Computations of the geographical distribution of the energy flux to mixing processes via internal tides and the associated vertical circulation in the ocean

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Abstract—The global flux of tidal energy to mixing processes via topographically generated internal waves is estimated utilizing gridded databases for bathymetry, vertical density stratification and barotropic tides together with a simple, local model for the generation of progressive internal tides at vertical steps in the ocean floor. Both the horizontal distribution of the energy flux to internal tides and its ocean mean are discussed. The computed oceanic mean value is $4.4 \times 10^{-4} \text{ W m}^{-2}$, a factor of about 2–3 greater than previous estimates (MUNK, 1966, Deep-Sea Research, 13, 707–730; BELL, 1975, Journal of Geophysical Research, 80, 320–327).

The global distribution of vertical diffusivity in the abyss is computed by assuming that topographically generated baroclinic motions dissipate locally and that the dissipation is distributed vertically according to an empirical law. Our results are linearly dependent on the flux Richardson number $R_f$. From the computed vertical diffusivities and the known vertical stratification we finally compute the global distribution of vertical velocities. Choosing a value of $R_f = 0.05$ we obtain an upward vertical transport in the interior of the ocean, at the 1000 m level, of about $1.5 \times 10^6 \text{ m}^3 \text{ s}^{-1}$, which agrees with WARREN's (1981, in: Evolution of physical oceanography, B. A. WARREN and C. WUNSCH, editors, 6–41) estimated rate of sinking from surface waters at high latitudes. Below the 1000 m level the upward transport increases and a maximum value of about $2.5 \times 10^6 \text{ m}^3 \text{ s}^{-1}$ is found at the 2000 m level, after which the transport decreases to about $8 \times 10^5 \text{ m}^3 \text{ s}^{-1}$ at the 4000 m level. This may be explained by the action of bottom currents. These currents entrain ambient water whereby the upward interior vertical transports tend to increase with depth. However, because of the entrainment of lighter ambient fluid the dense currents become less dense and only the most dense flows penetrate to the greatest depths.

1. INTRODUCTION

MUNK (1966) discussed the vertical circulation of the ocean below the thermocline by use of a stationary one-dimensional model in which vertical advection was assumed to be balanced by vertical diffusion for a conservative tracer. The renewal of bottom water was assumed to take place at great depths, inducing a constant vertical advective velocity $w$ (thus assuming vertical side walls). Utilizing the vertical diffusion equation in a diagnostic mode, Munk then determined the ratio between the vertical diffusivity $\kappa$ and $w$ from observed salinity and temperature profiles in the north Pacific. $w$ and $\kappa$ could then be

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determined separately using observed profiles of a non-conservative tracer, such as $^{14}$C, assumed to have known sources and sinks. Using $^{14}$C Munk estimated $w$ and $\kappa$ to $1.4 \times 10^{-5}$ cm s$^{-1}$ and $1.3$ cm$^2$ s$^{-1}$, respectively. These values of $w$ and $\kappa$ were to some extent, verified by considering the upward flux of radium from the sediments and the oxygen consumption in the ocean.

Munk estimated the rate of production of bottom water due to ice production in the Antarctic. About half of the estimated production was found to be required to penetrate northward at large depths to explain the estimated value of $w$. Munk also discussed different mechanisms that energetically may sustain turbulence in the abyss. The tides, via topographically generated internal tidal waves, were found to be the most likely major energy source. However, Munk found that this energy source alone could not account for the estimated rate of mixing.

In the real ocean new deep-water is added at all depths by dense bottom currents emanating from marginal seas. During penetration to depth, these currents will entrain ambient water from the ocean interior. Associated vertical velocities and transports may therefore vary significantly both horizontally and vertically. It is therefore questionable if one can describe the rate of vertical circulation in the ocean interior by only a single oceanic mean value. In that respect the vertical advection/diffusion model with constant vertical diffusivity and constant vertical velocity used by Munk (1966) may be too simple and even misleading.

Mixing performed by the dense bottom currents can be considerable (cf. Stigebrandt, 1987) which implies that the vertical mixing actually performed in the ocean interior (outside the dense bottom currents) may be less than estimated by Munk (1966). A model not accounting for mixing performed by dense bottom currents thus may overestimate the interior vertical diffusivity. This motivates a reconsideration of tidal dissipation, via topographically generated internal waves, as the major energy source for interior mixing in the ocean.

Munk (1966) estimated the energy flux from surface to internal tides in the deep ocean utilizing results from a linear model developed by Cox and Sandström (1962). In the Cox and Sandström model the sea floor topography is described using a spectral function for a wide band of wave lengths. Using a normal mode approach they calculated the fractional rate of conversion of barotropic tidal energy to baroclinic energy. By use of the Cox and Sandström model, assuming constant amplitude of the barotropic tide ($\alpha$) and constant buoyancy frequency ($N$), Munk estimated the horizontal surface mean of the energy flux to internal tides to $17 \times 10^{-4}$ W m$^{-2}$ in the North Pacific.

Bell (1975) studied the generation of internal waves due to the linear interaction between the barotropic tide and abyssal hills in an infinitely deep ocean. Bell's description of the bottom topography was similar to that used by Cox and Sandström although he did not resolve the solution into normal modes. Using a spectral estimate for the abyssal hill topography and constant values of $\alpha$ and $N$ Bell estimated the mean energy flux to internal tides in the North Pacific to $10 \times 10^{-4}$ W m$^{-2}$.

One of the present authors (Stigebrandt, 1976, 1979) suggested that diapycnal mixing in sill basins of fjords is mainly due to dissipating progressive internal tides. Stigebrandt and Aure (1989) presented additional evidence and showed that the rate of work against the buoyancy forces in sill basins of fjords is proportional to the energy flux to internal tides at fjordic sills. The latter was determined by a simple two-layer model where the sill topography was described by a single step, and it was assumed that the internal tides
generated at the sill are progressive, i.e. the reflexion of internal tides in the fjord is insignificant. Further evidence for the validity of the simple model for computations of barotropic to baroclinic energy conversions in fjords was presented by de Young and Pond (1989) who used measurements of the barotropic tidal energy loss in fjords, and a slightly modified model (Stacey, 1984) for internal wave generation. To the best of the present authors' knowledge there is no verified coupling between the energy flux to internal tides and the rate of work against the buoyancy forces for other types of ocean systems. The apparent validity of the simple model for computations of energy conversions in fjords encourages the use of similar models to compute the energy flux to internal tides in the open ocean.

In the present paper we will argue for the use of a simple model for the generation of internal tides due to the interaction between the barotropic tide and bottom topography, where the topography is described by a rectangular grid. The bottom is thus pictured as a large number of densely packed vertical pillars of different heights. Descriptions of the bottom slope as a single step has been widely used in models for the generation of internal tidal waves at the continental slope (e.g. Rattray, 1960; Rattray et al., 1969). Stigebrandt (1980) proposed a simple model for the generation of internal tides at a step-like topography which was applied to sills in fjords. For a linear stratification he showed that the energy flux into internal waves depends on the buoyancy frequency, $N$, and the ratio between step height $d$ and depth $H$.

In ray theory methods (e.g. Baines, 1982; Craig, 1989) the internal wave field at any point depends on the entire generation area. Using the evidence from fjords mentioned above, we will neglect the possibility for wave reflexion and consider each individual step as a generator of progressive internal waves that works independently of other generators. Internal waves generated at different steps may possibly organize themselves in wave fields coherent over much larger scales than the scale of a single surface element. This should influence the local "generators", and their strength may thus be increased or decreased. In general there should be an extensive mis-match between the wave fields from different generators, something that should be promoted by the three-dimensional character of natural slopes. A fraction of the internal wave energy will therefore probably dissipate close to the generation location, and the energy flux radiated by the large scale wave field may be substantially less than the total energy flux to baroclinic motions in the whole generation region. In fact, this was one of the conclusions drawn by de Young and Pond (1989) from extensive observations in fjords. Therefore we expect that ray theory methods, which probably are good in describing large scale internal wave fields for reasonably well-behaved bottom topography, in general should underestimate the total rate of energy flux from barotropic to baroclinic motions caused by the real ocean bottom. As already mentioned the only verified computations of the coupling between the rates of energy flux to internal tides and the vertical mixing have been made using local theory (see Stigebrandt and Aure, 1989). The arguments given here suggest that a local model is the correct type of model to use to compute the total rate of topographically induced energy flux from barotropic to baroclinic motions in the ocean. We have to know this flux to compute the associated vertical mixing.

Local models have the advantage of being tractable also for cases with realistic topography and stratification. In the present paper we extend Stigebrandt's model (Stigebrandt, 1980) to one with a non-constant buoyancy frequency and apply it to a multiple step case. Detailed, global information on step height $d$ and water depth $H$ can be
obtained from the gridded bathymetric database DBDB-5 that has the resolution 5' × 5'. Below we estimate the energy flux into internal tidal waves by applying the model to virtually all "steps" in the bottom topography. Information about the barotropic tide is obtained from the gridded tidal database GOTD (SCHWIDERSKI and SZETO, 1981) containing amplitudes and phases of the eleven most prominent harmonic tidal constituents on a 1° × 1° spherical grid system. The vertical stratification is obtained from the "Levitus database" which gives the annual mean of salinity and temperature at certain standard depths on a 1° × 1° global grid.

2. GENERATION OF INTERNAL WAVES AT A DEPTH DISCONTINUITY

In this section we will briefly discuss a simple local model for generation of internal waves in a stratified non-rotating ocean of variable depth.

We confine our interest to the generation process only, excluding all aspects of propagation and dissipation of internal waves. The only requirement is that the internal tides are progressive, to enable us to calculate the energy flux generated at a bottom step for a given set of parameters. As in STIGEBRANDT (1980) we assume that the barotropic forcing and the vertical distribution of the horizontal velocity normal to, and above, the step are known. Below the top of the step the normal velocity should vanish. This enforces a baroclinic response. Mathematically the sum of the barotropic and the baroclinic modes should satisfy the prescribed condition at the step.

Assume that the density ρ at the height z above the bottom is given by,

\[ \rho(z) = a - b/[1 + (H - z)/\delta], \]

(2.1)

where a and b are such that \( \rho(H) = \rho_1 \) and \( \rho(0) = \rho_2 (\rho_1 < \rho_2). \) \( \delta \) is at this stage an arbitrary parameter. Later on, \( \delta \) will be calculated to fit the theoretical density profile to observations. This type of expression for the vertical density distribution (see, e.g. SJöBERG and MÖRK, 1985) has the advantage of allowing an analytical solution to the normal mode problem. The buoyancy frequency \( N^2 \) (= \(-g(\rho_2 - \rho_1)/(\rho_0 H)\)) becomes

\[ N^2 = N_0^2(1 + H/\delta)/[1 + (H - z)/\delta]^2, \quad N_0^2 = g(\rho_2 - \rho_1)/(\rho_0 H). \]

(2.2)

In Fig. 1a and b the variation of density and buoyancy frequency with depth is shown for a few different values of the ratio \( \delta/H. \) In the ocean the ratio \( \delta/H \) has an average of about 10^{-1}. It is worth noting that as \( \delta/H \) approaches infinity a linear stratification is reached (\( N \) constant) and as will be shown below the results approach the results of ÖrSTRÖM (1980).

The equations are the same as in STIGEBRANDT (1980), describing long hydrostatic gravitational waves neglecting rotation and friction. Considering only free baroclinic waves, assuming a wave-like solution in the horizontal coordinate and in time, the normal mode solution to the vertical problem for the horizontal velocity becomes,

\[ U_n(z) = \left(1 + (H - z)/\delta\right)^{-1/2}(\sin \{v_n \ln [1 + (H - z)/\delta]\} + v_n \cos \{v_n \ln [1 + (H - z)/\delta]\}) \]

(2.3)

where

\[ v_n = n\pi/\ln (1 + H/\delta) \]

(2.4)

\[ k_n^2 = (v_n^2 + 0.25)\omega^2N_0^{-2}(1 + H/\delta)^{-1}\delta^{-2}. \]

(2.5)
$k_n$ is the horizontal wave number and $\omega$ the frequency which in our case will be given by the forcing frequency, i.e. the tidal frequency. The sign of $k_n$ is determined by the condition that wave energy propagates away from the step. In Fig. 2 $U_n$ (normalized) has been plotted for different values of $\delta/H$.

The baroclinic modes have two important properties: they do not supply any net horizontal transport, i.e.

$$\int_0^H U_n(z) \, dz = 0,$$

and they are mutually orthogonal, i.e.

$$\int_0^H U_n U_m \, dz = 0 \quad \text{for} \quad n \neq m.$$
As in Stigebrandt (1980) we assume a step of height \( d \) in the bottom profile at \( x = 0 \) and that the velocity profile above the step at \( x = 0 \) is known and given by \( f(z) \). Below \( z = d \) the normal velocity should vanish (the kinematic boundary condition). To fulfill this condition at \( x = 0 \) we add the barotropic and the baroclinic modes:

\[
\alpha e^{-i\omega t} + \sum a_n U_n(z) e^{-i\omega t} = \begin{cases} f(z) e^{-i\omega t} & \text{for } d < z < H \\ 0 & \text{for } z < d \end{cases} \text{ at } x = 0, \tag{2.8}
\]

where \( \omega \) is the frequency and \( \alpha \) is the tidal current amplitude in the absence of the sill. The transport must be the same with or without the sill, \( f(z) \) can therefore be written,

\[
f(z) = \alpha H/(H-d)g(z), \tag{2.9}
\]

where

\[
\int_d^H g(z) \, dz = (H-d).
\]

If normalizing \( U_n \) so that \( \int U_n^2 \, dz = H \), multiplying (2.8) with \( U_n \) and integrating the expression over the total depth an expression for the coefficients \( a_n \) is obtained,

\[
a_n = \alpha (H-d)^{-1} \int_d^H g(z) U_n(z) \, dz. \tag{2.10}
\]

This is then the general expression for the coefficients \( a_n \). To make progress we must know the function \( g(z) \). As to study the dependence of the coefficients on the ratios \( d/H \) and \( \delta/H \) we assume that \( g(z) \) is given by a step function,

\[
g(z) = \begin{cases} 1 & d < z < H \\ 0 & z < d \end{cases} \tag{2.11}
\]

Substituting for \( g(z) \) and \( U_n(z) \) in (2.10) gives after integration,

\[
a_n = \alpha \delta(H-d)^{-1} \left( \frac{2H}{\delta \ln (1 + H/\delta)(0.25 + \nu_2^2)} \right)^{1/2} \times \left[ 1 + (H-d)/\delta \right]^{3/2} \sin \left( \nu_n \ln \left[ 1 + (H-d)/\delta \right] \right). \tag{2.12}
\]

By letting \( \delta/H \to \infty \) we obtain directly Stigebrandt's (1980) solution for constant buoyancy frequency except for a factor of \( \sqrt{2} \), the difference depends on different normalization.

The energy in each mode is proportional to \( a_n^2 \) while the total internal energy density is proportional to \( \Sigma a_n^2 \). In Fig. 3a–c, \( a_n^2 \) is plotted as a function of wavenumber for \( d/H = 0.5 \) and three different values of \( \delta/H \). When \( \delta/H \) is increased the even numbered modes are suppressed and secondary maxima in the spectra disappear. As a result the modal energy distribution becomes a continuously decreasing function of modal number and the influence of the high modal numbers diminish. However, the total energy density stays constant independent on variations of the ratio \( \delta/H \) (Fig. 4a–e).

The fact that the total amount of internal energy density generated stays constant, independent of variation in \( \delta/H \), is most easily shown by rewriting (2.8),

\[
\Sigma a_n U_n(z) = \begin{cases} \alpha [H/(H-d)] - \alpha & d < z < H \\ -\alpha & 0 < z < d \end{cases}
\]

(2.13)
where the expression (2.11) has been used. If we take the square of this expression, and integrate over $z$ then,

$$\Sigma a_n^2 = a^2 \frac{d}{(H-d)}$$

(2.14)

if $U_n$ is correctly normalized. The total amount of internal energy density is thus only dependent on the amplitude $a$ and the factor $d/H$, not on $\delta/H$, the scale height of the stratification. This can be understood if one realizes that the generation process is due to the condition of vanishing, normal velocity at the sloping bottom (the kinematic condition).

2.1. The energy flux to internal waves generated by a bottom step

The energy flux through a vertical section is given by

$$F = \frac{1}{4} \int_z (pu^* + p^*u) \, dz \quad [\text{W m}^{-1}]$$

(2.15)

(see, e.g. LeBlond and Mysak, 1978). If now $u = \Sigma a_n U_n(z) \, e^{(kx - \omega t)}$, where $a_n$ is as given
above, then $p$ is given by $p = \rho \omega \Sigma a_n U_n/k_n e^{i(kx-\omega t)}$ and $F$, when averaged over $x$ and $t$, can be written

$$F = \Sigma F_n,$$

where

$$F_n = 1/2 \rho \omega (a_n^2/k_n H$$

$$= \frac{a^2 \rho N_0 \delta^2 [1 + (H - d/\delta)][1 + H/\delta]^{1/2} \sin^2 \left(\nu_n \ln \left(1 + (H - d)/\delta \right)\right)}{(1 - d/H)^2 \ln (1 + H/\delta)(1/4 + v_n^2)^{3/2}}$$

(2.16)

We also could have used the fact that for plane waves $F_n = E_n C_{gn}$, where $E_n$ is the energy density per horizontal unit surface area and $C_{gn} = \omega/k_n$ is the horizontal group velocity for mode $n$. It is easily seen that if $d$ is fixed, then
Distribution of energy fluxes to mixing processes

\[
F_n \rightarrow \frac{\alpha^2 \rho N_0 H^2 \sin^2 \left\{ n \pi (H - d)/H \right\}}{(1 - d/H)^2 n^3 \pi^3} \quad \text{as} \quad \delta/H \rightarrow \infty \quad \text{and} \quad (2.17a)
\]

\[
F_n \rightarrow 0 \quad \text{as} \quad \delta/H \rightarrow 0, \quad (2.17b)
\]

while if \(\delta\) is fixed,

\[
F_n \rightarrow \frac{\alpha^2 \rho N_0 H^2 (1 + H/\delta)^{1/2} \nu_n^3}{n \pi (\nu_n^2 + 0.25)^{3/2}} \quad \text{as} \quad d/H \rightarrow 1 \quad \text{and} \quad (2.18a)
\]

\[
F_n \rightarrow 0 \quad \text{as} \quad d/H \rightarrow 0. \quad (2.18b)
\]

The total energy flux will remain finite except when \(d/H\) approaches 1, then \(\Sigma F_n \sim \ln (n)\). However, when \(d/H\) becomes sufficiently close to 1 the conditions at the edge will become supercritical and no internal waves will be generated.

Although the total internal energy density generated is independent of \(\delta/H\) according to equation (2.14), the total energy flux through the vertical plane is not. The reason is that the group velocity is inversely proportional to \(n\), thus suppressing high modal influence. As \(\delta/H\) decreases more energy enters into high modes while \(\Sigma a_n^2\) stays constant, which consequently lessens the energy flux. This is reinforced by the fact that the group velocity itself becomes smaller with \(\delta\).

2.2. Effects of rotation

If including rotation by introducing a constant Coriolis parameter \(f\), assuming \(\omega > f\), the vertical mode solution will look exactly like the one already obtained although the waves become dispersive. The dispersion relation will change to,

\[
k_n^2 = (\nu_n^2 + 0.25)(1 + H/\delta)^{-1} \delta^{-2} (\omega^2 - f^2) N_0^{-2}.
\]

This means that the amplitude coefficients \(a_n\), and consequently the energy density, will be the same as before but the energy flux at the step will change by a factor of \((\omega^2 - f^2)^{1/2}/\omega\).

The effect of rotation is thus to disperse the waves, and thereby decreasing the energy flux. Since rotation affects the energy flux it will be included in the estimate of the global energy flux made below.

2.3. Effects of friction

In Stigebrandt (1980) it was shown that by choosing a linear velocity profile as a condition at the sill, instead of a step-function, the baroclinic response was increased. The imposed velocity profile satisfied a no-slip condition at \(z = d\), thus simulating the effect of bottom friction. However, there is no reason to believe that frictional effects influence the generation process mainly because the generation is assumed to take place on an infinitely short length-scale. Nevertheless, friction enters the problem indirectly because it is accounted for in the barotropic tidal model (Schwierski and Szeto, 1981). The generation of internal tidal waves can be viewed as a frictional process, caused by form drag, draining energy from the barotropic tide. In fact, if the internal wave drag is the dominating frictional drag upon the barotropic tide it should be possible to use the frictional estimates in the barotropic tidal model to check our estimates of the energy flux to internal waves. Unfortunately global estimates of frictional dissipation in tidal models often are not taken into account.
3. SOME ASPECTS OF THE MODEL’S VALIDITY AND ITS APPLICABILITY IN
THE OCEAN

The bathymetric database DBDB-5 can be thought to describe the bottom as consisting
of a large number of densely packed vertical pillars, each having four sides with an
approximately rectangular horizontal cross-section. How many of the four vertical sides
that generate internal energy flux will depend on the heights of neighbouring pillars. How
does the result depend on the grid size, \( A \)? All topographic features with length scales less
than \( a/N \), where \( a \) is the tidal current amplitude and \( N \) the buoyancy frequency, will be
neglected since we have assumed the baroclinic response to be hydrostatic. This require-
ment then sets a lower bound on \( \Delta \). We can find an upper bound on \( \Delta \) from the requirement
that \( d/H < 1 \) (which is required if not the internal energy flux should become infinite). This
condition can be expressed as \( \Delta < \frac{a}{N} \) where \( l = H/\delta H/\delta x \), the horizontal scale of variation
of \( H \). For typical oceanic conditions this means that \( \Delta \) must be chosen so that
\( 10^3 < \Delta < 10^5 \). DBDB-5 has a resolution of \( 5' \times 5' \), which gives a value of \( \Delta \sim 10^4 \), i.e. well
within the required bounds.

One may then ask to what degree the computational results will depend upon this
particular topographical resolution? For this purpose consider the energy flux \( \varepsilon \), in a case
with linear stratification, from a hill slope (e.g. a part of the continental slope) of width and
height \( L \) such that \( L = mA \). For simplicity we have assumed that the grid size \( A \) is the same
in both directions. The total height difference of the slope, \( H_j \), is then
\[ H_j = \sum \frac{d_j}{H_j} = \frac{H_j}{H_L} = \frac{d_j}{d_H} \]
where \( i, j \) denote the number of the grid. The energy flux \( \varepsilon_n \) from the area \( L \times L \) is then
\[ \varepsilon_n \sim \sum_i \sum_j \frac{H_j^2}{(H_j - d_j)^2} \sin^2 \left( \frac{n\pi d_j}{H_j} \right) \]
Since \( d_j/H_j \ll 1 \) we can write
\[ \frac{H_j^2}{(H_j - d_j)^2} \approx 1 + 2d_j/H_j. \]
Using this approximation, the series expansion \( \sin^2 (x) = x^2 \) and putting \( d_j = d_m \) and
\( H_j = H_m \), where \( d_m \) and \( H_m \) are average step height and average water depth respectively,
we obtain an approximative expression for \( \varepsilon \),
\[ \varepsilon \sim H_m^2(1 + 2d_m/H_m). \]
Since \( d_m \ll H_m \) it follows that the barotropic to baroclinic tidal energy conversion from a
given hill slope is only weakly dependent on the resolution of the topography \( \Delta = L/m \).

4. CALCULATIONS OF THE BAROTROPIC TO BAROCLINIC TIDAL ENERGY
FLUX IN THE OCEAN

If we now wish to calculate the barotropic to baroclinic tidal energy flux in the ocean, we
can use our findings above if we have knowledge of the distribution of oceanic depth, tidal
current amplitude and the vertical stratification. The bathymetric database DBDB-5
furnishes us with knowledge of the oceanic depth distribution with a resolution of \( 5' \times 5' \).
Tidal amplitude and phase, from which tidal current amplitude can be calculated, is
Table 1. Regional average of the energy flux to internal tides per unit horizontal surface area. $\epsilon_n$ represents the modal contribution and $\epsilon$ the total regional average. $F_{\text{tot}}$ is the total amount of energy generated in each region.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\epsilon_n$</th>
<th>$\epsilon$</th>
<th>$F_{\text{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($10^{-4}$ W m$^{-2}$)</td>
<td>($10^{-4}$ W m$^{-2}$)</td>
<td>($10^{10}$ W)</td>
</tr>
<tr>
<td>North Pacific</td>
<td>25</td>
<td>43</td>
<td>31</td>
</tr>
<tr>
<td>South Pacific</td>
<td>23</td>
<td>36</td>
<td>31</td>
</tr>
<tr>
<td>Indian Ocean</td>
<td>43</td>
<td>65</td>
<td>42</td>
</tr>
<tr>
<td>South Atlantic</td>
<td>16</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>North Atlantic</td>
<td>34</td>
<td>57</td>
<td>19</td>
</tr>
</tbody>
</table>

obtained from the GOTD global tidal model with a resolution of $1^\circ \times 1^\circ$. Information about the stratification is obtained from the “Levitus database” (Climatological Atlas of the World Ocean data, annual analyses, NODC) which have the same resolution as the GOTD. Before discussing any results we comment on some details in the calculations.

4.1. Prerequisites of the calculations

An accurate calculation of the energy flux requires a reliable method for estimation of the tidal currents, since the energy flux is proportional to the current amplitude squared. We have chosen to minimize the calculated energy flux by assuming the following relation between tidal amplitude ($A_0$) and current amplitude,

$$a = A_0(g/H)^{1/2},$$

i.e. we look upon the waves as long gravity waves in a non-rotating frame. Since the current amplitude in a rotating frame of reference is larger than in a non-rotating frame by a factor of $[1 + (f/\omega)^2]$, this approach minimizes the energy flux with respect to tidal currents.

The Levitus database (Climatological Atlas of the World Ocean Data, annual analyses, NODC) furnishes us with information about the actual density stratification represented by annual means at discrete depths in each $1^\circ \times 1^\circ$ box. By a three point LMS-method the profile (2.1) can be fitted to observed data, thereby obtaining values of $N_0$ and $\delta$.

From this discussion in 2.4, we conclude that the “best” choice for $g(z)$ is a step function. By this we neglect the effect of bottom friction and according to Stigebrandt (1980) the internal response becomes minimized.

In calculating the energy flux we have only considered water depths in excess of 1000 m. The equations used in the calculations are summarized in the Appendix. The energy flux generated at each step is calculated by multiplying $F$ with the step-width in both east–west and north–south directions. The calculations are done only for the $M_2$ tidal component since this is believed to be the most dominant. A cautious estimate of the internal wave energy flux due to all the other semidiurnal components indicates that they together only contribute about 20% of what comes from $M_2$ alone.

4.2. Results

The mean value of the energy flux to internal tides per unit horizontal surface area as function of mode number ($\epsilon_n$) in different regions is presented in Table 1. The amplitude
decrease with mode number $n$ is about $n^{-2}$, which differs from Munk (1966) who assumed a $n^{-1}$ dependence. By summing over 10 internal modes we obtain the regional average of the energy flux per unit surface area ($\epsilon$) (Table 1). For the North Pacific we note that the energy flux per unit surface area is $43 \times 10^{-4}$ W m$^{-2}$, compared to $17 \times 10^{-4}$ W m$^{-2}$ and $10 \times 10^{-4}$ W m$^{-2}$ estimated by Munk (1966) and Bell (1975), respectively. Although the present estimate is larger than previous ones it is of the same order of magnitude. Part of the difference is due to the fact that we use the observed $N$ and $A_0$ [for constant $N$ and $A_0$, using the same numerical values as Munk (1966) the energy flux per unit surface area in the North Pacific decreases to about $30 \times 10^{-4}$ W m$^{-2}$]. In a global estimate of the internal tidal energy flux from continental shelves and slopes Baines (1982) obtained a quite small figure compared to the estimates for the deep ocean mentioned above.

The global mean energy flux to internal tides per unit horizontal surface area, as computed here, has a value of $44 \times 10^{-4}$ W m$^{-2}$. This corresponds to a global energy flux of $1.3 \times 10^{12}$ W, which amounts to approximately 25% of the tidal energy dissipation required from astronomical observations (Cartwright, 1977).

In order to estimate the relative importance of different depth intervals with respect to the generation of internal tides we have plotted the area weighted mean energy flux per unit surface area as function of depth (Fig. 5). About 40–50% of the regional $\epsilon$ originates from the depth interval 1000–2000 m. Within this depth interval one finds the largest topographic gradients, the largest barotropic velocities and also usually the strongest stratification.

The computed energy flux per unit surface area (areal mean for each $1^\circ \times 1^\circ$ grid) is strongly governed by topography (Fig. 6). Maximum values of energy flux are found along continental margins and mid-oceanic ridges (e.g. the Mid-Atlantic Ridge) and around archipelagos (e.g. the Aleutian Islands).

5. INTERNAL TIDES AND MIXING IN THE OCEAN

The energy supplied to internal tides may be expected to eventually end up as turbulent energy. This may not necessarily happen in the vicinity of the generation site although we
Fig. 6. The geographical distribution of the energy flux $\varepsilon$ (W m$^{-2}$). The colour legend is as follows: red, $\varepsilon > 10^{-1}$; orange, $10^{-2} < \varepsilon < 10^{-1}$; yellow, $10^{-3} < \varepsilon < 10^{-2}$; green, $10^{-4} < \varepsilon < 10^{-3}$; white, $\varepsilon < 10^{-4}$. Black is land.
Fig. 9. The geographical distribution of vertical advection $\mathbf{w}$ (m s$^{-1}$) at the 1000 m level (a) and at 4000 m level (b). The colour legend is as follows: red, $w > 10^{-7}$; orange, $10^{-8} < w < 10^{-7}$; yellow, $10^{-9} < w < 10^{-8}$; green, $10^{-10} < w < 10^{-9}$; white, $w < 10^{-10}$ (m s$^{-1}$).
do not believe that the energy radiated by the locally generated internal tides is spread ocean-wide. Internal friction, baroclinic instabilities, critical layer absorption, destructive interactions between waves generated in neighboring areas and other phenomena will cause dissipation, probably on a regional scale (cf. De Young and Pond, 1989). Without knowledge about the horizontal length-scale over which dissipation occurs, i.e. the distance which typically is travelled by the internal tides, we arbitrarily assume that the internal tidal energy generated in each $1^\circ \times 1^\circ$ box of the ocean also is dissipated within the box. The discussion in the introduction supports this assumption.

Following Stigbrandt and Aure (1989) we can then compute the vertical diffusivity, which is due to dissipating internal tides, in each $1^\circ \times 1^\circ$ box. The vertical diffusivity $\kappa$ should be a function of the supply of turbulent energy and the vertical stratification. We can therefore express $\kappa$ as follows,

$$\kappa = \gamma q N^{-1}, \quad (5.1)$$

where $q$ is a non-dimensional function, $N$ is the buoyancy frequency and $\gamma$, which has the dimension velocity squared, is defined by

$$\gamma = \Phi/(M \rho_0). \quad (5.2)$$

Here $\rho_0$ is a reference density, $\Phi$ the mean specific rate of work performed by the turbulence against the buoyancy forces, and $M$ is the weighted average of the buoyancy frequency.

The mean specific rate of work $\Phi$ can be computed from knowledge about the flux of energy to internal tides. Assuming an efficiency factor $R_f$ (the flux Richardson number) for turbulence with respect to performed work against buoyancy forces, we compute regional means of $\Phi$ from the rate of energy supply to each $1^\circ \times 1^\circ$ box in the ocean.

$$\Phi = \Sigma \varphi F_u R_f / V, \quad (5.3)$$

where $\Sigma \varphi F_u$ is the sum of energy fluxes by the internal wave components to turbulence in a $1^\circ \times 1^\circ$ box. $\varphi$ is the inverted fraction of the total energy supply to internal tides provided by the $M_2$ component. $V$ is the volume of the box in which the internal tides dissipate:

$$V = \int_b^u A(z) \, dz, \quad (5.4)$$

where $A$ is the hypsographic function for each $1^\circ \times 1^\circ$ box, $b$ is the greatest depth within the box and $u$ is the 1000 m level.

The weighted average of the buoyancy frequency $M$ can be expressed

$$M = \frac{1}{V} \int_b^u NA \, dz. \quad (5.5)$$

The non-dimensional function $q$ in (5.1) is chosen to be

$$q = c(N/M)\theta, \quad (5.6)$$

where $\theta$ is an empirical constant and $c$ is defined by the integral condition

$$c \int_b^u (N/M)^{1+\theta} \, dz = V. \quad (5.7)$$

From the equations presented above it follows that we can compute $\kappa = \kappa(N)$ if we know the empirical constant $\theta$, the hypsographic function $A(z)$, the vertical stratification (giving $N$), the fractional conversion factor $\varphi$, the efficiency factor $R_f$ and the flux of...
Table 2. The horizontally integrated vertical transports $w_1$, $w_2$, $w_3$ and $w_4$ in Sverdrups ($10^6 \text{ m}^3 \text{ s}^{-1}$) and the oceanic mean values of the vertical diffusivities $\kappa_1$, $\kappa_2$, $\kappa_3$ and $\kappa_4$ in $10^{-5} \text{ m}^2 \text{ s}^{-1}$ through the 1000, 2000, 3000 and 4000 m level, respectively. The result is for $R_f = 0.056$

<table>
<thead>
<tr>
<th></th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$\kappa_1$</th>
<th>$\kappa_2$</th>
<th>$\kappa_3$</th>
<th>$\kappa_4$</th>
</tr>
</thead>
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<tr>
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<td>5.7</td>
<td>4.3</td>
<td>1.6</td>
<td>0.77</td>
<td>1.92</td>
<td>3.39</td>
<td>5.11</td>
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<td>2.5</td>
<td>1.2</td>
<td>0.33</td>
<td>0.78</td>
<td>1.34</td>
<td>1.99</td>
</tr>
<tr>
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<td>6.5</td>
<td>4.1</td>
<td>1.4</td>
<td>0.35</td>
<td>0.87</td>
<td>1.53</td>
<td>2.31</td>
</tr>
<tr>
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<td>2.5</td>
<td>2.4</td>
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<td>0.45</td>
<td>1.09</td>
<td>1.91</td>
<td>2.87</td>
</tr>
<tr>
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<td>7.2</td>
<td>2.7</td>
<td>0.85</td>
<td>2.04</td>
<td>3.53</td>
<td>5.28</td>
</tr>
</tbody>
</table>

energy to internal tides in each $1^\circ \times 1^\circ$ box of the ocean. Following Stigebrandt and Aure (1989) we choose the values $-0.5$, $1.18$ and $0.056$ for the constants $\theta$, $\varphi$ and $R_f$. Although Stigebrandt and Aure studied diapycnal mixing in fjords their value of $\theta$ probably holds for ocean conditions as well. The value of $\varphi$ is site-specific which means that, the value given by Stigebrandt and Aure may not hold for the ocean. However, the variation in $\varphi$ is expected to be small.

The value $0.056$ for $R_f$ might be in lower range of what is currently accepted. Gargett (1984) for example writes "... and $R_f \sim 0.20$ should be characteristic of systems in which diapycnal mixing results from internal wave breakdown to turbulence". However, since there are some uncertainties about the correct value of $R_f$, as discussed in Gargett and Holloway (1984), we have chosen to minimize $\kappa$ by choosing the smallest value of $R_f$ cited. However, it is a simple matter to adjust the results below to any other value of $R_f$.

Regional averages of $\kappa$ at four different depths are presented in Table 2. As discussed in Gargett (1984) most estimates of upper bounds for $\kappa$ suggest values $\sim 1.0 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ in the upper deep ocean (just below the main thermocline), which agrees with our results. As mentioned above, $\kappa$ is proportional to $R_f$. Letting $R_f$ equal 0.20 instead of 0.056, our $\kappa$-values would be a factor of about 4 greater than those given in Table 2. It also should be noted that a higher (less negative) value of $\theta$ would distribute more of the turbulence higher up in the water column.

It is of interest to note that there is a north-south discrepancy in the Atlantic and the Pacific (Fig. 7). The South Pacific and South Atlantic have about the same $\kappa$-values, a

Fig. 7. Horizontal average of vertical diffusivity $\kappa$ as function of depth $z$ for different ocean regions.
factor 2–3 less than in the northern hemisphere. This discrepancy is probably due to the bias in the north-south distribution of land and water.

To the best of the present authors' knowledge there is no reliable estimate, based upon direct measurements, of the mean vertical diffusivity in the ocean. Consequently, there is no obvious way to judge the veracity of our results. However, for a steady state there is a balance between vertical diffusion and vertical advection in the interior of the ocean. This may be utilized since we believe that the vertical transports in the ocean are better known than the vertical diffusivities. Assuming that the vertical density distribution satisfies a local steady state advection–diffusion balance (as in, for example, Gargett, 1984) and knowing the diffusivity at different depths we can estimate vertical velocities (and transports) at these depths if we calculate the local ratio between the vertical velocity \( w \) and the diffusivity \( \kappa \). The results are presented in Table 2 and in Figs 8 and 9.

If we sum the vertical transports through the 1000 m level for all oceans (Table 2, first column) we arrive at a vertical transport of about 15 Sv (1 Sv = \( 10^6 \) m\(^3\) s\(^{-1}\)). The rate of dense water entering the ocean from marginal seas and sinking below the 1000 m level seems to be in the range of 10–15 Sv (Warren, 1981). About two-thirds of this is produced in the North Atlantic and most of the rest is produced in the Antarctic. Thus, our results indicate that mixing performed by dissipating internal tides may be a major mixing agency, at least in the upper deep ocean. In addition our results indicate that the correct value on \( R_f \) in the ocean is \( \sim 0.05 \) rather than 0.2 since the latter would imply a vertical transport of about 60 Sv through the 1000 m level. We find this result quite comforting since it is...
difficult to explain why the conversion of internal tidal wave energy to mixing energy should be more efficient in the ocean than in fjords.

Below the 1000 m level the vertical transports increase and a maximum is found between 2000 and 3000 m depth, where the transports are about twice as large as at the 1000 m level (Fig. 8a). At the 4000 m level the vertical transport has decreased to approximately half of what was found at the 1000 m level (Fig. 8a, Table 2). There may be several reasons for why the vertical transports vary like this. It may indicate that the energy for mixing at the upper and lower boundaries of the abyss has other important sources than dissipating internal tides. However, that would invalidate the current estimates of the deep water supply (Warren, 1981). A far more attractive explanation is due to the origin and the behaviour of the dense bottom currents. These currents constitute the means by which the deep water can be renewed. They originate in the surface waters and are thus liable to variable prerequisites. As the currents propagate downwards they entrain ambient water whereby the volume flow increases. When the current has obtained neutral buoyancy it is believed to be interleaved in the interior of the ocean. Only the most dense flows or those which initially have the largest volume flows will penetrate all the way to the bottom of the abyss.

Stigbrandt (1987) found that the volume flow of the dense bottom currents in the Baltic Sea should increase by a factor of 6 before the water is finally interleaved in the deep-water. We also can estimate this factor of dilution by first calculating the vertical average of the vertical transport and then dividing it by the assumed value of the deep water supply (10–15 Sv). As an average we find that the volume flow of the dense bottom currents should increase by a factor of 2–4. The lower value in the ocean, as compared to the Baltic, is probably due to the fact that in the ocean rotational dynamics impede mixing and entrainment more than in the Baltic (cf. Stigbrandt, 1987).

6. DISCUSSION

The condition that the velocity component normal to the bottom should vanish (the kinematical boundary condition), is the ultimate reason for the topographical generation of internal waves. The present model satisfies the kinematical boundary condition everywhere at the bottom, thereby computing the “true” energy flux from the surface tide.

Destructive interaction among waves from different “generator elements” may lead to wave breaking and generation of turbulence close to the generation area. This should be particularly pronounced at real, three-dimensional topography. The energy radiated by internal waves from a large scale topographic generator should therefore, in general, be less than the energy flux estimated by our model. The difference should dissipate in the water column above the wave generating area. From this it follows that areas of baroclinic wave generation also should be areas of intense mixing. It should be underlined that the large scale internal wave field, i.e. the part that is not dissipated in the wave generation area, cannot be computed by the present model. For this task one has to use models of the type developed by Baines (1982) and Craig (1989).

For the modal description the energy flux \( \varepsilon_i \) away from a step of width \( \Delta \) is

\[
\varepsilon_i = \Delta \Sigma F_n = \Delta \Sigma E_n C_{gn}.
\]

We notice that the product \( \Delta E_n \) has the dimension of force and may be identified as the modal internal wave drag upon the barotropic current. With use of (2.14) the total drag force \( D \) can be written

\[
D = \rho \alpha^2 \Delta d \left[ H/(H - d) \right] = C_d \rho \alpha^2 \Delta.
\]
Here \( A = \Delta d \) is the area of the wave generating step and \( C_d \), which is defined by
\[
C_d = \frac{2H}{(H - d)},
\]
is the drag coefficient, which then is known exactly as long as the internal wave solution holds. It should be noted that the drag coefficient is independent of the vertical stratification and of the frequency of the oscillating surface tide.

The internal wave drag caused by the step is singular in nature since it occurs at the abrupt change of water depth (at the step). However, if we divide \( D \) by the horizontal surface area \( l\Delta \) of a "pillar", where \( l \) is the depth of the step, we obtain the mean wave stress \( \tau_i \) per horizontal unit surface area at each topographic step,
\[
\tau_i = \frac{D}{l\Delta} = \frac{\rho \alpha^2 H l (H - d)}{H - d} \frac{dH}{dx},
\]
where we have written \( d/l = dH/dx \). This might possibly be a useful parameterization of the internal wave stress for use in numerical barotropic tidal models. Inserting \( \rho = 10^3 \) (kg m\(^{-3}\)), \( \alpha = 10^{-1} \) (m s\(^{-1}\)), \( d = 10^2 \) (m), \( H = 10^3 \) (m) and \( l = 10^4 \) (m) we obtain an internal wave "stress" equal to about 1 dyne cm\(^{-2}\).

In order to model the vertical circulation in the ocean it is obviously necessary to accurately know the horizontal and vertical distributions of vertical diffusivity. Determining the vertical diffusivity in the ocean also is necessitated by the increasing concern to understand the dynamics of climate changes, for which the ocean most certainly plays an important role. The present investigation points to the need to determine the value of \( R_f \) for turbulence driven by dissipating internal tides. If \( R_f \) is equal to about 0.05, as suggested by fjord data (Stigebrandt and Aure, 1989), it seems that internal tides are the major energy source for vertical mixing in the ocean interior.

Since there are no accurate measurements, the predicted vertical diffusivity cannot be verified. However, in areas where one finds staircases in the vertical stratification molecular double diffusive mixing processes must dominate, as pointed out by Kelley (1988). Since double diffusion seems to be a fairly inefficient mixing process, one would expect to find staircases in areas with little mixing by internal tides. Our maps then could be used to predict where staircases are likely to occur.

Our results suggest that the upward vertical transport in the ocean interior has a maximum value at mid-depth in the abyss. The associated vertical velocities (Fig. 8b) have a similar vertical dependence although somewhat modified because of topographic effects (the horizontal surface area of the ocean changes with depth). The dynamical consequences of such a behaviour are several. The most important consequence being an equatorial flow in the upper deep ocean and an opposite, poleward, flow at great depths (as required by the vorticity equation). At the rim of the ocean this interior flow pattern would require an anticyclonic flow in the upper parts of the abyss and a cyclonic flow below. However, it is questionable if oceanic mean values of vertical velocity have any dynamical significance for the interior circulation, the reason being that the horizontal variation of the vertical velocity is of several orders of magnitude and that most upwelling occurs in boundary regions or near profound topographical features within the ocean. The implications for general ocean circulation modelling, of having \( a \) priori a certain distribution of upwelling, have (to the authors' knowledge) not yet been explored. However, one would expect that the mean ocean circulation would be dramatically different to that found in existing models (cf. Warren, 1981).

If the vertical diffusion in the deep ocean mainly is driven by dissipating internal tides then the vertical deep-water circulation in the ocean may be rather insensitive to changed
conditions on the sea surface. Accordingly, the rate of vertical circulation during, for instance, different phases of the glaciation cycle, might have been rather constant although the properties of the deepwater may have changed considerably.

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REFERENCES


Levitus Database, Climatological Atlas of the World Ocean Data, Annual analyses, NODC.


APPENDIX

A summary of the equations applying in the ocean

\[ N^2 = N_0^2 (1 + H/\delta)(1 + (H - z)/\delta)^{-2} \]

\[ N_0^2 = g \Delta \rho/\rho H \]
\( N_0 \) and \( \delta \) are calculated by fitting the theoretical profile to observations.

\[
\begin{align*}
\nu_n &= n\pi\ln (1 + H/\delta) \\
k_n^2 &= (v_n^2 + 0.25)(\omega/\nu_0)^2 \delta^2/(1 + H/\delta)(1 - (f/\omega)^2) \\
u_n &= (2H[\delta \ln (1 + H/\delta)(v_n^2 + 1/4)])^{1/2}[1 + (H - z)/\delta]^{1/2} \\
&\quad \times \{\sin (\nu_n \ln [1 + (H - z)/\delta])/2 + \nu_n \cos (\nu_n \ln [1 + (H - z)/\delta])\}
\end{align*}
\]

\[
\int_0^\delta u_n^2 \, dz = 1
\]

\[
\alpha + \Sigma a_n \mu_n = Ha/(H - d)\theta(z - d)
\]

\[
a_n = a[2H(\delta \ln (1 + H/\delta)(v_n^2 + 1/4))]^{1/2}(H - d)\delta \\
&\quad \times [1 + (H - d)/\delta]^{1/2} \times \sin [\nu_n \ln (1 + (H - d)/\delta)]
\]

\[
F = \Sigma F_n
\]

\[
F_n = 1/2a_n^2\rho \omega H k_n(1 - (f/\omega)^2).
\]