The use of oxygen as a test for an abyssal circulation model

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Abstract—Stommel's abyssal circulation velocities are used in the advective-diffusive-decay equation to calculate the dissolved oxygen distribution in an idealized world ocean basin. The use of a weighted difference scheme allows an extension of results obtained earlier to a broader range of values of the parameters for advection, decay, and eddy diffusion. Extreme cases of no diffusion and of no advection are used to interpret the results for intermediate values of the parameters. The optimal set of values found for the parameters are: horizontal eddy diffusion = \(6 \times 10^8\) cm\(^2\) sec\(^{-1}\), uniform upwelling velocity = \(1.5 \times 10^{-5}\) cm sec\(^{-1}\), oxygen consumption rate in abyssal waters = \(0.002\) ml l\(^{-1}\) yr\(^{-1}\), recirculation around the Antarctic Circumpolar Current = \(35 \times 10^6\) m sec\(^{-1}\). The optimal pattern is then compared with (an updated) observed oxygen distribution to determine the merits and deficiencies of the circulation model. The major deficiency appears to be the neglect of topographic effects on the abyssal circulation.

1. INTRODUCTION

In an earlier paper (Kuo and Veronis, 1970; hereafter referred to as I) we attempted to study the relative roles of advective and diffusive processes in the abyssal ocean by comparing the observed distributions of oxygen and \(^{14}\)C with the results of a calculation using the advective-diffusive equation together with Stommel's (1958) abyssal circulation model in an idealized world ocean basin. In that calculation the relative intensities of the advective and diffusive processes were adjusted to yield an oxygen distribution with gross features similar to the observed distribution. However, the numerical procedure used in the analysis could yield stable results only for a limited range of parameters. Following a suggestion of Manuel Fiadeiro of the Scripps Oceanographic Institution, we have since then made a series of calculations using a numerical procedure that makes it possible to obtain numerically stable results for the entire range of parameters. The numerical method and the results are presented here, together with a more up-to-date chart for the abyssal oxygen distribution.

The observed oxygen distribution at 4 km depth is shown in Fig. 1, where oxygen contours have been superimposed on a modified version of Defant's (1961) ocean depth chart. The contours in the Atlantic are taken partly from the chart produced by Stommel and Stroup (unpublished, but reported by Stommel and Arons, 1960b) and partly from Wüst's (1939) Meteor Atlas, the Indian Ocean values are from the atlas by Wyrtki (1971), and the Pacific values from a chart prepared by Reid (1971). The latter two include more detail than the Stommel-Stroup chart so they contain a denser network of contours.

In the Atlantic Ocean the maximum oxygen concentration occurs in the northwest region. Higher oxygen values exist at smaller depths in the Norwegian and Greenland

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Fig. 1. Distribution of observed dissolved oxygen in ml l$^{-1}$ at 4 km depth in the world ocean is plotted on a modified version of Defant's (1961) bathymetry chart. Data sources are given in the text. The stippled areas represent regions where the depth is less than 4 km. Where the data is too sparse for an unambiguous result (northwest Pacific and Mid-Indian) the contours are dashed.
seas, which are the source regions of oxygen in the North Atlantic. The role of western boundary currents is indicated by the high concentration (> 6 ml/l.) near the North American coast south to Florida and by the general decrease of concentration eastward. In the Antarctic Ocean, the concentration is lower (the high values in the Weddell Sea area do not appear in the chart). The minimum concentration is off the western coast of Africa between the equator and 15°S. The most significant feature of the distribution in the Atlantic is the more or less north–south orientation of the contour lines.

The distribution in the Indian Ocean shows the penetration of relatively high oxygen concentration northward from the Antarctic. In each of the sub-basins separated by the Mid-Indian Ridge, this penetration appears to be strongest on the western side. The smallest values of oxygen are in the northern part of the Arabian Basin. Generally, the values are lower than those in the Atlantic thus reflecting the fact that the water has aged since its exposure to the atmosphere in the polar regions of the Atlantic. An additional difference between the Atlantic and Indian distribution is the more nearly east–west orientation of the contours in the Indian Ocean. Even so, however, along a latitudinal circle the concentration is largest on the western side.

In the abyssal Pacific Ocean the oxygen concentration has values lower than in either of the other two oceans. Once again, the maximum values along a latitudinal circle tend to be on the western side. (In the South Pacific, submarine features north of New Zealand serve effectively as western walls.) The contours are oriented north–south in the North Pacific, east–west just north of the equator and are sloped toward the southeast in the South Pacific. Minimum values occur on the eastern side with the absolute minimum in the northeastern corner and quite low values off the coasts of Peru and Chile.

The goal in the present paper (as in I) is to use a circulation model together with the advective–diffusive equation for oxygen and to choose values for the parameters of the system to obtain a best fit with the observed oxygen distribution shown in Fig. 1. We interpret the degree to which the observed distribution can be reproduced as a measure of the validity of the approach and hopefully as a check on the circulation model. In addition, our hope is that we can obtain a measure of the importance of horizontal mixing in the distribution of tracers in the ocean.

The circulation model is a modified form of Stommel's abyssal circulation. The homogeneous abyssal ocean extends upward 3 km from the (flat) bottom, \( z = 0 \), to the base of the thermocline, \( z = H \). The flow is geostrophic everywhere except in boundary layers along the western sides of the ocean basins and along the northern boundaries of the Indian and Pacific oceans. These boundary layers are required only to satisfy mass continuity. The circulation is assumed to be driven by a horizontally uniform upward flux of fluid at the base of the thermocline. Since this upward flux serves as a sink of abyssal water, it is necessary that abyssal water be replenished. We have assumed sources of equal strength at the North Pole and at 40°W, 70°S in the Atlantic. Since the upward flow is not measurable, its value is one of the parameters to be determined.

The velocity pattern is easily calculated in the foregoing model when the velocity component normal to the eastern boundaries is taken to vanish. The velocity parallel to the western and northern boundary layers is assumed to be equal to the local boundary layer transport divided by the product of the depth and an assumed boundary layer thickness of 2.5° of longitude. In the present calculation the Antarctic Circumpolar Current is assumed to flow uniformly eastward and extends from 70°S northward to...
55°S. The eastward transport or 'recirculation' in the Antarctic Circumpolar Current has been estimated to be as low as $90 \times 10^6$ m$^3$ sec$^{-1}$ (Sverdrup, Johnson and Fleming, 1942) and as high as $265 \times 10^6$ m$^3$ sec$^{-1}$ (Reid and Nowlin, 1971). Since the bulk of the transport takes place in the upper kilometer, the abyssal recirculation, denoted here by $R$, may be as low as $25 \times 10^6$ m$^3$ sec$^{-1}$ or high as $75 \times 10^6$ m$^3$ sec$^{-1}$. We have made calculations with several values of $R$.

In Fig. 2 the trajectories of the abyssal flow together with the transports in the western boundary layers are shown in the idealized world ocean basin used in this study. The

![Diagram showing particle trajectories and transports](image)

**Fig. 2.** Particle trajectories for the abyssal circulation in the idealized world ocean are shown in the interior and in the boundary layers. A uniform recirculation is indicated by the arrows in the Antarctic Ocean. Sources of equal strength are located in the Atlantic at 90°N and at 40°W, 70°S and supply the abyssal water which is subsequently lost to the surface layer via uniform upwelling. Transport is in units of $\omega_0 a^2$.

Circumpolar Current extends 15° in latitude; otherwise, the model is the same as that used in I. For this geostrophic, hydrostatic model the velocity field is the same as that given by Stommel and Arons (1960a), viz.,

$$u = 2\omega_0 \frac{a}{H} \cos \phi (\lambda_B - \lambda)$$
$$v = \omega_0 \frac{a}{H} \tan \phi$$
$$w = \omega_0 (1 + \frac{z}{H}), \quad (1.1)$$

where $\lambda, \phi, z$ are coordinates directed eastward, northward and upward with velocity components $u, v$ and $w$, respectively; $\omega_0$ is the magnitude of the uniform upwelling.

*The recirculation is the subject of additional discussion in Section 4.*
velocity; \( H \) is the thickness of the abyssal ocean; \( a \) is the mean radius of the Earth and \( \lambda_B \) is the longitude of the eastern boundary for a basin.

Mass continuity requires stagnation points in the western boundary layers at 5°N in the Indian Ocean and 22.5°N in the Pacific Ocean with southward flow north of the stagnation points. Details of the circulation are given in I.

The advective–diffusive equation for dissolved oxygen is

\[
\mathbf{v} \cdot \nabla c^* + w \frac{\partial c^*}{\partial z} = K_v \frac{\partial c^*}{\partial z^2} + K \nabla^2 c^* + \nu^* , \tag{1.2}
\]

where \( \mathbf{v} = (u, v) \); \( c^* \) is the concentration of dissolved oxygen; \( K_v \) and \( K \) are vertical and horizontal mixing coefficients; \( \nabla^2 \) is the horizontal Laplacian; and \( \nu^* \) is the (constant) consumption rate of dissolved oxygen. The interior velocity field is given by equation (1.1). As mentioned earlier, the velocities in the boundary layers are determined from the boundary layer transports.

The concentration of oxygen is taken to be 6.5 ml/l. at the North Pole source in the Atlantic and 5.5 ml/l. at the South Atlantic source. At all side boundaries the normal derivative of concentration vanishes so that there is no flux of oxygen through these boundaries. In this respect the model differs from that in I, where we treated the concentration in the western boundary layers in an approximate manner which did not conserve the concentration when the consumption vanished.

A vertical average of (1.2) yields

\[
\mathbf{v} \cdot \nabla c + \frac{w_0}{H} (c^*|_{z=H} - c/H) = K_v \frac{\partial c^*|_{z=H}}{\partial z} + K \nabla^2 c + \nu , \tag{1.3}
\]

where \( \nu \) and \( c \) are now vertically averaged quantities. As in I, we shall assume that the terms arising from vertical convection and diffusion balance so that (1.3) becomes

\[
\mathbf{v} \cdot \nabla c = K \nabla^2 c + \nu . \tag{1.4}
\]

A horizontal average of (1.4) together with the boundary conditions of zero flux of fluid and of oxygen normal to the side boundaries leads to the balance

\[
< \frac{\partial w}{\partial z} > = < \nu > , \tag{1.5}
\]

where angular brackets denote horizontal averages. Thus the mean consumption, \(< \nu >\), of oxygen is balanced by the net vertical flux through the thermocline (the supply of \( c \) at the source must therefore exceed the loss via upward flow through the thermocline).

It is not possible for us to justify the detailed balance between vertical convection and diffusion required for the validity of (1.4). We simply assume that the primary horizontal variation can be determined from the manner in which horizontal advection and diffusion redistribute the oxygen from the source region. It is certainly possible that this assumption is wrong and even that sources associated with vertical processes dominate the horizontal distribution.

Equation (1.4) contains the two parameters, \( \nu \) and \( K \), neither of which can be determined simply. RILEY (1951) has proposed a value of \( \nu = 0.002 \) ml l\(^{-1}\) yr\(^{-1}\)
(or \(6.3 \times 10^{-11} \text{ ml l}^{-1} \text{ sec}^{-1}\)) for the consumption rate of oxygen in abyssal waters. This is about half the value proposed by Munk (1966). Since Riley’s estimate was based on calculations using a crude dynamical model, we would close the circle by adopting his estimate of \(v\) to test a dynamical model. Hence, we have experimented with several values of \(v\) in the following calculations.

The horizontal mixing coefficient, \(K\), is used to parameterize the process of exchange by small-scale (in both space and time) processes. Values of \(K\) extending over at least four orders of magnitude have been used in different studies associated with large-scale processes. \(K\) is the fourth adjustable parameter in the present study.

With four parameters at one’s disposal it ought to be possible to obtain almost any distribution.* However, each of the three terms in equation (1.4) contains one parameter and division by any one of them reduces the number of independent parameters to two. The recirculation is a third parameter but observational estimates impose a reasonably stringent condition on \(R\). Hence, there are basically only two independent parameters.

We can make use of equation (1.1) to non-dimensionalize the horizontal velocities by \(w_0a/H\) and the horizontal scales by \(a\). Thus

\[
v = w_0a/Hv', \quad a\nabla = \nabla',
\]

where primed quantities are dimensionless. Then the two independent parameters are the Peclet number, \(P = w_0a^2/HK\), which is a measure of advection to diffusion, and the dimensionless consumption parameter, \(\mu = va^2/K\).

In non-dimensional form equation (1.4) becomes (dropping the primes)\n
\[
P v \cdot \nabla c = \nabla^2 c + \mu.
\]

The dimensionless recirculation is

\[
p \equiv R/KH \delta \phi,
\]

where \(\delta \phi(= 15^\circ)\) is the north–south width of the Antarctic Circumpolar Current.

As we stated earlier, we have been able to obtain the distribution of \(c\) by a relaxation of a finite difference form of (1.7) when the velocity field is given by (1.1) in the interior and the boundary layer velocities are derived from the transports in the boundary layer. The boundary conditions on \(c\) are \(\partial c/\partial n = 0\) at all side boundaries. Here, \(\partial/\partial n\) corresponds to the normal derivative at a boundary. The world ocean is thus treated everywhere as a single system. In I we treated the western and northern boundary layers as separate regions.

When the Peclet number, \(P\), is \(0(1)\) or less, the solutions are easily obtained by ordinary relaxation methods from the centered difference approximation to (1.7). However, for \(P \gg 1\) the centered difference scheme does not yield a good approximation to (1.7) and the relaxation method does not converge. It is necessary in these cases to use a different numerical scheme. The problems encountered are more easily understood in the one-dimensional analogue to the convective–diffusive equation which we take up in the next section.

*As one wag put it, ‘With three parameters you can make an elephant; with four you can make it walk’.
2. THE WEIGHTED DIFFERENCE METHOD

The one-dimensional system that we consider is described by
\[
\frac{d^2c}{dx^2} + 2u \frac{dc}{dx} + vc = 0
\]
where \( u \) and \( v \) are arbitrary constants. The solution is
\[
c = \frac{e^{u(1-x)} \sin h \sqrt{(u^2 - v)x}}{2 \sin h \sqrt{(u^2 - v)}}.
\] (2.2)

For \( u^2 > v \) the solution is of exponential type (i.e. non-oscillatory) and for sufficiently large positive and negative values of \( u \) the distribution of \( c \) will exhibit sharp gradients near \( x = 0 \) and \( 1 \), respectively. Physically, the reason for these sharp gradients is easily appreciated. Strong convection from left to right corresponds to large negative \( u \) and the distribution will be predominantly determined by the fixed value at the left edge. Near \( x = 1 \) diffusive processes will adjust the value toward \( c = 1 \). Thus the distribution of \( c \) shows nearly zero values everywhere except near \( x = 1 \) where \( c \) rises sharply to \( c = 1 \). Similarly, strong convection from right to left (larger positive \( u \)) will yield \( c \approx 1 \) everywhere except near \( x = 0 \) where the distribution drops abruptly to \( c = 0 \).

Evaluating the derivatives by centered differences, i.e.
\[
\frac{dc}{dx} = \frac{(c_{n+1} - c_{n-1})}{2\delta}
\]
\[
\frac{d^2c}{dx^2} = \frac{(c_{n+1} - 2c_n + c_{n-1})}{\delta^2},
\] (2.3)
where \( \delta \) is the grid increment yields
\[
(1 + \delta u) c_{n+1} - (2 - v\delta^2) c_n + (1 - \delta u) c_{n-1} = 0
\]
\[
c_0 = 0, \quad c_n = 1.
\] (2.4)

Here \( c_n \) is the value of \( c \) at the point \( x = n\delta \), \( N \) is the number of grid intervals and \( n = 0, 1, 2, \ldots, N \).

The solution to (2.4) is
\[
c_n = \frac{r_1^n - r_2^n}{r_1^N - r_2^N},
\] (2.5)
where
\[
r_1 = \frac{2 - v\delta^2 + \sqrt{[4\delta^2(u^2 - v) + \delta^4v^2]}}{2(1 + u\delta)}
\]
\[
r_2 = \frac{2 - v\delta^2 - \sqrt{[4\delta^2(u^2 - v) + \delta^4v^2]}}{2(1 + u\delta)}.
\]

By writing \( r^n = e^{u1nr} \) one can easily show that the solution to (2.5) converges to (2.2) only for sufficiently small values of \( u\delta \). Thus, as \( u \) increases it is necessary to decrease \( \delta \).
to obtain convergence. If $u\delta$ is not sufficiently small, the solution is not only a poor
representation of the continuous system but it also changes character from exponential
to oscillatory. For reasons of computational economy it is often not feasible to choose $\delta$
as small as is required to obtain a good approximation to the continuous system. It is in
such circumstances that one obtains numerically unstable solutions when using
approximate methods such as relaxation.

To get around this problem, we introduce a weighted difference operator for the
convective term $u \frac{dc}{dx}$. In particular, we write the derivative $\frac{dc}{dx}$ as the sum of a
one-sided derivative to the left and a one-sided derivative to the right. Thus,

$$\frac{dc}{dx} = \theta \frac{c_{n+1} - c_n}{\delta} + (1 - \theta) \frac{c_n - c_{n-1}}{\delta},$$

where $0 \leq \theta \leq 1$. When $\theta = 1/2$, equation (2.6) reduces to the centered difference
scheme. For $\theta < 1/2$ the left-sided derivative is given more weight and for $\theta = 0$ or 1 the
derivative becomes completely one-sided.

The advantage of using (2.6) is that $\theta$ can be chosen to give proper weight to strong
convection from one side or the other. If $u$ also depends on $x$, then optimally $\theta$
should be chosen as a function of $x$. The disadvantage of using (2.6) is that the scheme involves
errors of $O(\delta)$ whereas the centered difference scheme is correct to $O(\delta^2)$.

Using (2.6) for the term $u \frac{dc}{dx}$, we obtain the finite difference system

$$(1 + u\theta\delta)c_{n+1} + [u\delta(1 - \theta) - 2 + v\delta]\{c_n + [1 - u\delta(1 - \theta)]c_{n-1}\} = 0,$$

$$c_0 = 0, c_N = 1.$$ (2.7)

A series of calculations for $|u| \leq 100$ and $|v| \leq 50$ was carried out and the value of $\theta$
which gave the best fit* to the analytic solution was determined for each case. By
adjusting $\theta$ we could restrict the maximum relative error to $3\%$ or less in all of our runs.
The most extreme case was for $u = 100$ with ten grid points so that $u\delta = 10$. For this
case the centered difference scheme yielded an oscillatory solution that differed from
the exponential, analytic solution by as much as $85\%$. With $\theta = 0.9$ the weighted
difference scheme produced a non-oscillatory solution with a maximum error of $3\%$.

In Fig. 3 we show the values of $\theta$ which produced the best fit to the analytical
solutions for several different values of $u$, $v$ and $\theta$. The results from the numerical
experiments are shown by asterisks and the solid curve (actually a broken line) is a mean
curve drawn through the experimental values. There is some scatter in the experiments
because of the wide range of $v$ ($0.2 \leq v \leq 50$) that was used.

PRICE, VARGA and WARREN (1966) have proposed the use of weighted derivatives
for the $u \frac{dc}{dx}$ term and have shown that the method leads to numerical stability for a
broader range of problems than the simple one treated here. A procedure similar to ours
has been used in numerical forecasting of weather. The weighted difference scheme has
sometimes been described as an artificial dissipation scheme because the equations can
be rewritten in a way that indicates that diffusion in the direction of flow is enhanced.

In our calculations for the oxygen concentration, we have used the weighted
difference scheme and have applied the values of $\theta$ given by the broken line in Fig. 3
and determined locally for the derivations in $v \nabla c$ in each direction. Then using an
over-relaxation method we could derive numerically stable results for all described

*By 'best fit' is meant the minimum value of the maximum relative error.
values of the parameters. For the calculations we used a square grid in longitude and latitude, i.e. $\Delta \phi = \Delta \lambda$. The finite difference form of (1.7) is then

$$\left\{ \frac{2}{\cos \phi_{i-j}} + 2 + P \left[ \frac{u_{i-j} \Delta \lambda}{\cos \phi_{i-j}} (1 - 2\theta) + v_{i-j} \Delta \lambda (1 - 2\sigma) \right] \right\} c_{i-j}$$

$$- \frac{1}{\cos^2 \phi_{i-j}} \frac{P u_{i-j} \Delta \lambda}{\cos \phi_{i-j}} c_{i+1,j} + \left[ \frac{1}{\cos^2 \phi_{i-j}} + \frac{P u_{i-j} \Delta \lambda}{\cos \phi_{i-j}} (1 - \theta) \right] c_{i-1,j}$$

$$+ \left[ 1 - \frac{\tan \phi_{i-j} \Delta \lambda}{2} - P v_{i-j} \Delta \lambda \sigma \right] c_{i,j+1}$$

$$+ \left[ 1 + \frac{\tan \phi_{i-j} \Delta \lambda}{2} + P v_{i-j} \Delta \lambda (1 - \sigma) \right] c_{i,j-1} + \mu (\Delta \lambda)^2,$$

(2.8)

where $\theta$ and $\sigma$ are the weighting factors for $\overline{\partial} / \overline{\partial \lambda}$ and $\overline{\partial} / \overline{\partial \phi}$, respectively, in the advection term. The subscripts $i$ and $j$ correspond to grid points in longitude and latitude, respectively. In our calculations we used the grid interval, $\Delta \lambda = \Delta \phi = 2.5^\circ$.

3. COMPARISON OF CALCULATIONS

In this section we shall discuss the results of the calculations and point out distinguishing features that can be associated with values of the parameters. Comparison with the observed distribution of oxygen will be made in the next section.

The results can be more easily understood if we first discuss two extreme cases, the one with no diffusion ($K = 0$ or $P = \infty$) and the one with no advection ($w_0 = R = 0$ or $P = \rho = 0$).

When there is no diffusion, equation (1.4) can be written in the form

$$\nu \frac{\partial C}{\partial \bar{y}} = \nu,$$

(3.1)
where $v_s$ is the horizontal velocity along a trajectory and $\frac{\partial}{\partial s}$ corresponds to a derivative along the trajectory. Equation (3.1) can be integrated to yield

$$c = c_0 + \int_{s_0}^{s} \nu \, ds,$$

where $s_0$ is the source point with concentration $c_0$ and $s$ is the distance from the source to the point under consideration.

For this case, the Weddell Sea source has no influence on the distribution, since the trajectories through that source do not penetrate into the ocean basins themselves. We have assumed here that the concentration is uniform across the Antarctic Circumpolar Current. The values of $c$ have been obtained from (3.2) by starting from the North Atlantic source and integrating along the trajectories to the northern edge of the Circumpolar Current. The value of $c$ is then averaged across the Circumpolar Current and the integration is continued. The oxygen inputs into the Indian and Pacific oceans are taken from the values at the corresponding longitude in the Circumpolar Current. The resulting distribution is shown in Fig. 4 for the case with $w_0 = 1.5 \times 10^{-5}$ cm sec$^{-1}$, $\nu = 0.002$ ml$^{-1}$ yr$^{-1}$ and $R = 3.5 \times 10^{13}$ cm$^3$ sec$^{-1}$. The most significant feature of the distribution is that the contour lines are oriented more or less north–south everywhere except near the northern boundaries of the Indian and Pacific oceans where recirculation of the flow in the northwest corners tends to decrease the concentration. This general north–south orientation of the contour lines is characteristic of the cases with high Peclet number. Our main interest here is in the orientation of the contour lines. The values of $c$ change with different values of $\mu$.

The second extreme case occurs when the convective velocity and the recirculation
both vanish so that the Peclet number vanishes. The only processes that act are diffusion and decay and equation (1.4) reduces to

\[ K \nabla^2 c + \nu = 0. \]  

(3.3)

The results for \( K = 10^7 \text{ cm}^2 \text{ sec}^{-1} \) and \( \nu = 0.002 \text{ ml l}^{-1} \text{ yr}^{-1} \) are shown in Fig. 5. Since the spread of oxygen takes place only by diffusion, the contours are oriented east–west in the ocean basins but there is a more complicated structure associated with the solution of the Poisson equation for the geometry of the Antarctic region.

Hence, we may observe that diffusion lends an east–west orientation to the distribution and advection orients the contours in the north–south direction. In all of the calculations reported below the orientation of the contours is determined completely by the value of the Peclet number. The actual values of \( c \) on the contours are determined principally by the value of \( \mu \).

Using the information gained from these extreme cases, we made a series of calculations and obtained a best fit to the observed oxygen distribution when we used the parametric values \( K = 6 \times 10^6 \text{ cm}^2 \text{ sec}^{-1}, w_0 = 1.5 \times 10^{-1} \text{ cm sec}^{-1}, \nu = 6.3 \times 10^{-11} \text{ ml l}^{-1} \text{ sec}^{-1} \) and \( R = 3.5 \times 10^{13} \text{ cm}^3 \text{ sec}^{-1} \). These values are quite close to the ‘best’ values proposed by Arons and Stommel (1967) using a much simpler model. The corresponding non-dimensional parameters are \( P = 3.4, \mu = 4.3 \) and \( \rho = 75 \). These will be referred to as the optimal values. The resulting distribution is shown in Fig. 6. It will be compared with the observed distribution in the next section.

We now show the changes brought about by altering the parameters from the optimal values. First, reducing \( P \) from 3.4 to 1.1 and keeping \( \mu \) and \( \rho \) fixed (this corresponds to decreasing \( w_0 \) with \( K, \mu \) and \( \rho \) fixed) yields the distribution shown in Fig. 7. The apparent age of the water is increased, i.e. \( c \) has values smaller than in the optimal case, and the contours are oriented more in the east–west direction. In Fig. 8 the only change from the optimal case is an increase of \( P \) (increase of \( w_0 \)) from 3.4 to 7.8. There is a significant rotation of the contours toward a north–south orientation. Because of the increased advection of oxygen there is less time for decay and the values of \( c \) are
Fig. 6. Same as Fig 4 but the values of the non-dimensional parameters are $P = 3.4, \mu = 4.3, p = 75$.

Fig. 7. Same as Fig. 6 but $P = 1.1$.

Fig. 8. Same as Fig. 6 but $P = 7.8$. 


increased. These two figures provide support for our statement that an increase (decrease) of $P$ rotates the contours toward a north–south (east–west) direction.

Keeping $P$ and $p$ fixed and changing $\mu$ yields the two distributions shown in Figs. 9 and 10. The contours are similar but the water appears older for larger $\mu$ and younger for smaller $\mu$, as one would expect.

![Fig. 9. Same as Fig. 6 but $\mu = 6.5$.](image)

![Fig. 10. Same as Fig. 6 but $\mu = 2.2$.](image)

The principal effect of increasing $R$ (or $p$) only is to increase the concentration in the Antarctic region. There is an associated increase of $c$ in the northeast corners of the Indian and Pacific oceans. A decrease of $R$ brings about a decrease of $c$ in the same regions. These changes can be seen in Figs. 11 and 12 which have $\rho = 32$ and $\rho = 120$, respectively. The relatively large decrease of $c$ in the northeast Pacific in Fig. 11 is due to the fact that a weak recirculation allows more decay so that the incoming water for the Pacific has substantially lower values of $c$. The longitudinal extent of the 5 ml l$^{-1}$ contour is very limited in Fig. 11. The contour extends around the globe in Fig. 12. This feature provides a good measure for choosing the optimal value of $\rho$. 
A simultaneous decrease of $P$ and $\mu$ with $\rho$ fixed leads to relatively unchanged values of $c$ but the contours tend to flatten. An increase of these two parameters rotates the contours to a steeper north–south orientation. The results are shown in Figs. 13 and 14.

Finally, in Figs. 15 and 16, we show the effects of a threefold decrease and increase respectively in $P$, $\mu$ and $\rho$. These changes correspond simply to a respective increase and decrease in $K$. The resulting distributions look much like those for the extreme cases when either diffusion or advection is absent. Hence, we can put definite bounds on the values of $K$ which lead to distributions significantly different from the purely advective and diffusive cases.

4. DISCUSSION OF RESULTS

The optimal values of the non-dimensional parameters leading to the distribution in Fig. 6 correspond to ratios of the parameters $K$, $w_0$, $v$ and $R$. Thus we can choose any values of $K$ and $w_0$ as long as the Peclet number is 3.4. The only real constraint on the dimensional parameters that is imposed by observation is the longitudinal extent of the
Fig. 13. Same as Fig. 6 but $P = 1.7, \mu = 2.2$.

Fig. 14. Same as Fig. 6 but $P = 6.8, \mu = 7.6$.

Fig. 15. Same as Fig. 6 but $P = 1, \mu = 1.2, \rho = 22$. 
Fig. 16. Same as Fig 6 but $P = 10$, $\mu = 12$, $\nu = 220$.

5.0 ml 1.−1 oxygen contour line. This contour extends approximately to the western side of the Pacific–Antarctic. (The 5.0 contour in the eastern side of the Pacific–Antarctic is probably associated with the relatively weak source of abyssal water in the Ross Sea which we have not taken into account here.) With this constraint, and the optimal value $\rho = 75$, the value of $K$ must be fixed at $6 \times 10^6$ cm$^2$ sec$^{-1}$. The values of $w_0$ and $\nu$ are then also determined to be those given for Fig. 6.

With the present circulation model, it is difficult to see how the Peclet number can be taken to be appreciably different from the present optimal value, although some freedom in the choice (maybe a factor of 2 one way or the other) is possible. Significantly larger values of $P$ steepen the Atlantic contours, but the observed contours are rather steep. The main objection to using larger $P$ values is that this would lead to considerably older water in the northeast Indian and Pacific than is observed. Hence, we must allow for a sufficiently strong diffusion to spread the tracer into these northeastern corners. Thus $P$ cannot be increased significantly. On the other hand, lower values of $P$ would flatten the contours in the Atlantic in contradiction to the observations. Hence, the combination of a magnitude for the recirculation restricted by the 5.0 ml 1.−1 oxygen contour, the degree of north–south orientation of the contour lines and the ages of the waters in the northeastern Indian and Pacific oceans leads to the optimal value of $P$, and, furthermore, to the individual values of $w_0$ and $K$.

With $P$ given it is not possible to alter $\nu$ by more than a fraction of the optimal value if the observed ages are to be preserved. We are therefore led to the optimal values given for Fig. 6.

The remaining figures are useful mainly to show the effects of varying the different parameters. The changes brought about in the flow patterns are evident from the graphs and the discussion in the previous section.

If we accept Fig. 6 as the best fit, our conclusions regarding the roles of the different processes are those given in I. The role of western boundary currents in transporting oxygen from the source regions to the different ocean basins is very clearly outlined by the decreasing concentration eastward from the western boundary currents which obviously carry the higher concentrations into each basin. The role of diffusion is evident from the flattening of the contours, particularly near the southern extremity of each basin. Hence, we conclude that the circulation in the abyssal waters can be deduced
from the observed oxygen distribution only if the distribution is interpreted in terms of a consistent set of dynamics together with mixing processes in the deep water. The recent observations of rather intense transient small-scale motions in deep water provide evidence to support the idea of horizontal mixing.

Although our calculations have not been carried out for all possible values of the parameters, we have attempted to consider the entire ranges of values that have been used by previous investigators. In some cases, such as the use of a substantially different value of \( \nu \) from the optimal, the overall changes are so extreme that one can easily narrow the range. In others, such as the value of \( R \), the effect on the ocean basins themselves is not large and one has to rely on a relatively local feature, as we did with the extent of the 5.0 ml l.\textsuperscript{-1} contour line, to narrow the acceptable range.

One can object to the use of constant values for \( K \), \( w_0 \) and \( \nu \) because these quantities probably do vary with geographical position. Even more serious, of course, is the use of a mixing coefficient to describe the effects of smaller scale processes. We can only admit to having used a highly oversimplified system and take refuge in the fact that the present state of the problem does not warrant more elaborate parametric representations of these processes. In any event, we can come to definite conclusions about the effects of the different processes and these should be helpful in future work.

When we started this study, one of our purposes was to test the validity of Stommel's abyssal circulation model. Although the model makes use of a consistent set of dynamics, it represents only a zero-order picture for a highly simplified system. It is evident from our study that an improvement of the circulation model is necessary. The most obvious need is to incorporate topographic effects. All of the ocean basins appear to be divided into sub-basins by the various ridges. The Mid-Atlantic Ridge is an effective barrier to communication between the eastern and western sides except through small passages such as the Romanche Trench and the one between the Guiana and the Cape Verde Basins. The Walvis Ridge and the Rio Grande Ridge also serve as barriers. The Mid-Indian Ridge divides the Indian Ocean into two sub-basins each of which appears to have its own effective western boundary. Inclusion of this ridge would increase the oxygen concentration in the eastern Indian Ocean and perhaps yield the minimum in the Arabian Basin. The Pacific, too, is broken up into a number of smaller basins.

The effects of these divisions can be seen in the observed oxygen distribution. The minima in the northwest Indian and in the eastern equatorial Pacific are two examples of effects due to ridges.

The overall calculated distribution of oxygen in the Pacific agrees least with the observations. Our results yield contours with the greatest north–south orientation in the equatorial regions and a more east–west slope in the polar regions. The observed contours slope nearly east–west in the equatorial regions. These differences are probably connected not only with the ridges but also with irregularities in lateral boundaries (or effective boundaries).

It would be very desirable to extend the abyssal circulation model to make it consistent with a complete model of ocean circulation, including the waters above the thermocline. We have made some progress in this direction but have not yet got to the point where one can incorporate the tracer distribution.

Finally, something should be said about where our study fits in with other efforts to interpret the distribution of tracers in terms of the circulation. In mathematical complexity the present approach lies between the simpler one which makes use of box
models (e.g. BROECKER and LI, 1970) and the much more complex one in which the
three-dimensional system is integrated numerically (e.g. HOLLAND, 1971). Our prefer-
ence for the present approach is based on our feeling that we can come to firmer con-
clusions about the roles of physical processes than is possible with box models, but, at
the same time, we want to avoid the uncertainties associated with massive numerical
integrations. All of these approaches contribute to our understanding. The choice which
one makes is a matter of taste and orientation.

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