Semidiurnal internal tides in a Patagonian fjord

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1. Introduction

Fjords are mid- to high-latitude estuaries formed by the advance of glaciers through mountainous terrain. They are narrow and deep and may contain one or more submarine sills (Farmer and Freeland, 1983; Inall and Gillibrand, 2010; Stigebrandt, 2012). The main mechanisms for mixing in fjords are thought to be linked to shear instabilities caused by wind stresses and internal waves/tides (Farmer and Freeland, 1983; Inall and Gillibrand, 2010) and Stigebrandt (2012) discuss how the barotropic tide supplies energy to internal waves. This energy is in turn lost to an increase in potential energy either through mixing or heat transfer; the latter not being likely because of the large specific heat capacity of water. The energy transfer from internal waves to turbulence and mixing is of particular importance to deep basin waters where wind-driven mixing has little effect. Therefore, the study of internal waves (tides) in fjords is basic and timely (Stigebrandt, 2012; Aguirre et al., 2010; Inall and Gillibrand, 2010; Drujhout and Maas, 2007; Allen and Simpson, 1998; Farmer and Freeland, 1983).

Internal waves are common in fjords because of the barotropic tide interacting with abrupt changes in bottom topography related to the presence of sills, as explained in, for example, Farmer and Freeland (1983) and St. Laurent et al. (2003). However, internal waves can also be generated at tidal frequencies when the barotropic tide interacts with river plumes (Nash and Moum, 2005; Stashchuk and Vlasenko, 2009).

In central Patagonia (47°S), cold and fresh estuarine water enters the fjord through glacial ice melt from Glacier Steffen and through discharge from the Baker River (Pérez-Santos et al., 2013, 2014). The interactions of these freshwater sources with the oceanic water produce haline stratification in the fjord, favoring the development of internal waves. The Baker River transports silt to the fjord, increasing water turbidity and reducing light. Most of
the silt is accumulated in the stratified layer, while some escapes to the bottom of the fjord. This accumulation of silt, together with accumulation of plankton and a typically sharp sound signal at the pycnocline, provide an opportunity to describe pycnocline undulations with the echo anomaly of a Doppler profiler (Valle-Levinson et al., 2001). Farmer and Freeland (1983) used a similar technique with echo-sounding in order to identify internal waves in Knight Inlet.

The objective of this study was to describe the presence of internal tides in the Steffen–Baker fjord and identify their triggering mechanism. This objective was addressed with acoustic measurements of current velocity and echo anomaly profiles collected from an acoustic Doppler current profiler (ADCP), and complemented by CTD profiles and sea level data. Observations revealed, for the first time in Patagonian fjords, the presence of semidiurnal internal tides. These findings are fundamental for the study of fjord physics because internal waves are one of the most dominant mechanisms for vertical mixing in these estuarine ecosystems (Monismith, 2010).

Methodology for this work is discussed in Section 2 with the location of the study site and an overview of data collection in Sections 2.1 and 2.2, respectively. Sections 2.3–2.6 detail data analysis techniques used on data presented in Section 2.2. Results are presented in Section 3, organized as follows: characteristics of the fjord will be discussed in Section 3.1 to determine tidal features such as the location of the pycnocline and internal wave generation. This subsection also details the wavelength, horizontal phase speed and the phase propagation angle of internal waves in this region. Section 3.2 presents analyses on acoustic data including Empirical Orthogonal Functions (EOF), spectral analysis and wavelet techniques. Section 3.3, focuses on the baroclinic velocity with EOF, spectral and wavelet analyses also applied to these data. Section 3.4 presents comparisons of Baker River discharge to the spatial and temporal location of internal waves derived from the echo anomaly as well as from the baroclinic velocity. Discussion and conclusions are presented in Sections 4 and 5, respectively.

2. Methodology

2.1. Study area

Central Patagonia, encompassing approximately 1000 km of the southwest coast of Chile (in a straight line), contains one of the world’s most extensive fjordic systems (Pantoja et al., 2011). The study area is located in central Chilean Patagonia (~47°S) and is a region with complex geographic features, e.g. the Martinez Channel, the Steffen fjords, the Baker River mouth and many islands and small channels (Fig. 1a). This extensive area located between the Northern and Southern Ice Field, receives ~3400 m³ s⁻¹ of fresh water from both precipitation (~2500 m³ s⁻¹, Dávila et al., 2002) and the Baker River (average flow rate of ~900 m³ s⁻¹, Cáceres and Gudiño, 2009).

This freshwater input from glacial melt and rivers to the fjords and channels of central Patagonia, as well as sizable pluvial influence, contributes to the formation of a thin surface layer (~5–10 m deep) characterized by low temperatures and salinity (Calvete and Sobarzo, 2011). This buoyant layer can be considered as...
river and estuarine water (REW; Aiken, 2012) with salinity and density range of 0–30 g kg\(^{-1}\) and 0–23 kg m\(^{-3}\), respectively (Fig. 4b). Below this layer, oceanic waters occupy the rest of the water column, represented mainly by the Modified Subantarctic Water (salinity 31–33 g kg\(^{-1}\), and mean temperature of 9.8°C) and Subantarctic Water (salinity >33 g kg\(^{-1}\) and mean temperature of 9.7 °C) (Sievers and Silva, 2008; Silva et al., 2009; Pérez-Santos et al., 2013) (Fig. 4b). A hydrographic profile displays a two-layer system, characterized by a strong pycnocline in which salinity changes from ~24 to 30 g kg\(^{-1}\) over a depth of 7–12 m (Silva and Calvete, 2002; Pérez-Santos et al., 2014). The intra-annual variability of stratification within Martinez Channel varies only slightly through a deepening of the pycnocline during austral summer and fall (Meerhoff et al., 2013; Aiken, 2012).

The geometry of Martinez Channel includes many islands, allowing for complex curvature throughout the fjord. Bathymetric charts indicate several possible sill/contraction locations located between the Pacific Ocean and the study area location within Martinez Channel (Fig. 2). This is expected since the open ocean is ~80 km westward of the study site. The closest sill location is located southwestward of Isla Berta, ~15.5 km from the study site location as shown in Fig. 2. The depth of the channel decreases to 71 m at this location, with surrounding depths nearing 300 m. This sill location would likely only affect flow moving in the southern channel. Another sill location is located to the east of Isla Irene (not shown). This sill location, approximately 31.3 km from the ADCP mooring, exhibits shallow depths between 65 and 90 m extended over the entire width of the channel.

The potential sills could not be verified in the field, but are explored here as potential locations for internal wave formation. Moreover, the Baker River mouth is closer than the possible sills to the sampling site and will also be investigated as a potential trigger for internal waves.

2.2. Data collection

Measurements were collected from a mooring anchored at the mouth of Steffen fjord (73.73°W, 47.82°S) located in Martinez Channel; ~6.2 km to the west of the Baker River mouth (Fig. 1a–c). Hourly velocity profiles were collected from an Acoustic Doppler Current Profiler (300 kHz Workhorse ADCP from Teledyne RD instruments), moored at ~40 m depth from March 8 to April 30 2009. The average of 180 pings, with ping interval of 20 s and vertical bin resolution of 1 m, produced a velocity standard deviation of 0.96 cm s\(^{-1}\). Water surface variations were recorded hourly with a SeaBird SBE 16plus SeaCat CTD. The same sensor began collecting data at 70 m depth after the passage of a storm that destroyed the surface buoys on March 24th (day 83) of 2009 and caused the SeaCat to fall below the depth of the ADCP (arrows in Fig. 1c). The CTD measured tidal ranges throughout this deployment, which varied from ~1.9 m during spring tides to ~1 m during neap tides (see Fig. 3a). It should be noted that the tidal data only encompasses March 25th to April 30th (days 84–120) while the velocity data extends from March 8th to April 30th (day 67–120) of 2009.

Salinity and temperature measurements were collected with a SeaBird SBE 16plus SeaCat CTD at 11 different locations along the eastern part of Martinez Channel up to ~100 m depth on May 9th, 2009 (day 122, 2009) (Fig. 1b). Table 1 details the location, time and maximum depth of each CTD cast. These CTD data were also used for estimates of internal wave modes, as well as modal speeds and wavenumbers. Measurements of Baker River discharge...
were collected by Chile’s general water department from (www.dga.cl) a flow station (Rio Baker Bajo Nadis; 47.5009°S, 72.9749°W) located ~85 km from the ADCP mooring at the mouth of Steffen fjord.

2.3. Tidal harmonic analysis

In order to determine the dominance of the principal tidal constituents in the study area a harmonic analysis was applied to the sea level time series obtained from the SeaBird 16plus SeaCat at the mooring location (Section 2.2). Tidal amplitude and phase of sea level records were computed using the harmonic method described by Pawlowicz et al. (2002), which considers the algorithms of Godin (1972, 1988) and Foreman (1977, 1978). Harmonic analysis included a standard error with a 95% confidence interval (Pawlowicz et al., 2002).

2.4. Detecting internal wave with echo anomaly from ADCP

Echo intensity (or its normalized version, acoustic backscatter) measures the reflection of the ADCP beam off of suspended particles varying with depth. Strong bottom to surface attenuation in the echo intensity required a normalization of the sound scatter. This was accomplished by applying the following transformation to the echo intensity, which will be denoted here, as $E_A$:

$$E_A = 10\log_{10}(ECHO) - \langle 10\log_{10}(ECHO) \rangle,$$

as done in Valle-Levinson et al. (2004) (Fig. 6b). The angle brackets denote a time mean and this normalization of the echo intensity will be referred to as the echo anomaly ($E_A$). A vertical gradient of the normalized echo intensity, or $E_A$, highlighted the possible presence of internal waves (Fig. 6c). A spectral distribution of $E_A$ with depth was determined to detect the depth-dependent structure of the variance in the dominant frequencies (Fig. 7). In addition, an Empirical Orthogonal Function (EOF) analysis of $E_A$ was performed to determine whether internal semidiurnal oscillations were identifiable in different modes, as shown in Fig. 8.

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Fig. 3. (a) Tidal amplitude records collected in Steffen–Baker region from March 25 to April 30, 2009. (b) Amplitude of principal tidal constituents (in bars) with confidence interval of 95% (in a dashed black line).
2.4.1. Empirical orthogonal function analysis

Empirical orthogonal function analysis is a standard data analysis tool borrowed from meteorology. It is a purely statistical technique that compacts spatial and temporal variability of data sets by decomposing them into multiple orthogonal functions (Emery and Thomson, 2004). Real-vector EOF analysis was used for the time series of echo anomaly and current velocity data acquired from the ADCP mooring (see Section 2.2). The goal of the EOF analysis was to determine if dominant modes of variability would exhibit a two-layer structure with respective temporal variation showing high energy at or near semi-diurnal frequencies. The setback of EOF analysis is that it will not necessarily indicate precise times when these frequencies occur. Therefore, wavelet analysis was invoked, as it does exactly this. For a more detailed description of the EOF analysis technique, see Appendix A.

2.4.2. Wavelet analysis

A wavelet analysis (e.g. Torrence and Compo, 1998) was applied to the echo anomaly, the vertical gradient of the echo anomaly, and the baroclinic velocity (Section 2.6). The spatial and temporal variability of the semi-diurnal energy could be found by extracting the amplitude and phase of the semidiurnal wavelet, which was calculated at each depth (see Fig. 9). This method of representing wavelet analysis results is called a power Hovmöller diagram (Torrence
and Compo, 1998). The advantage of the power Hovmöller is that the temporal variations in internal waves can be seen at different depths within the water column. This is not possible with a conventional wavelet or power spectral analysis, as wavelet analysis provides a measure of variations in power in two-dimensional frequency-time space and spectral analysis does it in the frequency–depth space. The wavelet analyses applied to these data presented positions where the power of one particular frequency, namely the $M_2$ harmonic constituent, was identified in depth–time space. For a more detailed description of the wavelet analysis technique applied to these data, see Appendix B.

2.5. Internal wave properties

Currently, there are two methods available for describing internal waves: vertical mode theory and ray theory. Neither method is...
ideal for describing internal waves in fjords, as the first method assumes that the bottom is horizontal and flat, and the latter assumes wavelengths are small in comparison to depth. However, both of the above mentioned methods have been shown to successfully describe internal tidal waves in fjords (Cushman-Roisin and Svendsen, 1983; Allen and Simpson, 1998) and both will be explored in this study.

2.5.1. Vertical mode theory

Buoyancy frequency characterizes the water column stratification and provides the maximum frequency of internal waves:

\[ N^2 = \frac{g}{\rho_o} \frac{\partial \rho}{\partial z} \]  

(1)

where \( g \) is the gravitational acceleration, \( \frac{\partial \rho}{\partial z} \) is the water column stratification (or change in potential densities of vertically adjacent seawater parcels with respect to depth) and \( \rho_o \) is a reference water density (1025 kg m\(^{-3}\)). The buoyancy frequency was calculated from salinity and temperature measurements up to 98 m depth and along Martinez Channel (Figs. 1 and 4d). In addition to calculating \( N^2 \), the along-channel mean of the buoyancy frequency was calculated to give one value at each depth \( \left( N^2 \right) \).

A linear internal wave equation can be formulated using the along-channel mean buoyancy frequency, \( N^2 \) (Gill, 1982; equation 6.10.2). Consider a horizontal, two-dimensional, linear internal progressive wave travelling in the \( x \)-direction with frequency, \( \omega \), namely \( u(x, z, t) = U(z) \cos(kx - \omega t) \) and \( w(x, z, t) = W(z) \sin(kx - \omega t) \), where \( k \) is the unknown horizontal wavenumber, \( u \) is the horizontal velocity and \( w \) is the vertical velocity. It has been shown in Phillips (1977), that the vertical velocity, \( w \), satisfies the linear internal wave equation:

\[ \frac{d^2 w(z)}{dz^2} + k^2 \left( \frac{N^2(z) - \omega^2}{\omega^2} \right) w(z) = 0, \]  

(2)

for a horizontal, flat bathymetry. The above equation together with boundary conditions \( w = 0 \) at \( z = 0, -h \), and \( \rho, N^2 \) and \( \omega \) prescribed, constitutes an eigenvalue problem with an infinite set of solutions described by eigenvalues, \( k \), and eigenfunctions, \( w(z) \). The eigenfunctions describe the vertical structure of the vertical velocity and are more commonly called the vertical modes of the internal waves.
waves. From continuity, an equation for the horizontal velocity is found as:

\[ u(z) = \frac{1}{k} \frac{dw(z)}{dz} \]  

(3)

The vertical structure of both \( u \) and \( w \) obtained from Eqs. (2) and (3) are heavily dependent upon the buoyancy frequency profile \( (N^2) \).

The frequency of the semidiurnal internal waves, \( \omega \), was denoted as \( \omega = 2\pi/(12.42 \times 3600) \) s\(^{-1}\). Using the known values of \( \omega \) and \( N^2 \) to solve Eq. (2), the internal wavelength was then found for mode \( i \) internal waves as:

\[ \lambda_i = \frac{2\pi}{k_i} \]  

(4)

where \( k_i \) is the mode \( i \) horizontal wavenumber. The phase speed for mode \( i \) internal waves was then found as \( c_i = \frac{\omega}{k_i} \).

Vertical mode theory is only valid for a flat, horizontal bottom. Therefore, in order to utilize this method for a sloping bed, as found in fjords, ray theory must be instated to show that the angles along

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**Fig. 9.** Power Hovmöller diagrams showing the semidiurnal signal obtained from wavelet analysis of (a) echo anomaly (db\(^2\) s\(^{-2}\)) where (b) is the time average of (a) and (c) is the depth average of (a); (d) the vertical gradient of echo anomaly (db\(^2\) s\(^{-2}\) m\(^{-2}\)), where (e) is the time average of (d) and (f) is the depth average of (d); and (g) baroclinic velocity (cm\(^2\) s\(^{-2}\)) where (h) is the time average of (g) and (i) is the depth average of (g).
which energy propagates are greater than the bottom slope (Le Blond and Mysak, 1978).

2.5.2. Ray theory
Internal waves have a dispersion relation where their frequency depends only on the angle, $\theta$, made between wave rays and the vertical axis. This is quite different from surface waves whose dispersion relation depends only on the magnitude of the wavenumber. The dispersion relation of internal waves can be expressed as (Gill, 1982; Farmer and Freeland, 1983):

$$\omega^2 = N^2 \frac{k^2}{k^2 + m^2} = N^2 \cos^2 \theta.$$  \hspace{1cm} (5)

The frequency, $\omega$, the buoyancy frequency, $N$, and the wavenumber, $k$, are all known quantities and therefore the vertical wavenumber, $m(z)$, can be determined as $m(z) = \pm k N^2 \omega \sqrt{\frac{z}{N^2}}$, as shown in Cushman-Roisin and Svendsen (1983). In the case where $|m| > |k|$, the phase propagates in a nearly vertical orientation, while energy propagates in nearly the horizontal plane. Ray trajectories, or phase propagation angles, are defined from Eq. (5) as

$$\theta = \cos^{-1} \frac{\omega}{N}.$$  \hspace{1cm} (6)

and energy propagates along a ray having slope, $\varphi = \frac{\pi}{2} - \theta$, always perpendicular to the direction of phase propagation (Farmer and Freeland, 1983).

2.5.3. Internal tide generation mechanisms
There are three potential internal tide generation mechanisms. First, there are potential sill locations throughout Martinez Channel which will be explored as internal tide generation sites. Second, tidal currents interacting with the sloping shore at the head of a fjord can generate internal tidal waves as in Farmer and Freeland (1983). In these cases, the internal Froude number at the river mouth (or more commonly used at the sill crest), is given by,

$$Fr_I = \frac{u}{c_i},$$  \hspace{1cm} (7)

where $u$ is the along-channel velocity and $c_i$ is the phase speed of internal wave mode $i$. If $Fr_I > 1$, the supercritical internal flow will inhibit development of any wave behavior. However, if $Fr_I < 0.3$, internal tides would be expected to be generated. Finally, for the third mechanism, recent studies by Nash and Moun (2005) and Stashchuk and Vlaskenko (2009) have described how the Columbia River plume generates internal waves during the beginning ebb stages of the tidal cycle. These waves are generated at the river plume front when the flow goes from supercritical to subcritical. The internal waves are trapped until the internal wave phase speed surpasses the river plume velocity, and the internal waves are then shed from the plume front. For this to be the generating mechanism in Martinez Channel, the internal Froude number given in Eq. (7) (with $u$ now the frontal velocity) would have to be $>1$ at the river mouth. Moreover, the internal waves would be shed once their wave speed surpassed that of the river plume front.

2.6. Internal wave detection from baroclinic velocity

The east–west ($u$) and north–south ($v$) velocity components measured by the ADCP were rotated to have orientations along and across the Martinez Channel, respectively (denoted $u_*$ and $v_*$). The depth means of both the rotated along- and cross-channel velocities were subtracted from each rotated velocity component, $u = u_* - \overline{u}$, $v = v_* - \overline{v}$, to give the anomaly from the depth mean, or the baroclinic velocity components (along-channel component shown in Fig. 11a). This technique is sufficient when velocity measurements are available for the entire depth of the water column (Aguirre et al., 2010). These data include the upper ~40 m of the water column. While the fjord is 283 m deep at the location of the ADCP mooring, this method was utilized because velocities below the available measurements (deeper than 40 m) are minimal in comparison to surface velocities and therefore contribute little to the depth mean velocity. Also, the hydrographic variables of the fjord (salinity, temperature, density) change very little below 40 m depth as shown in Fig. 4. The barotropic tide typically penetrates the entire water column; therefore, the baroclinic tide can cause amplification of tidal variability in a restricted portion of the water column, in particular at the pycnocline (Bengtsson et al., 2012).

Once the baroclinic velocity was obtained, wavelet and EOF analyses were applied to these data (Figs. 9g–i and 12). Both analyses were applied as done for the echo anomaly. The purpose of the wavelet analysis was to determine the time and depths that exhibited the largest contribution from the $M_2$ constituent (Fig. 9g); and compare these depth–time locations to those found in the wavelet of the echo anomaly. Analysis of EOF was applied to the baroclinic velocity in order to determine whether dominant spatial modes include two-layer structures. Also, this analysis was used to verify whether the temporal variability of the dominant modes portray a semidiurnal signal. The two features, two layer structure and semidiurnal variability, are typical of internal tide behavior.

3. Results

3.1. Fjord characteristics and internal wave properties

Harmonic analysis of the sea level data (Fig. 3a) revealed dominance of the semidiurnal principal lunar constituent, $M_2$ (12.42 h), with amplitude of 0.52 m. Other semi-diurnal constituents, the principal solar, $S_2$, and larger lunar elliptic $N_2$, had amplitudes of 0.10 and 0.25 m, respectively. The diurnal constituents, luni-solar, $K_1$ and principal lunar, $O_1$, were <0.15 m (Fig. 3b). Overall, the tidal regime in the Steffen–Baker region is mixed with semi-diurnal dominance as the form factor $F = (|K_1 + O_1|)/(M_2 + S_2)$ is 0.36.

Conductivity, temperature and depth measurements obtained from CTD profiles along the eastern portion of Martinez Channel featured a pycnocline between ~8 m and ~15 m depth (Fig. 4). Hydrographic properties changed insignificantly below ~40 m, relative to the upper 40 m (Fig. 4). These data allowed for determination of density anomaly and buoyancy frequency values as shown in Fig. 4c and d.

3.1.1. Internal wave modes

Estimates of internal wave properties, including wavelength, phase speed and angle of propagation, were made possible by the CTD estimates taken along Martinez Channel in May of 2009 combined with normal mode theory and ray theory. These estimates are made on properties that internal waves could have if present in the fjord. Therefore the estimates do not guarantee the existence of internal waves.

Internal wave normal modes and corresponding wavenumbers were calculated introducing the semi-diurnal tidal frequency and $N^2$ into Eq. (2), with results shown in Fig. 5. The average along-channel density anomaly (Fig. 5a) showed the pycnocline between ~5 m and 15 m depth. The vertical profile of the horizontal velocity, shown as the blue line in Fig. 5b, corresponds to the first internal wave mode. The wavenumber associated with mode 1 internal waves was found to be $k_1 = 0.12$ km$^{-1}$, which implies a wavelength of $\lambda = 52$ km, with a phase speed of $c_i = 1.16$ m s$^{-1}$. Highest scaled amplitudes were restricted to the upper water column, with weak amplitudes below ~15 m depth (below the pycnocline).
The vertical profile of vertical velocity for mode 1 showed highest values between 10 and 20 m depth (Fig. 5c, blue line). Moreover, the vertical profile of horizontal velocity for mode 2 internal waves, shown as the red line in Fig. 5b, displays small amplitudes throughout the water column. Internal wave mode 2 produced a wavenumber \(k_2 = 0.3 \text{ km}^{-1}\) and hence a wavelength of \(\lambda = 21 \text{ km}\), with a phase speed of \(c_2 = 0.47 \text{ m s}^{-1}\). The respective vertical profile of the vertical velocity indicated small amplitudes above \(\sim 12 \text{ m depth}\), with a sign change occurring at this depth. Amplitudes increased below 12 m, and peaked between 40 and 60 m depth (Fig. 5c, red line).

The vertical wavenumbers for both modes 1 and 2, \(m_1 = 102 \text{ km}^{-1}\) and \(m_2 = 251 \text{ km}^{-1}\), respectively, were much greater than their corresponding horizontal wavenumbers, \(k_1\) and \(k_2\). This guarantees that the phase propagates almost vertically while the energy propagates almost horizontally, as explained in Section 2.5.2. Indeed, the angle of phase propagation was found to be \(\phi = 87.93^\circ\), indicating energy (i.e., the wave form) propagates at an angle \(\theta = 90^\circ - \phi = 2.09^\circ\). The along-channel bottom slope, calculated from bathymetry charts, was found to be slightly less (\(\sim 2^\circ\)) than the slope of the ray characteristics (angles along which energy propagates), therefore justifying the use of normal mode theory, which is only valid for flat-bottom channels. These theoretically derived results will now be compared to internal wave observations found in the ADCP data collected in Martinez Channel.

3.2. Echo anomaly analysis

Sporadic patches of elevated signal, \(\sim 2 \text{ db}\), were identified around the pycnocline (8–15 m depth) within the echo anomaly data (Fig. 6b). This elevated signal, along with the vertical undulation of the pycnocline, was suggestive of internal wave packets. After calculation of the vertical gradient of the echo anomaly, the oscillations found around the pycnocline clearly indicated internal waves (Fig. 6c). The most prominent pulses were identified in the largest negative and positive values of the vertical gradient of the echo anomaly. Well-defined internal wave packets were observed, for example, between days 70 and 73, 87 and 89, and 112 and 117 (Fig. 6b and c). The reason for the sporadic nature of internal wave occurrences will be explored after determining whether these internal waves in Martinez Channel can be identified as internal tides.

Spectra of the echo anomaly for all depths sampled indicated high variance at both the lowest frequencies (\(<0.5 \text{ cycles per day}\)) and at semi-diurnal frequencies, as shown in Fig. 7. However, the highest energies at 2 cpd were located between \(\sim 8\) and 17 m depth, therefore not penetrating the entire water column. A band of diurnal energy is found between \(\sim 13\) and 30 m depth, but was not as prominent as the spectral energies found at lower frequencies and at the semi-diurnal frequency.

The EOF analysis applied to the echo anomaly (Fig. 8) showed mode 1 explaining 62.6% of the total variance. Its spatial structure (eigenvectors) consisted of a profile with values of the same sign (Fig. 8a). Temporal variations, often called the principal component, of mode 1 (which will be denoted by PC 1) did not show distinct temporal variability (Fig. 8b). A power spectrum of PC 1 revealed a peak at the diurnal frequency but no semi-diurnal peak (Fig. 8c). Modes 2 and 3 of the EOF analysis explained 16.3% and 8.5% of the variance, respectively. The spatial structure of mode 2 indicated a two layer structure while mode 3 indicated a three layer spatial structure. The principal components (Fig. 8d and e) showed no immediate temporal pattern, as with mode 1. However, spectra of these modes displayed a significant peak of high energy at 2 cpd (Fig. 8c).

The power Hovmöller of the echo anomaly showed largest semi-diurnal amplitude occurring at depths near the pycnocline for both the echo anomaly and the vertical gradient of the echo anomaly, with a 95% confidence interval indicated by the white contour lines (Fig. 9a–f). For the echo anomaly the wavelet semi-annual amplitude reached depths up to 40 m (Fig. 9a). However, most energy was restricted to above 20 m as shown in Fig. 9b. The wavelet of the vertical gradient of the echo anomaly displayed highest amplitudes restricted to the first 20 m of the water column and at infrequent time intervals.

The power Hovmöller, derived from wavelet analysis of the baroclinic velocity, showed largest amplitudes in sporadic packets near the pycnocline, similar to those found in the wavelet analyses of the echo anomaly and its vertical gradient (Fig. 9g). The regions

![Fig. 10](image_url)

Fig. 10. (a) Phase (degrees) acquired from power Hovmöller diagram of echo anomaly from day 85 to 90 of 2009, between 8 and 20 m depth. (b) Same as (a) for vertical gradient of echo anomaly. (c) Same as (a) for baroclinic velocity.
with most prevalent periods of semidiurnal influence found in the wavelet of the baroclinic velocity were between days 72 and 73, 87 and 90 and 112 and 117. However, there were regions with statistically significant semi-diurnal energy below the pycnocline region (<20 m depth).

The phase of the wavelet was calculated in addition to the semidiurnal amplitude. The phase of the power Hovmöller of both the echo anomaly and the vertical gradient of the echo anomaly showed 180° vertical phase shifts around the pycnocline (8–15 m depth) as seen between days 85 and 90 (Fig. 10a and b). For
example, on day 88, the phase was 180° above 12 m water depth and on the same day the phase was found to be 0° below 12 m depth, as seen in Fig. 10a. The phase of the power Hovmöller of the baroclinic velocity indicated a 180° phase shift in the pycnocline during periods of largest wavelet amplitude (Fig. 10c). This was the same behavior seen in the phase of the wavelet analysis of the echo anomaly and the vertical gradient of the echo anomaly.

3.3. Baroclinic velocity

The along-channel velocity component indicated highest current velocities in the top ~20 m of the water column as shown in Fig. 11a. This was reinforced by the time averaged profile, which indicated the main flow direction was out-fjord, with average velocities reaching 4 cm/s at the surface (Fig. 11b). At the pycnocline, the average velocity profile became up-fjord with average speeds reaching 0.5 cm s⁻¹. The along-channel component of the baroclinic velocity (depth averaged profile shown in Fig. 11c subtracted from along-channel velocity in Fig. 11a) showed current patterns similar to the along-channel velocity and reaching ±20 cm s⁻¹ in the first ~25 m of the water column (Fig. 11d). Vertical shear of the baroclinic velocity was strongest (±10 s⁻¹) at the same depths and times as that of the baroclinic velocities, yet was restricted to a smaller near-surface depth range (~8–18 m) (Fig. 11g).

Further analysis of the baroclinic velocity included the application of EOF decomposition to these data. EOF mode 1 explained 27.6% of the variance with a spatial structure depicting a two-layer flow in the along-channel direction (Fig. 12a). In this mode the flow direction switched at approximately 15 m depth. The temporal variability of mode 1 (PC 1) displayed fluctuations with no obvious dominant period (Fig. 12b). The power spectrum of PC 1 exhibited a broad, yet significant peak centered around two cycles per day (Fig. 12c). Modes 2 and 3, which explained 14.7% and 11% of the variance, respectively (Fig. 12d and e), displayed a three-layer structure with the first, and most drastic, flow direction change occurring at the pycnocline and the second at ~22 m depth. Spectral analysis of PC 2 (Fig. 12c) contained a significant peak at two cycles per day, similar to mode 1, yet mode 3 displayed no such peak.

3.4. River discharge

The Baker River is one of the largest rivers in Chile, in terms of discharge, with freshwater output reaching 2000 m³ s⁻¹ (Aiken, 2012). Hourly measurements of these discharge values were collected during the ADCP deployment and are shown in Fig. 13 (blue dotted line). Even though a small time difference was observed between the maximum Baker River discharge and maximum positive value of echo anomaly, attributed to the distance between stations (~85 km), the correlation coefficients showed highest values at 11 m depth ($r^2 = 0.77$ and 95% confidence interval of [0.74, 0.80]) (Table 2). Using bathymetric charts and the hourly Baker River discharge measurements, estimates of the internal Froude numbers were calculated for mode 1 internal waves. At the Baker River mouth (head of the fjord) with a discharge value of 1200 m³ s⁻¹, $Fr_1 = 1.03$, indicating the river plume experiences nearly critical flow conditions.

4. Discussion

4.1. Echo anomaly

The echo intensity (the raw signal from the ADCP) and the echo anomaly provided the first indication of internal wave activity in Martinez Channel (47.86°S), central Patagonia (Fig. 6a and b). A spectrum of these data showed high variance at two cycles per day restricted to the upper water column, between 10 and 15 m depth (Fig. 7), which suggested tidal forcing amplified in the pycnocline. The high variance found at the lowest frequencies was attributed to atmospheric forcing. Further evidence of semidiurnal oscillations near the pycnocline was provided by modes 2 and 3 of the EOF analysis of the echo anomaly (Fig. 8).

Both the vertical and temporal structure of EOF modes 2 and 3 (Fig. 8a) indicated semidiurnal internal waves. Mode 2 evidenced internal waves with a two-layer vertical structure separated at the pycnocline (between 12 m and 18 m depth) and a statistically significant semidiurnal peak in the spectra of PC 2 (Fig. 9a and c–d). Also, the spatial structure of EOF mode 1 was similar to the dynamic mode 1 internal wave vertical profile of horizontal velocity shown in Fig. 5b (blue line). Mode 3 evidenced internal waves by a
three-layer vertical structure with middle layer between 11 m and 22 m depth, which envelops the pycnocline region. Also, for mode 3, spectra of PC 3 displayed the semidiurnal tidal frequency. In summary, semidiurnal internal waves were found in modes 2 and 3 of the EOF analysis but not in mode 1, either because of the sporadic nature of the fluctuations throughout the time series or because of the effects of wind. The wavelet analysis helped to better understand the sporadic nature of the semidiurnal signal in the echo anomaly.

Power Hovmöller diagrams derived from the wavelet analyses of both the echo anomaly and the vertical gradient of the echo anomaly produced semidiurnal signal where one would expect, namely the same depths and instances as those of the internal waves (Fig. 9a–f). Time averages of power at each depth (Fig. 9b and e) showed highest values in the upper ~20 m of the water column for both the echo anomaly and its vertical gradient. Although some semidiurnal was seen below the pycnocline in the echo anomaly, the majority remained within it. The corresponding phase of the Power Hovmöller of the echo anomaly and the vertical gradient of the echo anomaly produced a 180° phase shift at times and locations where semidiurnal energies were most pronounced (Fig. 10a and b). In other words, periods of increased semidiurnal wavelet amplitude of the echo anomaly and its vertical gradient were accompanied by vertically sheared internal wave oscillations at the pycnocline, which was identified in the phase.

Each analysis performed on the echo anomaly provided evidence of the presence of semidiurnal internal waves located around the pycnocline in Martinez Channel. Results from EOF and wavelet analyses applied to the baroclinic velocity provided further evidence of semidiurnal internal tides around the pycnocline.

4.2. Baroclinic velocity

The baroclinic velocity together with its vertical gradient depicted vertically sheared flow moving in opposite directions around the pycnocline, as seen, for example, between days 87 and 90 and depth ranges of ~10–20 m (Fig. 11d and g).

Semiannual amplitudes of the baroclinic velocity shown in a power Hovmöller diagram (Fig. 9g) coincided with spatial and temporal regions of strong (positive and negative) vertical shear, providing evidence that the modulations found around the pycnocline were semidiurnal internal tides (Fig. 9g–i). As with the phase derived from the power Hovmöller of the echo anomaly, the phase derived from the baroclinic velocity indicated a 180° phase shift within the pycnocline, describing wave horizontal velocities (Fig. 10c). The time average of the power Hovmöller, shown in Fig. 9h, clearly indicated that the majority of semidiurnal energy was found in the upper 20 m of the water column, although there was statistically significant signal below this depth. The power Hovmöller analyses of the echo anomaly, the vertical gradient of the echo anomaly, and the baroclinic velocity presented highest semidiurnal amplitudes at approximately the same depths and times (Fig. 9). These locations of highest energies (Fig. 10), showed 180° phase shifts with depth, reinforcing the presence of semidiurnal internal waves within the pycnocline.

Using EOF decomposition on the baroclinic velocity, the spatial structure of mode 1 represented a two-layer flow modulated at semidiurnal periods near the pycnocline. The spatial structure of EOF mode 1 was similar to that of the horizontal velocity of dynamic mode 1 internal waves (Figs. 5b and 12a). The same was also found for mode 2 of the EOF analysis of the echo anomaly. The time series of EOF principal component 1 showed a broad peak at 2 cpd demonstrating semidiurnal influence, therefore indicating that the baroclinic flow was modulated by the semidiurnal tide. EOF mode 2 described a three-layer flow which was similar in structure to the vertical profile of dynamic mode 2 internal wave horizontal velocity (Fig. 5b, red line). Both EOF modes 1 and 2 indicated a multi-layered flow structure varying at the semidiurnal frequency, as seen in EOF modes 2 and 3 of the echo anomaly.

The power Hovmöller and EOF analyses of the baroclinic velocity and echo anomaly suggested the same result: multi-layer flow influenced by semidiurnal frequencies near the pycnocline. These findings substantiated that the sporadic packets of baroclinic velocities and echo anomaly signal found around the pycnocline were indeed semidiurnal internal tides.

4.3. Internal wave characteristics and Baker River discharge

Measurements collected in Chilean Patagonia indicated the presence of semidiurnal internal waves, and theoretical calculations reinforced this observational evidence. Ray theory implied that the internal wave energy travelled almost fully in the horizontal plane, or in other words, the internal wave frequency was small.

Table 2: Correlation of echo anomaly and river discharge.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$r^2$</th>
<th>Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.69</td>
<td>[0.65, 0.73]</td>
</tr>
<tr>
<td>11</td>
<td>0.77</td>
<td>[0.74, 0.80]</td>
</tr>
<tr>
<td>12</td>
<td>0.75</td>
<td>[0.71, 0.78]</td>
</tr>
</tbody>
</table>

Fig. 13. Baker River discharge values (blue dotted line) and echo anomaly at 11 m depth (brown line). Brown dashed line indicated a 3 day running mean and brown solid line indicated a 12 h running mean of the echo anomaly during period of March–April 2009. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
compared to the maximum value of \( N \) found around the pycnocline. The significance in this finding is derived from simple arguments, where \( \cos^{-1}(\theta) \sim 90^\circ \) implies \( \theta \sim 0 \) and so \( N \) must be appreciably larger than \( \alpha \). Physically, \( N \gg \alpha \) indicates that internal wave energy propagates nearly horizontally, but will remain practically inside the pycnocline (Fig. 4d), and the phase propagates almost vertically. The angle of energy propagation in Martinez Channel was found to be \( \theta = 2.09^\circ \), a similar value to that found in Knight Inlet by Farmer and Freeland (1983) and in Skjomen Fjord by Cushman-Roisin and Svendsen (1983). Internal wave energy travelling in the horizontal plane points to the internal wave generating mechanism being either mode 1 lee waves modulated from a sill location, tidal interaction with the head of the fjord (sloping shore) (Rattray, 1960) or tidal interaction with the Baker River plume (Nash and Moum, 2005).

As no data were collected over the potential sill locations, and only one mooring was available, it was not possible to determine the direction of propagation (in-fjord or out-fjord), so the internal tides could indeed be lee waves generated over a sill. However, with the sill locations many kilometers away from the mooring site and considering the complex geometry of Martinez Channel, it is likely that internal tidal energy would be dissipated by the time of arrival at the mooring location (Farmer and Freeland, 1983). Nonetheless, the internal tides, although sporadic in occurrence, were strikingly apparent in the observational data from the ADCP mooring.

River discharge measurements obtained during the ADCP deployment were scarce between days 74 and 84 of 2009 with little or no measurements available. However, internal wave packets coincided with high river discharge pulses (Figs. 6 and 13) on days 85–120 of 2009. The pulses of internal tides appeared either simultaneously or within one day after the pulse of high river discharge. As the river discharge measurements were collected ~85 km east of the ADCP mooring, a time lag between river discharge peaks coincided with high river discharge peaks (Figs. 6 and 13) on days 85–120 of 2009. The pulses of internal tides appeared either simultaneously or within one day after the pulse of high river discharge. As the river discharge measurements were collected ~85 km east of the ADCP mooring, a time lag between river discharge peaks and internal wave events was not unusual.

The correlation coefficients calculated between the river discharge and the echo anomaly revealed that pulses of internal tides near the pycnocline were most likely related to pulses in river discharge. Further qualitative comparison of Baker River discharge values to the echo anomaly is shown in Fig. 13. A three-day running mean of the echo anomaly at 11 m, a depth with high buoyancy frequency values, revealed co-variability with high river discharge pulses. Therefore, river discharge pulses did not only determine the depth of the pycnocline (Fig. 4), but were the likely drivers of internal waves in Martinez Channel.

The internal Froude number calculated for mode 1 internal waves was found to be \( Fr_1 = 1.03 \), indicating critical flow conditions at the Baker River mouth. This rules out the potential for the sloping shore at the head of the fjord to be the generating mechanism, as critical conditions will not allow upstream propagating wave conditions. The value \( Fr_1 = 1.03 \) was calculated for a discharge of 1200 m³ s⁻¹, which is considered a high river discharge event (Fig. 13). This indicates that the river plume flow only becomes supercritical when discharge exceeds this value. Hence, internal waves would not be generated for lower discharge values, as wave frontal growth would not take place when the internal Froude number drops below 1 (Nash and Moum, 2005). Indeed, packets of internal tidal waves were found exactly when the river discharge exceeded 1200 m³ s⁻¹, verifying that the river plume produced the internal tidal waves (Figs. 6, 9 and 13). This finding confirmed that semi-diurnal internal waves could be generated by the vertical displacement of the pycnocline in a range of ~10–20 m, contributing to the vertical mixing of physical and bio-chemical water properties.

### 5. Conclusions

Analyses of ADCP echo intensity anomaly and velocity profiles showed the presence of semi-diurnal internal tides for the first time in Patagonian fjords. These internal tides were found near the pycnocline (~8–15 m depth) during periods highlighted by wavelet, EOF and spectral analyses. The power Hovmöller diagram of the echo anomaly, the vertical gradient of the echo anomaly, and the baroclinic velocity were all in good agreement, showing semi-diurnal oscillations at the same depths and times. The analysis also showed wavelet phases that indicated variations in opposite directions (±180°) above and below the pycnocline. The first three modes of the EOF analysis of the baroclinic velocity, and mode 2 and mode 3 of the EOF analysis of the echo anomaly, featured two layer spatial variability with spectra of principal components showing two cycle per day fluctuations.

The appearance of internal tides in this record was sporadic throughout the time series owing to the Baker River discharge. High correlation values \( r^2 = 0.77 \) suggested that internal tides in the Martinez Channel were related to high discharge pulses of the Baker River. Also, the internal Froude number \( Fr_1 = 1.03 \) implied that high pulses in river discharge, rather than interaction of barotropic tide with a sill or contraction found in the surrounding region, triggered these internal tides. The generation of internal tides by river discharge pulses is also a scarcely reported finding in fjords.

### Acknowledgements

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### Appendix A. Empirical orthogonal analysis

The time series of current data, which varies spatially with depth, can be written as \( u_n(t_m) \), where \( 1 \leq m \leq N \) is the spatial interval and \( 1 \leq n \leq M \) h is the time interval; \( M \) represents the last hourly measurement captured from the ADCP mooring (~1680 measurements). In order to apply an EOF analysis on these data, \( u_n(t_m) \) at any depth \( n \), must be expressed as the sum of 40 orthogonal spatial functions (eigenfunctions), denoted \( \alpha(z_n) = \alpha_{mn} \), such that

\[
\alpha_m(t_m) = \sum_{i=1}^{40} [\mathbf{a}_i(t_m) \mathbf{\alpha}_n],
\]

where \( \alpha_{mn} \) is the amplitude of the ith orthogonal mode at time \( t_m \) (Emery and Thomson, 2004; Kaihatu et al., 1998). The weights, \( \mathbf{a}_i(t_m) \), or time amplitude, indicate how the spatial modes, \( \alpha_{mn} \), vary with time.

The spatial modes or eigenvectors are required to be orthogonal (independent of each other); therefore, the following orthogonality condition is established:

\[
\sum_{n=1}^{40} \mathbf{\alpha}_n \mathbf{\alpha}_m = \delta_{mn},
\]
where \( \delta_{ij} = \begin{cases} 0, & j \neq i \\ 1, & j = i \end{cases} \) is the well-known Kronecker delta function. Another required condition is for the time amplitudes to be uncorrelated, which implies

\[
\hat{a}_i(t_m) \hat{a}_j(t_n) = \delta_{ij} \delta_{mn},
\]

where the over-bar in Eq. (A3) denotes a time average and \( \beta_i = \hat{a}_i(t_m)^2 \) describes the variance in each orthogonal mode. Eq. (A3) guarantees uncorrelated time variability; hence together, Eqs. (A2) and (A3) secure orthogonality.

Taking the covariance matrix for the current velocity data, \( C_{nt} = u_n(t_m)u_t(t_n) \), 1 \( \leq t \leq 40 \), multiplied by the eigenvectors and summed over all modes, \( l \), the following equation is derived,

\[
\sum_{l=1}^{40} U_l(t_m) \hat{u}_l(t_m) x_{m \lambda n} = \beta_l x_{m \lambda n},
\]

which describes the \( \lambda \)th mode at some depth, \( n \). Eq. (A4) is also known as the canonical form of the eigenvalue problem which can be written in the following more compact form (if the time mean at each depth is removed from the covariance).

\[
(C - \beta I) A = 0,
\]

where \( I \) is the identity matrix, \( C \) is the covariance matrix and \( A \) is the eigenfunction matrix. For Eq. (A5) to have a non-trivial solution it is required that, \( \det(C - \beta I) = 0 \). From linear algebra it is clear that when expanded, this determinant gives a polynomial of the 40th degree, whose \( 40 \) eigenvalues satisfy the relation, \( \beta_1 > \beta_2 > \ldots > \beta_{40} \), indicating that the percent variance explained by a mode is ordered according to its eigenvector. This is a monotonically decreasing sequence where the first mode contains the highest variance. The solution of the eigenvalue problem in Eq. (A5) will provide all 40 eigenfunctions. Once these are known the final step is determine the weighted temporal amplitudes for each mode as (Kaihatu et al., 1998)

\[
a_l(t_m) = \sum_{n=1}^{40} U_l(t_m) x_{m \lambda n}.
\]

The EOF analysis method is effective for two main reasons. First, the first few modes may contain the majority of the total variance, and second, each mode contains a potential description of physical processes containing both spatial and temporal scales.

### Appendix B. Wavelet analysis

A Morlet wavelet, which is a non-orthogonal, complex wavelet function, was used in this analysis, namely

\[
g(t) = \pi^{-\frac{1}{4}} e^{i\omega t} e^{-\frac{t^2}{2}},
\]

where \( \omega \) is the nondimensional frequency (taken here to be \( \omega = 6 \) as in Torrence and Compo (1998)) and \( t \) is a nondimensional time. The wavelet function (B1) is convolved with a discrete Fourier Transform, \( \hat{e}_n \), of the echo anomaly data (the method was applied to the vertical gradient of the echo anomaly and the baroclinic velocity) namely,

\[
\hat{e}_n = \frac{1}{N} \sum_{n=0}^{N-1} e^{i 2\pi n \omega n},
\]

where \( m = 0, \ldots, N - 1 \) is the frequency index. The convolution of the wavelet function and Eq. (B2) gives the wavelet transform,

\[
T_n(s) = \sum_{m=0}^{N-1} \hat{e}_n \hat{g}^*(s02m) e^{i 2\pi n \omega n \lambda}. \tag{B3}
\]

where complex conjugation is indicated by \( \hat{g}^* \), and the wavelet scaling is denoted by \( s \), and

\[
\alpha_{nm} = \begin{cases} \frac{2\pi m}{\lambda}, & m \leq n \\frac{\pi}{\lambda} \\ \frac{-2\pi m}{\lambda}, & m > n \frac{\pi}{\lambda} \end{cases}
\]

Now, with Eqs. (B3) and (B4), one can calculate the continuous wavelet transform (for all scales, \( s \)) at all times steps, simultaneously. As done in Torrence and Compo (1998), the choice of scale is given as fractional powers of two, namely \( s_n = s_02^k \), where \( k = 0, 1, \ldots, K \), determines the largest scale, as \( K = \delta^k \log_2(\hat{g}^2) \) and \( s_0 \) is the smallest scale. After the scale is established, the wavelet power spectrum can be calculated. This power spectrum is defined as \( |T_n(s)|^2 \), and because both the wavelet transform and function are complex, \( |T_n(s)|^2 \) can be split into real and imaginary components, where \( |T_n(s)|^2 \) is the amplitude and tan \( \theta = [\text{IM}(\text{RE})] \) represents the phase \( \text{IM} \) and \( \text{RE} \) denote the imaginary and real parts of \( T_n(s) \), respectively.

In order to arrive at the power Hovmöller diagram, the wavelet power spectrum of the echo anomaly is calculated in the same manner as described above, and this is done at each depth (up to 40 m). The wavelet power of the echo anomaly at the semi-diurnal period is extracted and concatenated into a two-dimensional contour plot with depth on the \( y \)-axis. The 95% confidence level was computed using the lag-1 autocorrelation calculated at each depth. The conventional estimator for lag-1 autocorrelation is given as

\[
r_1 = \frac{\sum_{n=1}^{N-1} (X_n - \bar{X})(X_{n+1} - \bar{X})}{\sum_{n=1}^{N-1} (X_n - \bar{X})^2}, \tag{B5}
\]

where \( i \) indexes through the time step and \( \bar{x} \) is the time average.

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