \leq L, \) \hspace{1cm} (2)

and boundary conditions

\( c(0, t) = g(t) \quad 0 < t \leq T, \) \hspace{1cm} (3a)

\( c(L, t) = h(t) \quad 0 < t \leq T, \) \hspace{1cm} (3b)

where \( f, g \) and \( h \) are known functions, while the function \( c \) is unknown. \( u \) is the velocity in \( x \) direction and \( D \) is the dispersion coefficient. Note that \( u \) and \( D \) are considered to be positive constant values.

In general, explicit finite difference methods for the numerical solution are the restriction of the grid size and/or time step due to stability requirements. These restrictions require extremely small values for time step. So, these are impractical methods for most problems [40].

On the other hand, implicit finite difference schemes require the solution of large number of simultaneous linear algebraic equations direct or iterative solution at each time step. The required number of iterations to meet the desired accuracy may become large, particularly for large time increments and small space grid size [41].

In this study, a user-friendly and flexible solution technique is proposed instead of compact high-order schemes to reduce the numerical diffusion which is one of the most important problems encountered in the numerical solution of ADE and to increase the sensitivity of the results. A solution by using the iterative spreadsheet simulation technique is obtained without the necessity of using the suggested solution technique and time-bound ADE’s matrix algebra.

The performance of the proposed model is compared with the analytical and numerical solutions given in the literature.

3. Numerical solution

The solution domain of the problem is covered by a mesh of grid-lines

\( x_i = i \Delta x \quad \ldots \ldots \quad i = 0, 1, 2, \ldots, M, \) \hspace{1cm} (4)

\( t_n = n \Delta t \quad \ldots \ldots \quad n = 0, 1, 2, \ldots, N, \)

where \( x_i \) and \( t_n \) are parallel to the space and time coordinate axes. The constant spatial and temporal grid-spacing are \( \Delta x = L/M \) and \( \Delta t = T/N. \)

Using a forward-difference representation for the time derivative \( t = n \Delta t \)

\[
\frac{\partial c}{\partial t} = \frac{c(i,n+1) - c(i,n)}{\Delta t}
\]

A third-order scheme which is modified in respect to time dimension has been used for second term of ADE as given in [28]. Note that, for the boundary points, a four-point upwind formula may be written such that either point to the left or to the right is considered in the finite difference approximation.

Consider the following approximations of the derivatives in the advection–diffusion equation which incorporate temporal weight parameter \( \phi \) as follows:

Near the left boundary, i.e., for \( i = 2 \)

\[
u \frac{\partial c}{\partial x} = \frac{u}{6 \Delta x} \{ \phi[-11c(i,n+1) + 18c(i+1,n+1) - 9c(i+2,n+1)] \\
- (+1 - \phi)[-11c(i,n) + 18c(i+1,n) - 9c(i+2,n) + 2c(i+3,n)]\}
\]

(5a)

Interior nodes of the solution domain, i.e., for \( i = 3, \ldots, M - 2 \)
\[ \frac{\partial c}{\partial x} = \frac{u}{6 \Delta x} \left\{ \phi [c(i - 2, n + 1) - 6c(i - 1, n + 1) \right. \\
+ 3c(i, n + 1) + 2c(i + 1, n + 1)] \\
+ (1 - \phi) [c(i - 2, n) - 6c(i - 1, n) + 3c(i, n)] \\
+ 2c(i + 1, n) \left\} \right. \\
\] (5b)

Near the right boundary, i.e. for \( i = M - 1 \)
\[ \frac{\partial c}{\partial x} = \frac{u}{6 \Delta x} \left\{ \phi [-2c(i - 3, n + 1) + 9c(i - 2, n + 1) \right. \\
- 18c(i - 1, n + 1) + 11c(i, n + 1)] \\
+ (1 - \phi) [-2c(i - 3, n) + 9c(i - 2, n) \\
- 18c(i - 1, n) + 11c(i, n)] \right\} \\
\] (5c)

and the second derivatives occurring in diffusion term is considered using a central-difference representation as follows:
\[ -D_x \frac{\partial^2 c}{\partial x^2} = - \frac{D_x}{\Delta x^2} [c(i - 1, n) - 2c(i, n) + c(i + 1, n)] \] (6)

Substituting Eqs. (4), (5) and (6) into Eq. (1).
\[ c(i, n + 1) = c(i, n) - \frac{1}{6} \phi [18c(i + 1, n + 1) - 9c(i + 2, n + 1) + 2c(i + 3, n + 1)] \\
- (1 - \phi) [-11c(i, n) + 18c(i + 1, n) - 9c(i + 2, n) + 2c(i + 3, n)] \\
+ \left[ \frac{Cr}{Pe} [c(i - 1, n) - 2c(i, n) + c(i + 1, n)] \right] \/
\left[ 1 - \frac{11}{6} \phi \right] \] (7a)

for \( i = 2 \)
\[ c(i, n + 1) = c(i, n) - \frac{1}{6} \phi [6c(i - 1, n + 1) - 2c(i - 2, n + 1) + 2c(i - 1, n + 1)] \\
- (1 - \phi) [c(i - 2, n) - 6c(i - 1, n) + 3c(i, n) + 2c(i + 1, n)] \right) \left[ 1 + \frac{1}{2} \phi \right] \] (7b)

for \( i = 3, \ldots, M - 2 \)
\[ c(i, n + 1) = c(i, n) - \frac{1}{6} \phi [-2c(i - 3, n + 1) + 9c(i - 2, n + 1) - 18c(i - 1, n + 1) \\
+ (1 - \phi) [-2c(i - 3, n) + 9c(i - 2, n) - 18c(i - 1, n) + 11c(i, n)] \\
+ \left[ \frac{Cr}{Pe} [c(i - 1, n) - 2c(i, n) + c(i + 1, n)] \right] \left[ 1 + \frac{11}{6} \phi \right] \] (7c)

for \( i = M - 1 \) may be written where \( Cr = \frac{u\Delta x}{\lambda} \), Courant number and \( Pe = \frac{\lambda}{x_o} \) is Peclet number.

4. Model development

The general structure of solution domain for the ADE-TUSS can be seen in Fig. 1. The equations in the cells are easily generated as much as it is required depending on \( \Delta x \), which are the size of grids in the solution domain.

The ADE-TUSS model is divided into rectangular grid intervals both in \( x \) direction and time dimension in order to carry out the iterative spreadsheet calculations. It takes \( L, T, u, \Delta t, \Delta x, D \) and \( x_0 \) values as input parameters, where, \( x_0 \) is the coordinate of the centre of Gaussian pulse. The other terms are previously given. The constants used in the spreadsheet representation of ADE-TUSS model are given in Fig. 1.

In between Column E and I, coordinates, initial values, previous time step solution, numerical and analytical solutions term are described respectively. The columns length in the ADE-TUSS model is formed automatically with the help of a basic macro according the grid space.

The flowchart of ADE-TUSS can be seen in Fig. 2. The ADE-TUSS consists of two loops as in Fig. 2; inner loop and outer loop. The inner loop computes the concentration value for the given time, then, the outer loop controls the time step. When time reaches the maximum level (\( T \)), the ADE-TUSS has been completed to calculate and the output of it can be seen as graphical interface of the spreadsheet.

The solution of the ADE-TUSS model is carried out based on Eqs. (7a)-(7c) in the following spreadsheet format.

\[ H_3 = (G3 - (1/6) * CS818 * (CS812 * (18 * H4 - 9 * H5 + 2 * H6) + (1 - CS812) * (-11 * G3 + 18 * G4 - 9 * G5 + 2 * G6)) + CS819 * (G2 - 6 * G3 + 4 * G4 + 2 * G5)) / (1 - (1/6) * CS818 + CS812) \] (8a)

\[ H4 = (G4 - (1/6) * CS818 * (CS812 * (H2 - 6 * H3 + 2 * H5) + (1 - CS812) * (G2 - 6 * G3 + 3 * G4 + 2 * G5)) + CS819 * (G3 - 2 * G4 + G5)) / (1 + (1/2) * CS818 + CS812) \] (8b)


where \( H3 \), which is an intersection of the eighth column (H) and third row (3), represents the concentration in a cell. Related terms are used in solving Eqs. (7a)-(7c) has been shown in Eqs. (8a)-(8c) as a spreadsheet format.

Eqs. (8a)-(8c) are represented near the left boundary, interior nodes of the solution domain and near the left boundary node, respectively.

Before starting the ADE-TUSS model, firstly it is necessary to enter the initial and boundary condition values. In this respect, initial values given in Fig. 1 are entered to the...
F column and boundary conditions are to the first and last cell of the H column.

Eqs. (8a)–(8c) are written to the related cells in the solution domain shown in Fig. 1 and the equation expressing the analytical solution is written to the I3 cell. After the completion of this process, the formulas in the H4 and I3 cells are copied and pasted to the relevant cells in the H and I columns with the help of a basic macro and the calculation is started.

The values obtained for the calculated time step are copied and pasted to the G column which shows the values in the previous time step and this procedure is repeated until the time index in the outer loop reaches the value, as shown in Fig. 2. Detailed explanation about how the spreadsheet can iterate the equations can be seen from Ref. [36].

5. Numerical applications

Example 1. The analytical solution to the one-dimensional advection–diffusion in a region bounded by 0 ≤ x ≤ 1 is taken from [39,40] and given as:

\[ c(x, t) = \frac{0.025}{\sqrt{0.000625 + 0.02t}} \exp\left( -\frac{(x + 0.5 - t)^2}{(0.00125 + 0.04t)} \right) (9) \]

with initial conditions

\[ c(x, 0) = \exp\left( -\frac{(x + 0.5)^2}{0.00125} \right) \]

and boundary conditions

\[ c(0, t) = \frac{0.025}{\sqrt{0.000625 + 0.02t}} \exp\left( -\frac{(0.5 - t)^2}{(0.00125 + 0.04t)} \right) \] (11a)

\[ c(1, t) = \frac{0.025}{\sqrt{0.000625 + 0.02t}} \exp\left( -\frac{(1.5 - t)^2}{(0.00125 + 0.04t)} \right) \] (11b)

The values of the various parameters used are \( D = 0.01 \text{ m}^2/\text{s}, \ u = 1 \text{ m/s}. \) The space step and time step are taken to be \( \Delta x = 0.02 \text{ m} \) and \( \Delta t = 0.004 \text{ s}, \) respectively.

As can be seen from Fig. 3a, while greater values than the analytical solution are obtained for the \( \phi = 0 \) value, smaller values than the analytical solution are obtained for \( \phi = 1. \) As can be seen from Eq. (5), \( \phi = 0 \) value responds to the explicit expansion of the advection term and the \( \phi = 1 \) value responds to the implicit expansion. For this reason, in order to increase the accuracy of the numeric solution, different experimentations have been made for the values of \( \phi \) in between 0 and 1. Following these experimentations, it is seen that there is a very good agreement between the numeric solution values and the analytical results for the \( \phi = 0.5 \) value (see Fig. 3b).

It is noted that the \( \phi = 0.5 \) value responds to the expression of the advection term with the Crank–Nicolson scheme. Similar results have been obtained in the first study realized by the author for the numerical solution of ADE [39]. In the mentioned study, while the error value is \( 9.8 \times 10^{-4} \) for \( X = 0.5 \) and \( T = 1, \) in this study the error
value for the same input values has been obtained as $5.8 \times 10^{-4}$. This value is reported to be $3.9 \times 10^{-3}$ in [40]. In between the 0.025–1 of the Courant number, the error values for the different grid dimensions are given in Table 1. As can be seen from Table 1, the sensitivity of the ADE-TUSS model shows an excellent agreement with the analytical value as the grid sizes reduce.

As the Courant number reduces, numeric solutions show a good agreement with the analytical solution for all of the $\phi$ values (see Fig. 4).

However, as can be seen from Table 2, CPU time increases for the small Courant numbers and grid sizes.

**Example 2.** The analytical solution to the one-dimensional ADE of a Gaussian pulse of unit height, centred at $x_0 = 1$ in a region bounded by $0 \leq x \leq 9$ as given in Refs. [6,30] is

$$c(x,t) = \frac{1}{\sqrt{4t + 1}} \exp \left[ -\frac{(x - x_0 - ut)^2}{D(4t + 1)} \right]$$  \hspace{1cm} (12)

The initial condition is given by

$$c(x,0) = \exp \left[ -\frac{(x - x_0)^2}{D} \right]$$  \hspace{1cm} (13)

and the boundary condition at the two ends at any time $t$ is obtained by substituting $x = 0$ and $x = 9$, respectively, in Eq. (12), where $u$ is the velocity in the $x$ direction, $x_0$ is the centre of the initial Gaussian pulse, $D$ is the diffusion coefficient in the $x$ direction and $t$ is the time coordinate. The values of the various parameters used are $D = 0.005$, m$^2$/s and $u = 0.8$ m/s.
As in Example 1 and 2 has been solved numerically by using the input values given in literature with the aim of testing the performance of the ADE-TUSS model. As can be seen from Figs. 5a and 5b, while greater values than the analytical solution are obtained for the \( \phi = 0 \) value in this example as in Example 1, smaller values than the analytical solution and fairly different values from the analytical solution are obtained for \( \phi = 1 \). As can be seen from Fig. 5b, a very good agreement with the analytical result has been obtained when the mentioned example is solved for \( \phi = 0.5 \).

In between 0.016 and 3.2 of the Courant number, the error values for the different grid dimensions are given in Table 3. As can be seen from Table 3, the sensitivity of the ADE-TUSS model shows an excellent agreement with the analytical value as the \( \phi = 0.5 \) and the grid size reduces. It is seen clearly that the third-order upwind scheme proposed in Table 3 gives better results than the second-order central scheme suggested by the author in the earlier study.

It is noted that the same problem is solved in [30] by using the third-order upwind explicit scheme given in [28] and much less accurate results are obtained. In the ADE-TUSS model, more accurate results are obtained by modifying the same scheme in the time dimension. Moreover, different schemes such as implicit, explicit and Crank–Nicolson are solved with one algorithm by easily changing the \( \phi \).

In addition, the sensitivity of the numerical

Table 1
A comparison of error values of different solution techniques for Example 1 (\( x = 0.5, T = 1 \) s)

<table>
<thead>
<tr>
<th>( Cr )</th>
<th>Analytical solution</th>
<th>Implicit solution (Ref. [39])</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BTCS Upwind Crank–Nicolson</td>
<td>( \Delta x = 0.02 )</td>
</tr>
<tr>
<td>0.025</td>
<td>0.1741</td>
<td>1.8E–03 3.0E–02 7.8E–04</td>
<td>2.6E–04</td>
</tr>
<tr>
<td>0.050</td>
<td></td>
<td>2.9E–03 3.1E–02 7.8E–04</td>
<td>3.0E–04</td>
</tr>
<tr>
<td>0.100</td>
<td></td>
<td>4.9E–03 3.2E–02 8.8E–04</td>
<td>3.8E–04</td>
</tr>
<tr>
<td>0.167</td>
<td></td>
<td>7.4E–03 3.3E–02 8.8E–04</td>
<td>5.0E–04</td>
</tr>
<tr>
<td>0.200</td>
<td></td>
<td>8.8E–03 3.4E–02 9.8E–04</td>
<td>5.8E–04</td>
</tr>
<tr>
<td>0.400</td>
<td></td>
<td>1.6E–02 3.4E–02 1.4E–03</td>
<td>1.1E–03</td>
</tr>
<tr>
<td>0.500</td>
<td></td>
<td>1.9E–02 3.4E–02 1.6E–03</td>
<td>1.5E–03</td>
</tr>
<tr>
<td>1.000</td>
<td></td>
<td>3.3E–02 5.1E–02 2.9E–03</td>
<td>NA</td>
</tr>
</tbody>
</table>
solution can easily be seen as [39] whereby the powerful
graphic interface of spreadsheets.

It is seen from Fig. 6 that the different \( \phi \) values give
similar results for the small values of the Courant number.
However, as it can be seen from Table 4, the CPU time
increases drastically for the small Courant numbers and the
grid sizes.

6. Conclusions and limitations

In this study, a user-friendly and a flexible solution algo-

rithm is proposed for the numerical solution of the one
dimensional advection–diffusion equation. The proposed
solution algorithm is based on the description of ADE by
using the third-order upwind scheme for advection term
and second-order central finite difference representation.

By changing the proposed ADE-TUSS model and only
the time weighting factor \( (\phi) \), the result can be obtained as
explicitly or implicitly. In order to test the proposed model,
two problems which have a numerical and analytical solu-
tion in the literature, have been solved.

<table>
<thead>
<tr>
<th>Cr</th>
<th>Grid size</th>
<th>CPU time (s) for Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta x = 0.0250 )</td>
<td>( \Delta x = 0.0125 )</td>
</tr>
<tr>
<td>0.016</td>
<td>80</td>
<td>257</td>
</tr>
<tr>
<td>0.032</td>
<td>41</td>
<td>138</td>
</tr>
<tr>
<td>0.064</td>
<td>21</td>
<td>70</td>
</tr>
<tr>
<td>0.080</td>
<td>17</td>
<td>56</td>
</tr>
<tr>
<td>0.160</td>
<td>9</td>
<td>31</td>
</tr>
<tr>
<td>0.320</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>0.640</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>1.600</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 3
A comparison of error values for Example 2 \((x = 5 \text{ m}, T = 5 \text{ s})\)

<table>
<thead>
<tr>
<th>Cr</th>
<th>Analytical solution</th>
<th>Implicit solution (Ref. [39])</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BTCS</td>
<td>U'wind</td>
<td>Crank–Nicolson</td>
</tr>
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In both examples, while greater values than the analytical solution are obtained for the ADE-TUSS model value $\phi = 0$ (explicit scheme), smaller values than the analytical solution are obtained for the $\phi = 1$ value (implicit scheme). Results have shown that very good agreement with the analytical solution are obtained even in the high Courant numbers in both examples for the $\phi = 0.5$ value (Crank–Nicolson scheme).

While the numerical diffusion is prevented, the ADE-TUSS results and the analytical results show a good agreement for the two presented examples. The performance of the ADE-TUSS model is better than the presented examples in terms of errors.

With the change of the input parameters in the ADE-TUSS model, the change in the model results can easily be observed graphically. Thus, the effect of the model parameters on the model accuracy and stability can be seen visually.

The present method may not be applied for solving the set of equations if the grid number exceeds the maximum size of spreadsheets.

While the FDM schemes used in this study are implicit, the iterative calculation property which is one of the most important properties of spreadsheets is used for solving the algebraic equation system. Thus, it is shown that spreadsheets are used to solve ADE by using high-order schemes. Note that, the proposed implementation may be adapted to the two-dimensional ADE and similar PDEs. Moreover a source and/or sink term may be added to the ADE-TUSS implementation.

References

[34] Anderson MP, Bair ES. The power of electronic worksheet: modeling without programs in BASIC. Amsterdam: Elsevier; 1986.

