Geostrophic Adjustment Over Submarine Canyons

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The geostrophic adjustment of a stratified coastal current in the presence of a submarine canyon is considered with a mathematical model in which the vertical structure of the fluid is handled with a "level" technique that represents vertical gradients by finite differences. Two situations are investigated in detail: a two-level system where one level represents the shelf and one the canyon, and a three-level system with two levels over the shelf and one in the canyon. There are four important length scales in the adjustment process: the initial width of the coastal current, the width of the canyon, and the internal and external radii of deformation. For each vertical mode, the shorter of the radius of deformation for that mode and the width of the coastal current determines the distance over which the perturbing influence of the canyon decays. For typical shelf situations, the external mode decays with the width of the coastal current, while the internal modes decay with the internal radius of deformation. The width of the canyon determines the strength of the cross-canyon flow and thus, the strength of the canyon's effect on the overlying coastal current, with the interaction becoming smaller as the canyon width becomes smaller than the current width or the radius of deformation. As the cross-canyon flow becomes small, the importance of the geostrophic balance decreases and the internal density gradients become more important in balancing pressure gradients. Therefore even in the case of flow over a narrow canyon, the isopycnals at the top of the canyon are distorted and there will be some residual circulation on the shelf that is forced by the presence of the canyon.

INTRODUCTION

The effects of submarine canyons on the dynamics of persistent flows on continental shelves have only recently been considered [Freeland and Denman, 1982; Klinck, 1988] in spite of numerous observations of currents in and near submarine canyons [Inman et al., 1976; Shephard et al., 1979; Han et al., 1980; Hotchkiss and Wunsch, 1982; Mayer et al., 1982; Freeland and Denman, 1982; Freeland et al., 1984; Hickey et al., 1986; Hunkins, 1988]. Many of these observational studies focused on tides or internal waves and the effect of these relatively high-frequency currents on sediment distribution and resuspension. Several recent studies (discussed below) describe measurements that demonstrate the existence of longer time scale (and time mean) currents in and around submarine canyons. Moreover, these studies show that canyons can influence circulation on the adjacent continental shelf.

The first of these studies, a diagnostic model of the Hudson Shelf Valley [Han et al., 1980], was used to investigate the circulation in the New York Bight forced by various wind stress patterns. Model results showed that the near-surface velocities everywhere in the bight were influenced mainly by wind stress and sea surface slope, while the velocities at the bottom near the canyon (Figure 6 in the work by Han et al. [1980]) were clearly aligned with the canyon topography. The second study [Freeland et al., 1984] presented observations on the continental shelf near Vancouver Island that showed a persistent deflection of the coastal flow during the summer in the vicinity of a small submarine canyon. This flow pattern (interpreted as an eddy trapped by the canyon topography) and the dynamics involved are analyzed in detail by Freeland and Denman [1982]. The third study [Hickey et al., 1986] described multiyear observations of currents and suspended sediments in Quinault Canyon off the U.S. northwest coast that revealed a correlation between flow along the canyon axis and the alongshelf circulation. The phase of this correlation is such that the pressure gradient due to the geostrophically balanced coastal flow forces upwelling or downwelling in the deep parts of the canyon. Supporting evidence for this forced vertical circulation is found in temperature, salinity, and transmissivity measurements which indicate vertical rather than horizontal motion of water. The final study [Hunkins, 1988] considers a set of approximately year-long current measurements near the head of and in Baltimore Canyon which shows the existence of persistent up- and downcanyon flows. Currents near the surface (Figure 7 in the work by Hunkins [1988]) are not strongly affected by the submarine canyon. However, currents within a few meters of the bottom seem to align with isobaths. One explanation is that the flow is following isobaths, although an alternative hypothesis is that the turning is due to a bottom Ekman layer. The mean current below the level of the shelf in the head of Baltimore Canyon is clearly different from the alongshore flow or that due to a bottom Ekman layer. All of these observational studies support the existence of some coupling of shelf and canyon circulations.

The purpose of this study is to investigate the general character of these interactions. Of particular concern are the horizontal length scales that control the current-canyon interaction. The approach taken here is to consider a geostrophic adjustment problem which reveals the important length scales of the interaction without requiring that the transient initial value problem be solved in detail. For a review of the open ocean adjustment problem, see Blumen [1972] or Gill [1982, pp. 191–203]. Other examples of uses for this technique are in analyzing the circulation produced by surface cooling [Stommel and Veronis, 1980] and in calculating the structure of a tidal front [van Heijst, 1985].

The present study considers the geostrophic adjustment of a stratified, rotating fluid which is initially flowing on a shelf perpendicular to a rectangular submarine canyon. The vertical structure of the flow is represented in terms of "levels"
Model Formulation

The geostrophic adjustment of a stratified rotating fluid is considered with linear inviscid dynamics. The model domain is an infinitely wide flat-bottomed shelf with a straight rectangular channel cut into the bottom which represents a submarine canyon (Figure 1). The initial flow on the shelf is chosen to be at right angles to the canyon and to be geostrophically balanced by a sloping free surface. Furthermore, the shelf flow is chosen to have a trigonometric structure in the along-canyon direction (the flow is "banded") with some inherent width scale. The initial circulation in the canyon is zero.

Geostrophic adjustment occurs by radiation of rotationally modified gravity waves, but it is possible to obtain directly the steady state that will result from a given initial current distribution [Gill, 1982, pp. 191-203]. In the absence of viscosity, fluid parcels will conserve potential vorticity which allows the direct calculation of the final steady state.

The presence of both variable fluid depth and continuous stratification greatly complicates the analysis of the adjustment problem. Traditionally, the dynamics are posed in terms of a multilayer fluid. However, it is crucial for the present calculation to allow the dynamics in the canyon to be decoupled from that of the shelf. If the canyon is taken to be a layer by itself, then, when water spills out of the canyon, the location of the edge of the "canyon" water must be calculated-a difficult problem. To avoid this problem while retaining separate dynamics for the canyon, the model equations will be used in a "level" format [Pedlosky, 1979, p. 396], which is best thought of as a finite difference representation of the vertical gradients in the governing primitive equations.

At least three levels are required for an adequate representation of a stratified shelf flow reacting to the presence of a submarine canyon, i.e., there must be two levels over the shelf and one level in the canyon. The governing equations in terms of nondimensional variables for inviscid flow in the region over the canyon (0 < x < l, where l (=L/(gD)^1/2) is the nondimensional width of the canyon) are

\[ \frac{\partial u_1}{\partial t} = +v_1 - \partial \eta/\partial x \]  
\[ \frac{\partial v_1}{\partial t} = -u_1 - \partial \eta/\partial y \]  
\[ \frac{\partial \eta}{\partial t} = -\delta_1(\partial u_1/\partial x + \partial v_1/\partial y) - \delta_3(\partial u_3/\partial x + \partial v_3/\partial y) \]  
\[ \frac{\partial u_3}{\partial t} = +v_3 - \partial \eta/\partial x - e_2 \partial \rho/\partial x \]  
\[ \frac{\partial v_3}{\partial t} = -u_3 - \partial \eta/\partial y - e_2 \partial \rho/\partial y \]  
\[ \frac{\partial \rho}{\partial t} = -\delta_3(\partial u_3/\partial x + \partial v_3/\partial y) - \delta_5(\partial u_5/\partial x + \partial v_5/\partial y) \]  
\[ \frac{\partial u_5}{\partial t} = +v_5 - \partial \eta/\partial x - e_2 \partial \rho/\partial x - e_4 \partial \rho/\partial x \]  
\[ \frac{\partial v_5}{\partial t} = -u_5 - \partial \eta/\partial y - e_2 \partial \rho/\partial y - e_4 \partial \rho/\partial y \]  
\[ \frac{\partial \rho}{\partial t} = -\delta_5(\partial u_5/\partial x + \partial v_5/\partial y) \]  

where the numerical subscripts refer to the depth level (Figure 1). The variables \(u, v\) refer to velocity components in the \((x, y)\) directions, \(\eta\) is the departure of the sea surface from its mean level, and \(\rho\) is the departure of the density from the background value of \(\rho_0\). The location in the vertical of the variables is indicated in Figure 1. The variables are nondimensionalized (the dimensional variables have an asterisk superscript) as

\[ (u^*, v^*) \sim U \quad (x^*, y^*) \sim (gD)^{1/2} \quad \eta^* \sim \eta_0 \quad \rho^* \sim -\eta_0 \partial \rho/\partial z \]

where \(U\) is the speed of the initial flow and \(D (=H_1 + H_3 + H_5)\) is the total depth over the canyon, \(f\) is the Coriolis
parameter, and g is the acceleration of gravity. The parameters in (1) are the nondimensional thicknesses of the levels \( \delta_i = H_i / D \), for \( i = 1, 3, 5 \) and the stratification parameters \( \epsilon_i = (H_i / \rho_0) (d \rho / dz) \), for \( i = 2, 4 \), where \( \rho(z) \) is the temporal and spatial mean stratification.

Away from the canyon \( (x < 0, x > 1) \), the flow is represented by only two levels and the governing equations come from (1) by setting \( u_5 = v_5 = P_4 = 0 \) and \( \epsilon_4 = \delta_4 = 0 \).

The vorticity equation for each level is obtained by cross-differentiating the momentum equations and substituting from the continuity equation. The resulting equations are integrated in time to give

\[
\zeta_i + \frac{1}{\delta_i}(-\eta + p_2) = C_1(x, y) \\
\zeta_i + \frac{1}{\delta_i}(-\eta - p_2) = C_3(x, y) \\
\zeta_i - \frac{1}{\delta_i}p_4 = C_5(x, y)
\]

where \( \zeta_i = (\nu_i)_{x} - (\omega_i)_y \) is the relative vorticity for level \( i \) and the integration constants, \( C_i \), are determined from the initial conditions.

The governing equations for the final adjusted state are obtained from the divergence of the momentum equations at each level, using the continuity equation and assuming steady state, whence

\[
\nabla^2 \eta + \frac{1}{\delta_1}(-\eta + p_2) = C_1(x, y) \\
\nabla^2 \eta + \frac{1}{\delta_2}(-\eta + p_2 + \epsilon_2 p_1 + \frac{1}{\delta_1} \eta) = C_3(x, y) - C_1(x, y) \\
\n\nabla^2 \eta - \frac{1}{\delta_3}p_4 = C_5(x, y) - C_3(x, y)
\]

where \( \delta_i = (\nu_i)_{x} - (\omega_i)_y \) is the relative vorticity for level \( i \) and the integration constants, \( C_i \), are determined from the initial conditions.

The boundary conditions for (3) are that the geostrophically balanced initial state remains unchanged away from the perturbing influence of the submarine canyon. The length scale of this decay will be determined in the course of this analysis. The other condition is that the normal flow \( (u_i)_{x} \) at the walls of the canyon must be zero, which is equivalent to demanding that \( \eta + \epsilon_3 p_2 + \epsilon_4 p_4 = 0 \) along \( x = 0, 1 \).

The initial state is geostrophically balanced flow over the shelf and no flow in the canyon, or

\[
u_1(t = 0) = \cos(\mu y) \\
\nu_3(t = 0) = 0 \\
\eta(t = 0) = -(1/\mu) \sin(\mu y) \\
p_2(t = 0) = 0
\]

The potential vorticity for the initial state is

\[C_1 = (\mu + 1/\delta_1) \sin(\mu y) = C_1 \sin(\mu y) \]

Given the trigonometric structure of the forcing, the solution to (4) and (5) can be found in the form

\[
\eta(x) = \hat{\eta}(x) \sin(\mu y) \\
p_2 = \hat{p}_2(x) \sin(\mu y)
\]

The \( x \) structure of the solution on either side of the canyon is obtained by integrating

\[
a^2 \hat{\eta} / a^2 x - (\mu^2 + (1/\delta_1)) \hat{\eta} = \hat{C}_1
\]

and retaining only the decaying solution. For \( x < 0 \) the solution is

\[
\hat{\eta} = A \exp[(\mu^2 + (1/\delta_1))^{1/2}x - \frac{\hat{C}_1}{\mu^2 + 1/\delta_1}]
\]

for \( x > 1 \),

\[
\hat{\eta} = D \exp[-((\mu^2 + (1/\delta_1))^{1/2}(x - 1)) - \frac{\hat{C}_1}{\mu^2 + 1/\delta_1}]
\]

The equations governing the flow over the canyon (4) are coupled so it is convenient to convert to normal modes (after...
the fashion of Veronis and Stommel [1956]). A modal amplitude is defined as $M = \eta + \alpha \eta _z$, where $\alpha$ is a constant to be determined. The governing equation for $M$ is obtained by multiplying the second equation in (4) by $\alpha$ and adding to the first. In the process of reducing to only modal variables, a consistency condition (a quadratic equation) appears that specifies the form of $\alpha$ as

$$\alpha _\pm = \frac{(\delta _1 + (1 - \eta _2)\delta _3)}{2\delta _1} \left( -1 \pm \left( 1 + \frac{4\delta _2\delta _3^2}{(\delta _1 + (1 - \eta _2)\delta _3)^2} \right)^{1/2} \right)$$

and the resulting modal equation is

$$d^2M/dx^2 - (\mu _\pm + \beta _\pm )M \pm = \tilde{\alpha} \delta _1 \beta _\pm$$

where $\beta _\pm = (1 - \alpha _+/\varepsilon _2)/\delta _1$. These two modes are the familiar barotropic (plus) and baroclinic (minus) modes of a two-level system. The two $\beta$ values are the reciprocals of the square of the nondimensional radii of deformation for the two modes. The surface elevation and density are recovered from the modal variables as

$$\eta = \frac{\alpha _+ M_+ - \alpha _- M_-}{\alpha _+ - \alpha _-} \quad \rho _2 = \frac{M_+ - M_-}{\alpha _+ - \alpha _-}$$

The solution to the modal equation (8) is

$$M_\pm = B_\pm \exp - ((\mu _\pm + \beta _\pm )^{1/2}x) + C_\pm \cdot \exp ((\mu _\pm + \beta _\pm )^{1/2}(x - l)) - \frac{\tilde{\alpha} \delta _1 \beta _\pm}{\mu _2 + \beta _\pm}$$

This particular form of the solution is chosen to show that the disturbances within the canyon decay away from the canyon edges with a nondimensional scale determined by the smaller of $\mu ^{-1}$ (the width of the initial current) and $\beta _\pm ^{-1/2}$ (the radius of deformation for each mode).

The six integration constants ($A, B_\pm, C_\pm, D$) in (6), (7), and (9) are determined from the requirement that both $\eta$ and $d\eta/dx$ be continuous at $x = 0$ and $l$ and that $\eta + \varepsilon _2\rho _2 = 0$ along the edges of the canyon ($x = 0, l$). These conditions lead to a $6 \times 6$ system of linear equations which can be solved, but neither the equations nor the solution are displayed here as they do not add significantly to the discussion.

Results

The character of the steady circulation for the two-level model is apparent from expressions (6), (7), and (9). There are four nondimensional length scales in the solution: $\mu ^{-1}$, $\delta _1$, $\beta _\pm ^{-1/2}$, and $\beta _\pm ^{-1/2}$, which represent (1) the $y$ structure of the initial flow, (2) the external radius of deformation away from the canyon, (3) the external, and (4) internal radii of deformation over the canyon, respectively. For parameter values appropriate for typical shelf systems (shelf depths of 100 m, a density gradient of 1 kg/m$^3$ per 100 m), the width scales for coastal currents are 10–50 km, the external radius of deformation is 200–400 km, while the internal radius of deformation is 2–10 km. Thus the nondimensional parameters representing the external mode (recall that this scale is used to nondimensionalize the horizontal variables) are $O(1)$, while $\mu ^{-1}$ is $O(10^{-1})$ and the internal scale is $O(10^{-2})$. Therefore for typical continental shelf situations, the disturbance on either side of the canyon will decay with the scale of the initial current. Over the canyon, the internal mode will decay away from the canyon walls with the internal radius of deformation while the external mode will decay with the scale of the forcing current.

This structure of the solution (note that dimensional variables are presented in the figures) is seen most easily for the case of a very wide canyon relative to the forcing or internal scale (Values of the other parameters are given in Appendix A). For a 200-km-wide canyon, the cross-canyon flow (Figure 2a) in the upper level is slower over the canyon than away from the canyon, and it is slowest at the edges of the canyon. The perturbing effect of the canyon decays on the shelf away from the canyon with the forcing scale (50 km, in this case). Over the canyon, $\rho$ and $v$ decay with the shorter, internal radius of deformation.

During adjustment, the water in the canyon is pushed down the initial pressure gradient which causes upwelling at places where the surface elevation in the initial state was lowest and downwelling where it was highest. Upwelling stretches the vortex tubes in the canyon creating cyclonic vorticity, as is evident from $v_3$ at the edges of the canyon (Figure 2a). The mass redistribution reduces the slope of the free surface creating cyclonic (anticyclonic) vorticity in level 1 on the shelf side of the canyon wall in places where the free surface rises (falls). The isopycnal surface over the canyon near the wall rises sharply compressing vortex tubes creating anticyclonic vorticity right at the wall. An estimate of the vertical displacement of the isopycnal over the canyon is given by $\rho _2/d\delta _2/dz$; a density anomaly of 0.2 kg/m$^3$ against a background stratification of 2 kg/m$^3$ in 100 m gives a vertical displacement of 10 m. The surface elevation rises by only 0.75 cm so vortex tubes are being compressed.

For a narrower (and more typical) canyon (10 km, Figure 2b), the magnitude of the distortion to the initial flow is slightly larger with the flow over the canyon being a little slower and the free surface rising slightly compared to the previous case. The external mode in the canyon is inhibited because the canyon is narrower than either of the decay scales for the external mode. The rising isopycnal creates a cyclonic vortex within the canyon and an anticyclone above the canyon.

If the canyon width is reduced to 1 km (narrower than the internal radius of deformation, Figure 2c), the perturbation to the initial flow is quite small, while the density perturbation is relatively large. There is still a cyclone in the canyon and an anticyclone over the canyon but the strengths are much reduced over the previous case (Figure 2b). The initial flow is only slightly affected by the presence of the canyon as is evident by the small changes in $u_1$ and $\eta$. The density perturbation is about twice the strength of the previous case, but there is very little $x$ gradient. In this case, the canyon is so narrow that $v_3$ is small and the geostrophic balance in the along-canyon direction is inhibited. The pressure gradient due to the free surface slope is balanced mainly by the contrary slope of the internal density surface. That is, everywhere over the canyon, $(\eta + \varepsilon _2\rho _2)$ tends toward zero and the pressure gradient in the canyon becomes small.

As the canyon continues to narrow, the flow in the lowest layer goes to zero and the density perturbation at the interface grows until $\rho _2 = -\eta /\varepsilon _2$. For this case (parameters given in Appendix A), the density perturbation would be 0.44 kg/m$^3$ (about double the disturbance in Figure 2c). The circulation in the canyon goes to zero as the canyon width decreases to zero. Therefore as a submarine canyon becomes narrower (by a factor of about 2) than the internal radius of deformation, it ceases to have much influence on the overlying coastal flow. However, there is a redistribution.
Fig. 2. The $x$ structure of the steady (adjusted) solution in a two-level model for the parameters given in Appendix A. The $y$ structure of the solution is obtained by multiplying $u$ by $\cos(\mu y)$ and $v$ and $\rho$ by $\sin(\mu y)$. The initial flow is 0.1 m/s with a $y$ structure like $\cos(\pi y/50)$ km. The basic stratification over the canyon is 2 kg/m$^3$ per 100 m giving an internal radius of deformation of 3.5 km. The upper level is 50 m thick, while the canyon is 25 m thick. Dashed lines indicate the location of the canyon walls. Three cases are shown for different canyon widths: (a) the canyon is 200 km wide; (b) the canyon is 10 km wide; and (c) the canyon is 1 km wide.

of density which means that there should be a density signal in the vicinity of even a narrow canyon.

The major problem with this simple case is that the disturbance away from the canyon must decay with the smaller of the current forcing scale or the external radius of deformation. In order to consider the effect of a canyon on a stratified shelf, the shelf flow must be represented by at least two levels, which is the three-level case considered in the next section.

**THREE-LEVEL ADJUSTMENT**

*Model Equations*

Given the insight obtained from the previous two-level solution, it is relatively straightforward to analyze the three-level model. Since the shelf flow is represented by two levels, both the internal and external modes can be active. This situation is more representative of conditions on continental shelves.
The governing equations for this case are given in (3). The initial conditions are barotropic, geostrophic flow over the shelf and no motion in the canyon. As before, the variables are taken to be proportional to $\sin(\mu y)$, that is, $\eta(x, y) = \tilde{\eta}(x) \sin(\mu y)$, and similarly for the other variables.

It is possible to separate (3) into three normal modes over the canyon and solve the resulting modal equation analytically as was done in the previous section. However, this analysis becomes quite cumbersome because of the necessity of solving a cubic equation for the modal parameters and because of the relatively large number (10) of boundary and matching conditions that must be imposed; a numerical solution is obtained instead. The separation into modes of two-level flow has been accomplished in previous sections, so the shelf equations are given in (8) and the solutions are given in (9).

In order that the coupling of the flow at the canyon wall occur simply, the two-level modal separation is also used over the canyon. A third mode is defined to be $\tilde{\tilde{\alpha}} = \tilde{\alpha} + E^2$, which vanishes at the edges of the canyon to satisfy the no normal flow condition. The resulting coupled equations over the canyon ($0 < x < l$) are

$$
\frac{d^2 \tilde{M}_+}{dx^2} - \left( \mu^2 + \beta_+ + \frac{\alpha_+ (\epsilon_2 - \alpha_-)}{\delta_4 (\alpha_+ - \alpha_-)} \right) \tilde{M}_+ = -\frac{\alpha_+}{\delta_4 E^2} \left( \frac{\tilde{\tilde{\alpha}} - 1}{(\alpha_+ + \alpha_-)} \right) \tilde{M}_- + C_1 + \frac{\alpha_+}{\epsilon_2} \left( \tilde{\tilde{C}}_3 - \tilde{C}_1 \right)
$$

$$
\frac{d^2 \tilde{M}_-}{dx^2} - \left( \mu^2 + \beta_- + \frac{\alpha_- (\alpha_+ - \epsilon_2)}{\delta_4 (\alpha_+ - \alpha_-)} \right) \tilde{M}_- = -\frac{\alpha_-}{\delta_4 E^2} \left( \frac{\tilde{\tilde{\alpha}} - 1}{(\alpha_+ + \alpha_-)} \right) \tilde{M}_+ + \frac{\alpha_-}{\epsilon_2} \left( \tilde{\tilde{C}}_3 - \tilde{C}_1 \right)
$$

$$
\frac{d^2 \tilde{\tilde{\alpha}}}{dx^2} - \left( \mu^2 + 1 \right) \frac{1}{\delta_4 E^2} \tilde{\tilde{\alpha}} = \frac{(\alpha_+ - \epsilon_2) \tilde{M}_+ - (\alpha_- + \epsilon_2) \tilde{M}_-}{\delta_4 E^2 (\alpha_+ - \alpha_-)}
$$

where the initial potential vorticities are $\tilde{C}_1 = \mu + 1/\delta_1 \mu$, $\tilde{C}_3 = \mu$.

Imposing the boundary conditions far from the canyon presents a bit of difficulty for the numerical calculation. The actual condition is that the disturbance must decay on either side of the canyon for the solution to be energetically consistent. Typically, the numerical domain is extended far enough that the far field values can be specified. A better boundary condition can be obtained from the analytical solutions on either side of the canyon. The solution to (8) has a constant forced term along with one growing and one decaying exponential term. For the region $x < 0$, the solution which obeys the far field boundary condition is

$$
\tilde{M}_+ = A e^{(\mu^2 + \beta_+) y} - \tilde{C}_1 + (\alpha_+ + \epsilon_2) \left( \tilde{\tilde{C}}_3 - \tilde{C}_1 \right) \frac{(\alpha_+ + \epsilon_2)}{(\mu^2 + \beta_+)^{1/2}}
$$

However, this is also the solution to the first order equation

$$
\frac{d \tilde{M}_+}{dx} = (\mu^2 + \beta_+)^{1/2} \tilde{M}_+ = \frac{\tilde{C}_1 + (\alpha_+ + \epsilon_2) (\tilde{C}_3 - \tilde{C}_1)}{\mu^2 + \beta_+} \frac{(\alpha_+ + \epsilon_2)}{(\mu^2 + \beta_+)^{1/2}}
$$

This first-order equation is then the proper “reduced” equation to use as the boundary condition at $x = -X$, which is one edge of the numerical grid. An analogous first-order equation can be constructed for the region $x > l$ which is used at $x = X$, at the other end of the numerical grid.

These boundary conditions allow only the decaying solution away from the canyon without requiring that the numerical domain extend far enough away from the canyon that the perturbation effects are effectively zero. For the cases considered here, the domain would have had to extend about 500 km (2 external radii of deformation) on either side of the canyon. While not an impossible situation, there is no need to solve such a large numerical problem if only a few tens of kilometers in the center of the domain are of interest.

Each of the ordinary differential equations in (10) is converted to an algebraic equation by using the central difference representation for the second derivative. The resulting tridiagonal linear system is solved for the grid point values [Lindzen and Kuo, 1969]. The boundary conditions are (1) $\tilde{T} = 0$ at the grid points representing the canyon walls and (2) the first and last equations of the linear system are the central difference representation of the “reduced” equations (e.g., equation 11) which allow only the decaying solutions.

The coupled system (10) is solved iteratively with a two step procedure. First, the two modal equations for $\tilde{M}_\pm$ are solved for given $\tilde{T}$ and then a new $\tilde{T}$ is obtained using the new $\tilde{M}_\pm$ values. The process begins with $\tilde{T} = 0$ which is consistent with the boundary conditions on $\tilde{T}$ at the edges of the canyon.

Convergence of the iterative process is measured by the root-mean-square change in the solution for each equation over one iteration normalized by the root-mean-square magnitude of the solution. For narrow grid spacing (0.5 km) and narrow canyons (less than 10 km), the process converges in fewer than 10 iterations for an error less than $10^{-10}$. For wider canyons (up to 100 km), convergence can take several hundred iterations to obtain the same error reduction. At this level of convergence, the governing equations are satisfied and potential vorticity is conserved to better than one part in $10^6$.

Results

The three-level versions (Figure 3) of the previous two-level solutions (Figure 2) are considered. Parameter values for these cases are given in Appendix B. For a very wide canyon (200 km, Figure 3a), the disturbance at each of the canyon walls is independent. There are two scales evident in the solution (compare $u_1$ or $v_1$); the scale of the initial current and the internal radius of deformation. The external mode decays with the current width, while the internal mode decays with the internal radius (also evident in the structure of $\rho_2$ and $\rho_3$). Furthermore, the flow is noticeably bottom trapped; the largest density gradients and speeds are within the canyon. There is vertical shear in the currents on the shelf with the flow in level 3 being stronger than that in level 1. However, the character and amplitude of the solution is not markedly different from the two-level result. The density distortion at the top of the canyon ($\rho_3$) and the free surface are nearly identical in the two- and three-level calculations.

The solutions for a 10-km canyon (Figure 3b) and a 1-km canyon (Figure 3c) are similar to the two-level results. The major difference between them is that the flow over the canyon is weaker in level 1 than in level 3 indicating bottom trapping of the disturbance.

The real benefit of the three-level results is in indicating the lateral scale of the disturbance on the shelf side of the canyon.
canyon walls. For the case with a 10-km canyon (Figure 3b), the density perturbation ($\rho_2$) decays away from the canyon with the internal radius of deformation (2.45 km in this case). However, the flow fields which were initially barotropic, decay with the longer scale associated with the width of the initial current. For a narrow canyon (Figure 3c), there is a mound of density over the canyon which is about two internal radii in diameter. In all cases, the density anomaly decays away from the canyon edges with the internal radius of deformation.

The resulting flow patterns are easily understood in terms of vortex stretching where there is now the complication of an intermediate level. For example, if the canyon is 10 km wide (Figure 3b), then the isopycnals just above the canyon rise about 10 m, while the isopycnals higher in the water rise by only 2 m. Therefore the cyclonic vortex in the canyon, created by motion of the isopycnal at the top of the canyon, is as strong as in the two-level case, while above the canyon, there is an anticyclone, which becomes weaker at greater distances above the bottom.

The two-dimensional structure of the solution for a 10-km canyon is best seen in a plan view (Figures 4a–4f). In each of these figures, $u$ ($v$, $\eta$, $\rho$) is symmetric (antisymmetric) about the $x$ axis (at the bottom of the figure), while $u$, $\eta$, $\rho$ ($v$) is symmetric (antisymmetric) about a line parallel to the $y$ axis along the center of the canyon. The horizontal velocity vectors in level 1 and level 3 (Figures 4a–4f) at the right edge of the figure are the unmodified initial conditions while closer to the canyon, there is a strong cyclonic turning. A strengthened current exists along the edge of the canyon in level 3 (Figure 4b) geostrophically driven by the internal density gradient. The strongest currents (Figure 4c) are within the canyon along the walls. The free surface (Figure 4d) has clearly risen over the canyon and the $x$ gradient there is about half of its initial amplitude. The doming of the mid-depth isopycnals over the canyon (Figure 4e) is evident along with the localized nature of the disturbance. There is a clear tendency for the largest density gradients to exist near the canyon walls. Since this canyon is about four internal radii wide, the peaks in the density perturbation are well separated. For a narrower canyon (relative to the internal radius), these peaks will merge into a single density mound. Finally, the strongest density disturbance exists in the canyon (Figure 4f) and is strongly confined to the walls.

**Implications of the Results**

Since coastal currents rarely exist as alternating bands of currents, it is appropriate to consider the extension of these "ideal" solutions to the case of a single coastal current flowing over a submarine canyon. Assuming that the coastal jet is created (or accelerated) by an impulsive addition of
momentum, then what might be the adjusted steady state? From the solutions presented here, the adjusted state would have a dome of higher density water on the left of the current (looking downstream) and low density on the right. There would be a cyclone in the canyon on the left of the current with a bottom trapped anticyclone above the canyon. The converse would exist on the right of the current. In using these extensions to the present analysis it is assumed that the canyon has a flat bottom, which is almost never the case. However, the vortex generation arguments can be extended to account for a sloping bottom such that a stronger circulation would be observed over the shallower parts of the canyon. Since there will be some viscous damping of these induced circulations (say, by a bottom Ekman layer) then only the stronger flows are likely to remain after adjustment, and these flows will be created over shallow water because the stretching there will be largest. So, one might expect the density signal of the current-canyon interaction to be most apparent on the shoreward side of the coastal current. There are likely other important processes that must exist in canyons of finite length so it is not fruitful to extrapolate these solutions any farther than these general statements.

Comparison With Observations

There are five studies that provide observations of the circulation in and around canyons that can be used to compare with the results of the three-level model. However, unlike the model, none of the observations are for an
adjusted steady state circulation. Consequently, comparisons between model and observed distributions are not always conclusive.

The best comparison that can be made is with the persistent eddy that exists over a small canyon south of Vancouver Island [Freeland and Denman, 1982]. This feature seems to be the result of a seasonally varying southward flowing coastal current. The deep circulation of the eddy (50/100m dynamic topography as shown in Figure 9 of Freeland and Denman [1982]) is cyclonic with a diameter of 30-40 km. This compares well in size and direction of circulation with the three-level model which indicates that there should be upwelling on the left side (looking downstream) of a coastal current and that the upwelling should drive a cyclonic eddy on the shelf near the canyon. The model also predicts that there should be an anticyclone in the upper water column, but the observations presented by Freeland and Denman [1982] are not sufficient to show the existence (or absence) of this circulation.

Two studies in the Mid-Atlantic Bight of the U.S. east coast [Mooers et al., 1979; Church et al., 1984] show the influence of Wilmington Canyon on the southward coastal flow in this region. Comparison of the model results with these observations is not good. The model predicts downwelling in the canyon and an anticyclone on the shelf. The temperature data [Mooers et al., 1979] indicates the exist-
ence of upwelling in the canyon and a cyclonic flow around the canyon. To further confuse the Mooers et al. [1979] study, two warm-core eddies were observed at the shelf break which bracketed the canyon and appeared to strongly influence the flow in the region. Finally, a set of surface drifters 60 km north of the canyon indicated northeastward surface flow which is counter to that expected in this area. The second observational program focused completely on Wilmington Canyon and reinforced the conclusions of the first study by showing that there is a cyclonic eddy over the canyon (0/80 m dynamic topography, Figure 7, Church et al. [1984]). This second study took place when there were no warm-core eddies near the canyon, but the character of the flow near the canyon does not seem markedly different from that observed by Mooers et al. [1979]. On careful consideration of Figure 7 [Church et al., 1984] it appears that the flow over the canyon is mainly offshore and that the pressure gradient over the canyon is actually such that deep water would be pushed up the canyon. The cyclone at the head of the canyon may then be due to downwelling (and vortex stretching) or simply the result of flow turning to follow topographic contours around the canyon. These data do not contradict the adjustment theory presented here, but do suggest that the oceanographic situation over Wilmington Canyon is not well represented by this simple model.

An extensive set of long-term current meter observations in and near Baltimore Canyon in the Mid-Atlantic Bight of the U.S. east coast [Hunkins, 1988] offers a picture of the mean circulation over a canyon. The near-surface mean flow seems to be unaffected by the canyon [Hunkins, 1988, Figure 7], except at one place over the head of the canyon. Observations from deeper levels (6 m above the bottom [Hunkins, 1988, Figure 9]) show upcanyon flow in the center of the canyon and down canyon flow at the canyon head. The upwelling within the canyon is the reverse of that expected due to the pressure gradient from the southwestward coastal flow. Hunkins proposes that the upcanyon mean flow is due to rectification of the tidal circulation. Because no hydrographic data or dynamic topography maps are presented in this study, the details of the circulation on the nearby shelf can not be discussed. These observations do not correspond to the model, but the existence of stratification in the canyon, which is not included in the model, may influence the resulting circulation pattern.

Conditions in and near Quinault Canyon off the northwest ern U.S. coast have been measured with current meters and hydrographic surveys [Hickey et al., 1986]. The focus of this study was on resuspension of sediments, but there is evidence that the deep flow along the axis of the canyon is correlated (at the 4- to 5-day period) with fluctuations of the near surface alongshore flow. Hickey et al. [1986] proposed that the deep flow in the canyon responds to the pressure gradient produced by the geostrophically balanced alongshore flow. This mechanism is the same as the one proposed here for the initial geostrophic adjustment in the model. The observations from Quinault Canyon show no indication of a persistent eddy at the head of the canyon, but the canyon is quite wide (30-40 km) so the geostrophic balance within the canyon may not be strongly inhibited and upwelling at the head of the canyon may not occur. It also appears that the poleward undercurrent at the shelf break enters the canyon and travels around the rim (occasionally forming eddies). This externally forced current may have a stronger effect on the circulation in the canyon than stretching of vortices due to vertical motion of isopycnals during adjustment.

These four situations range from narrow to wide canyons (compared to the internal radius of deformation) and the correspondence to the adjustment model is good in the first case, becoming less clear in the other cases. Only the situation observed near the canyon off Vancouver Island is related to the simple situation considered in the model. For the other canyons, the dynamics of pressure driven flow and vortex stretching seem to apply, but the overall dynamical...
It is not possible, or desirable, to show a large number of numerical solutions for the three-level equations for various values of the governing parameters. However, it is useful to understand how the solutions depend on various parameters, so simulations will be described in terms of the area-averaged energy of the initial and final states.

It is well known that only a portion of the energy of the initial state remains after adjustment [Gill, 1982, pp. 191-203; van Heijst, 1985], the remainder having been radiated as gravity waves. In the classical problems with a displaced free surface, the initial energy is only potential energy, while in the present analysis, the initial energy is both kinetic (mainly) and potential. The open ocean adjustment problem analyzed by Gill [1982] always results in one third of the potential energy lost by the initial state remaining as kinetic energy in the adjusted steady state. For adjustment over a canyon, there will be a range of values for the ratio of initial to final energy; a very narrow canyon will have little influence on the initial flow and the initial and final states will have almost the same energy resulting in an energy ratio of nearly one. The wider canyon cases will be more like the open ocean adjustment in which considerable energy is radiated.

The intent of this section is to show from energy calculations how the solution is affected by the internal and external radii of deformation, the width scale of the initial current and the width of the canyon. To this end, a series of cases is considered for fixed stratification and layer thickness (hence for fixed internal and external radii of deformation) but for a variety of current and canyon widths. The external radius of deformation is several hundred kilometers and is always larger than any other scale in the problem. As is evident from analysis of the two-level solutions, it is the shorter length that controls the width of the resulting flow, in any case. The internal radius does play a role but it will remain fixed at a value of 2.45 km for the parameters given in Appendix B.

Expressions for conservation of nondimensional kinetic energy come from (1) by multiplying the x and y momentum equation by \(u_i\) and \(v_i\), respectively, for each level \(i = 1, 3, 5\) and integrated over a level (by multiplying by the layer thickness, \(\delta_i\)). The potential energy expressions are obtained by multiplying the continuity equations by \(\eta_i\), \(e_3\psi_2\), and \(e_6\psi_4\), respectively. The sum of these equations yields conservation equations for total energy, where the conserved quantities are

\[
KE = \sum_{i=1,3,5} \frac{(u_i^2 + v_i^2)\delta_i}{2}
\]

\[
PE = \frac{1}{2} \left( \eta_i^2 + e_3\mu_2^2 + e_6\mu_4^2 \right)
\]

and the total energy is the sum of \(KE\) and \(PE\).

A useful quantity to consider is the total energy extracted from the initial state and how much of this energy remains in the final circulation. To obtain the total energy, the expressions for \(KE\) and \(PE\) must be integrated over an area; however, the initial state contains infinite potential and kinetic energy because the flow extends to infinity. It is logical to integrate in the y direction over one wavelength of the solution, that is, from \(y = 0\) to \(2\pi\mu\). The integral in the x direction is more of a problem, since the energy will be arbitrarily large for integrals over larger distances. The solutions, however, decay away from the canyon walls with a scale determined by the deformation radii and the length scale of the initial current. Therefore the choice is made that the x integrals will extend on either side of the canyon a distance equal to twice the longer of these scales. Energy integrals are divided by the area so that the final results will be in energy density. Tests (not given here) using the two-level analytical solutions confirm that the numerical integrals over the finite domain defined above were within 1% of the true value of the integral over an infinite domain.

The first energy consideration is how much energy is lost in the adjustment process as a measure of the perturbing influence of the canyon. The ratio of final energy density to the initial energy density for a variety of current and canyon widths for the three-level simulations (Figure 5(left)) shows that as the canyon and current become wider, there is a much more disturbing effect of the canyon on the initial flow. For the widest canyons shown here, about 30% of the initial energy has been radiated. For narrow canyons or narrow coastal currents, the canyon has only a small effect on the initial flow.

A second energy consideration is how much of the kinetic energy lost by the initial flow goes to potential energy (Figure 5(right)). Even though narrow canyons have a smaller total energy extraction, the largest amount of the extracted energy (50%) goes into potential energy (distortion of the density surfaces). As the canyon becomes wider, much less of the energy goes into potential (down to 10%). There is some evidence that a canyon with a width a little less than one internal radius (2.45 km in this case) produces a little more potential energy than a canyon that is slightly wider or narrower. However, the most efficient production of potential energy is clearly in canyons that are much narrower than the internal radius.

This energy ratio is larger than the value of one third obtained in the classical adjustment problem [Gill, 1982], although in the classical case it is potential energy that is released and kinetic energy that is created. The likely reason for this larger retention of energy is that the canyon walls allow internal pressure gradients in the x direction to exist in the final state which are not allowed in the open ocean adjustment problem (see Gill [1976] for analysis of adjustment in an ocean of finite width).

**DISCUSSION AND SUMMARY**

The purpose of this study is to consider the effect of submarine canyons on persistent coastal flows; of particular interest are the length scales of this interaction. The direct way to obtain this information is to analyze the geostrophic adjustment of an initially unbalanced current in the presence of a submarine canyon. This use of the adjustment problem has a long history [Rossby, 1938; Cahn, 1945; Bolin, 1953; Blumen, 1972; Gill, 1976; Stommel and Veronis, 1980; van Heijst, 1985] and the various aspects of geostrophic adjustment are summarized by Gill [1982, pp. 191-203].

Density stratification is included in a simple way by allowing the flow to exist as levels, that is, the velocity and density are known only at discrete depths and the vertical
derivatives in the governing equations are represented by finite differences. Two situations are considered in detail: a two-level system, for which one level represents the shelf flow and one level represents the canyon, and a three-level system, for which two levels represent the shelf flow. The two-level model is solved analytically and the various decay scales are seen explicitly. The three-level model is solved, ultimately, by numerical integration and the various length scales are determined by inspection of the solution.

There are four important length scales in the simulations: the internal and external radii of deformation, the width of the canyon, and the width of the initial current. For a given dynamical mode, the length scale with which perturbations decay is the shorter of the radius of deformation for that mode and the width of the initial current. For the external mode, the decay scale is always the width of the current it is unlikely that a coastal current will be several hundred kilometers wide. For the internal modes, the governing scale is the internal radius which is always smaller than (or at most equivalent to) the width of coastal currents.

The width of the canyon determines the character of the current-canyon interaction. For very wide canyons (compared to the internal radius or the width of the initial current), the steady circulation is trapped to the edges of the canyon, the external mode decays with the width of the current and the internal mode decays with the internal radius (e.g., Figure 3a). As the canyon narrows, the external mode in the canyon is inhibited (i.e., the geostrophic flow is reduced) because the pressure gradients are restricted, since the canyon is narrower than either of its allowed scales (e.g., Figure 3b). As the canyon continues to narrow, the internal mode is inhibited (e.g., Figure 3c). Eventually, for an infinitesimally wide canyon, there is no circulation in the canyon and the density gradient at the top of the canyon balances the surface pressure gradient. Even though there is very little circulation in the canyon, the canyon does have an effect on the shelf since the density is displaced to balance the pressure gradient at the top of the canyon. Also, there will be a residual circulation (an "eddy") on the shelf (with a radius about equal to the internal radius of deformation) due to the presence of the internal density gradients.

At some point, viscous effects, which are neglected here, will become important and will halt any influence of the canyon on the shelf flow. Such issues must await studies with viscous effects included.

A final observation is that geostrophic adjustment over a canyon results in a larger percentage of the initial energy being retained in the final state than is calculated for open ocean adjustment. Part of the reason is that motions are allowed that are trapped to the canyon walls, a situation that does not occur in an open ocean adjustment problem.

**Appendix A: Parameters for the Two-Level Canyon Simulations**

\[
g = 9.8 \text{ m/s}^2 \quad H_1 = 50 \text{ m} \quad H_3 = 25 \text{ m}
\]

\[
u_0 = 0.1 \text{ m/s} \quad f = 10^{-4} \text{ s}^{-1} \quad \frac{d\phi}{dz} = 0.02 \text{ kg m}^{-4}
\]

**Appendix B: Parameters for the Three-Level Canyon Simulations**

\[
H_1 = 25 \text{ m} \quad H_3 = 25 \text{ m} \quad H_5 = 25 \text{ m}
\]

\[
g = 9.8 \text{ m/s}^2 \quad u_0 = 0.1 \text{ m/s} \quad f = 10^{-4} \text{ s}^{-1}
\]

\[
\frac{d\phi}{dz} = 0.02 \text{ kg m}^{-4}
\]

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**References**


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