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Review of the Partition of Tidal Energy in Five Canadian Fjords

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ABSTRACT


The physical processes in a number of Canadian fjords have received considerable study over the years. The fjords tend to be tidally energetic and can be highly stratified because of freshwater runoff. Observations from instruments such as cyclocomets (profiling current meters), acoustic Doppler current profilers (ADCPs) and S4s (InterOcean vector averaging current meters) have yielded observations of the circulation over almost the entire water column. Combined with simulations from analytic and numerical models, these observations have allowed the partition of tidal energy in the fjords to be examined in some detail. The net tidal energy flux into a fjord (i.e., the rate at which energy is removed from the surface tide), and the cause of the energy flux, can vary significantly between fjords. The energy can be removed from the surface tide by boundary friction or by the generation of internal motions (i.e., the internal tide, hydraulic jumps etc.). In this paper the partition of tidal energy in five Canadian fjords and the techniques used to determine the partition will be reviewed. The analysis techniques will be described in some detail and, where possible, the relationship between the various analysis techniques will be pointed out. The fjords discussed are Observatory Inlet, Knight Inlet, Burrard Inlet/Indian Arm, Sechelt Inlet and the Saguenay Fjord.

ADDITIONAL INDEX WORDS: Tidal inlets, tides, internal tides, coastal oceanography, Canada.

INTRODUCTION

Fjords are plentiful along the west coast of Canada, and are found in the Arctic and eastern Canada also. Their scientific study dates basically from the early 1900's, although much earlier observations/comments can be found in the journals of the original European explorers. BANCROFT (1913) discussed the geological origin of the west coast fjords. The oceanography of a number of west coast fjords, e.g., Quatsino Sound (HUTCHINSON and LUCAS, 1927); Howe Sound, Indian Arm, and Bute Inlet (HUTCHINSON and LUCAS, 1931); Jervis, Sechelt and Toba Inlets (CARTER, 1932, 1934); Saanich Inlet (CARTER, 1934); Nootka Sound (TULLY, 1937); Alberni Inlet (TULLY, 1949)] was studied in the first half of the twentieth century. It became recognized that the tides, winds and freshwater runoff could all contribute significantly to the currents in the fjords even though the observations were largely limited to temperature and salinity, and perhaps other scalars such as dissolved oxygen. A picture developed, similar to that of fjords elsewhere in the world, of (1) a seaward flowing surface layer (∼ 5–10 m thick and caused by river runoff) that entrains saltier water from below as it moves seaward, (2) a compensating inflowing intermediate layer, and (3) a lower less active layer below and landward of fjord's sill.

A major advance in our knowledge of west coast fjords occurred with the studies of PICKARD and ROGERS (1959) and PICKARD (1961, 1963, 1975). PICKARD and ROGERS (1959) presented some of the first current measurements for a deep inlet. Longitudinal and transverse currents in Knight Inlet were made over the sill and in much deeper water near Tasmakstum Island (Figure 1C). They documented the influence of freshwater runoff, the winds and the tides on the currents in the surface layer, noting that the tendency for the surface layer to flow seaward could be reversed by the winds and the tides. They observed that the tidal currents over the sill were relatively uniform with depth and close in value to what one would predict from continuity and the tidal height variations in the inlet. In the basin, however, the tidal currents were not uniform with depth, and the timing of the maximum and minimum velocities in the upper part of the water column was not what one would predict from the tidal height variations. They noted that the isotherms in the basin oscillated with the tidal period and that these oscillations might be due to internal waves which could influence the current structure.

PICKARD (1961, 1963) described the physical oceanography of a number of west coast fjords. PICKARD (1961) reported on 39 mainland inlets, and PICKARD (1963) reported on 25 inlets along the west coast of Vancouver Island. PICKARD (1961) observed the presence of internal waves with periods of 1 to 4 minutes and amplitudes of up to 5 m in the upper layers, and with the semi-diurnal tidal period and amplitudes of 5
Figure 1. Plan views and longitudinal sections of (A) Indian Arm, (B) Observatory Inlet, (C) Knight Inlet, and (D) Sechelt Inlet. [Adapted from De Young and Pond (1989) and Tinis (1995).]
to 75 m deeper down. He noted that these waves appeared to be quite common and that their influence 'must be borne in mind' when making measurements of water properties in a fjord.

The 1970s marked the time when studies containing a significant quantitative and theoretical component began to appear. Drunkwater and Osborn (1975) made measurements of temperature, salinity and oxygen in Rupert and Holberg Inlets, which share a common connection, Quatsino Sound, to the ocean. They found that the inlets tended to be well mixed but with large monthly variations in their observed temperature, salinity and oxygen. They found that mixing due to the tides in Quatsino Sound could provide the energy needed to cause the observed changes in T, S and O2; only a small fraction of the net flux of tidal energy into the inlets was required to cause the observed changes, consistent with the fact that most of the available tidal energy would be dissipated and would not contribute to mixing. Their calculation of the net tidal energy flux into the inlets may have been the first such calculation for a Canadian fjord.

As the importance of mixing caused by the tides in fjords was realized, more effort was made to observe and explain the flow of stratified tidal currents over fjord sills. Pioneering work in making such observations (e.g., Farmer and Smith, 1978; Farmer and Smith, 1980a,b) used high frequency echo sounders to map the displacement of isopycnals near fjord sills. Farmer and Freeland (1983) presented a thorough review of the physical processes in fjords, and they pointed out that progress in our understanding of fjords would require a better understanding of processes, in particular the scales and intensities of turbulent flows and the mechanisms by which energy is fed into such flows and ultimately distributed.7 And in fact after the 1970's the number of west coast fjords being studied dramatically decreased as the research emphasis changed from being primarily observational to being one of understanding the underlying physical processes.

Until recently the Saguenay fjord (Figure 2), which lies along the north shore of the Saint Lawrence Estuary in Quebec in eastern Canada, was not as well studied as west coast fjords. Schaffer et al. (1990) reviewed the physical and other aspects of the oceanography. The Saguenay has an inner sill, as well as a sill near its mouth, and one aspect of its circulation that has been studied in more detail than that of similar west coast fjords is the exchange of water between its two main basins (Stacey and Gratton, 2001; Belanger, 2004). Before this work, the most recent work published in the refereed literature was that of Siebert; Trites, and Reid (1979), who studied the exchange of water between the Saguenay and the Saint Lawrence Estuary. They found that the Saguenay was flushed more frequently than other fjords, with an estimated flushing time of 65 days for the outer basin even though the outer sill is no more than about 30 m deep. This is a much shorter time than for many other fjords, and they speculated that the energy source for this rapid flushing comes from the generation of internal waves through the interaction of the barotropic tide with topography. Schaffer, Smith, and Cote (1990) presented evidence (salinity and temperature distributions; bottom sediment composition; suspended, particulate matter distributions) from a number of sources that there tends to be higher turbulence levels and more mixing in the outer than in the inner basin. Subsequent observational and theoretical studies in the 1990s have addressed these questions and they will be discussed in this review paper.

The purpose of this paper is to review research that has been done on some tidally-forced aspects of the circulation in Canadian fjords since the 1980s, a time when the number of fjords being actively studied was much less than in the 1960s, but for which the measurements became much more detailed with the goal of better understanding and explaining quantitatively the underlying physics. All but one of the fjords is located on the west coast of British Columbia. The four British Columbian fjords are Observatory, Knight, and Sechelt Inlets, and the fjord formed by Indian Arm and Burrard Inlet. The other fjord that will be discussed is the Saguenay fjord.

All of these fjords have strong tides, which tend to dominate the circulation in many respects. Mixing in these fjords tends to be dominated by the net flux of barotropic tidal energy into the fjords, but how this tidal energy is partitioned can vary significantly between the fjords, and within the fjords the net energy flux can vary as the stratification varies with changing river runoff etc. How this tidal energy is partitioned in each fjord will be reviewed. Also, the techniques used to determine how the energy is partitioned will be discussed in some detail.

**Fjords Descriptions**

**Knight Inlet**

Knight Inlet (Figure 1C) has been studied more intensively than any other Canadian fjord. It has a narrow (inner) sill about 70 km from its head that is about 70 m deep. The first tributary at about 100 km has a shallow connection to Knight Inlet and there is another (outer) sill (not shown) about 65 m deep located about 130 km from the head. The average width of the inlet is about 2.5 km and it reaches a maximum depth of about 550 m inside the inner sill and of about 200 m between the inner and outer sills. The tides are mixed but dominated by the semi-diurnal M2 constituent which has an amplitude of about 1.5 m. The dominant diurnal constituent is Kr with an amplitude of about 0.5 m. During spring tides the current velocities over the inner sill can exceed 1 m/s. The rate of fresh water runoff from the Klinaklini River at the head of the fjord varies significantly during the year, being highest during the spring and summer when a distinct surface layer 5–10 m thick develops. The rate of runoff varies from about 100 m3/s in winter to over 500 m3/s in summer.

**Burrard Inlet/Indian Arm**

Vancouver, Canada's third largest city, abuts Burrard Inlet which is basically an extended sill for the fjord called Indian Arm (Figure 1A). Indian Arm reaches a maximum depth of about 220 m, but Burrard Inlet is never deeper than about 65 m and much of the inlet is 25–35 m deep. First and Second Narrows in Burrard Inlet are locations of intense mixing, particularly during spring tides. Burrard Inlet and Indian Arm
Figure 2. Plan view and longitudinal section of the Saguenay Fjord. [Adapted from Stacey and Gratton (2001) and Belanger (2004).]
are each about 20 km long. The main source of fresh water runoff is the Bunten Power Houses, located on the eastern shore of Indian Arm, with an average runoff of about 40 m³/s. The tides are mixed semi-diurnal/ diurnal; \( M_2(K_1) \) has an amplitude of about 0.93 (0.85) m.

**Sechelt Inlet**

This inlet (Figure 1D) is located about 50 km north-west of Vancouver, and opens into Jervis inlet (not shown). Two other inlets, Narrows Inlet and Salmon Inlet, branch off from Sechelt Inlet. Sechelt Inlet has a maximum depth of about 275 m and a very shallow sill at its mouth of 5–20 m depth. It is about 35 km long and 1–2 km wide. A shallow sill (about 15 m depth) separates Sechelt and Narrows Inlets but there is no sill between Sechelt Inlet and Salmon Inlet. The mean freshwater input is estimated to be about 100 m³/s. The amplitude of the \( M_2(K_1) \) tide is about 1 (0.9) m.

**Observatory Inlet**

Observatory and Portland Inlets (Figure 1B) are located just south of the Alaska Panhandle and are separated by a sill approximately 50 m deep and located about 50 km from the open ocean. Portland Inlet is about 5 km wide, about twice as wide as Observatory Inlet. Observatory Inlet has a maximum depth of about 500 m while Portland Inlet deepens continuously away from the sills towards the open ocean. At its ‘head’ Observatory Inlet branches into two arms, Alice Arm and Hastings Arm. The major source of freshwater runoff is the Nass River which flows into Portland Inlet about 5 km from the sill, and it has a mean annual input of about 1000 m³/s (Figure 4a). The amplitude of the \( M_2 \) tide is about 1.9 m.

**Saguenay Fjord**

The Saguenay Fjord (Figure 2) is about 120 km long and 1–7 km wide. At its ‘head’ it branches, into Ha! Ha! Bay and the North Arm. It has two main sills, an outer (inner) sill about 30 (60) m in depth. The outer sill is located at the mouth and the inner sill is about 20 km further up-fjord. The outer (inner) basin has a maximum depth of about 250 (275) m. The major source of runoff is the Saguenay River with a mean input of about 160 m³/s. The amplitude of the \( M_2(K_1) \) tide is about 1.6 (0.24) m. The next largest semi-diurnal constituent \( S_2 \) is significantly larger than \( K_1 \) at about 0.5 m.

The net flux of barotropic tidal energy (i.e., the power \( P \)) through any cross-section of a fjord is given by

\[
P = \int \int_{\text{area}} \rho g \overline{u} \eta \, ds
\]

for the flux of the constituent with angular frequency \( \omega \) and amplitude \( \eta_0 \), where for small \( \epsilon \) and \( \phi_i \) one has

\[
\sin(\epsilon) \approx \frac{\phi_i}{1 + \frac{S_i - S_2}{S_1 + S_2}}
\]

where \( \rho \) is the water density, \( g \) the acceleration due to gravity, \( u \) the barotropic tidal velocity and \( \eta \) the sea-surface height (Garrett, 1975). (The overbar represents a time average, and the integral is over the cross-sectional area.) For a pure standing wave \( u \) and \( \eta \) are in quadrature and the integral (1) has a value of zero. In many fjords each diurnal and semi-diurnal tidal constituent is indeed almost a standing wave, and most of the energy that enters the fjord during one stage of the tide ends up leaving during another stage. However, even though the percentage of tidal energy staying in a fjord is typically small, it still often represents a much larger net energy flux than that due to the winds and freshwater runoff.

If \( \eta(t) = \eta_0 \sin(\omega t) \) is the tidal height for a single constituent, where \( \eta_0 \) is the amplitude and \( \omega \) is the angular frequency, and \( u(t) = u_0 \cos(\omega t - \epsilon) \) is the corresponding barotropic velocity at a given cross-section [note that \( u(t) \) is not in quadrature with \( \eta \) by the (usually but not always small) angle \( \epsilon \)], (1) becomes

\[
P = \rho g \frac{\eta_0 u_0 A}{2} \sin(\epsilon)
\]

where \( A \) is the cross-sectional area. Drinkwater and Osborn (1975) used (2) to estimate the energy flux into Rupert and Holberg Inlets. Also, Stacey and Gratton (2001) used an equivalent expression to calculate the energy flux into the Saguenay Fjord, using the output from a numerical model. They also calculated a higher order correction to (2) that takes into account the mean flow velocity and the fact that the cross-sectional area changes as the tide goes up and down. The correction was found to be small.

The problem with (2) is that sufficiently accurate velocity observations are not always available to provide \( u_0 \) and \( \epsilon \). The surface height is far easier to measure. In almost all tidally energetic fjords more energy is removed from the barotropic tide in the vicinity of the sill than elsewhere and, if one has measurements of the sea-surface height on either side of the sill, one can estimate the rate at which energy is removed from the barotropic tide. It turns out that an estimate of the energy removed from the barotropic tide up-fjord of a given cross-section can be made if a few approximations are made, the exact form of the approximations being dependent on how the surface height varies along the fjord.

In many fjords the amplitude and phase of each diurnal and semi-diurnal constituent varies by only a few percent along the fjord. When this is the case, and assuming all of the energy removed from the tide is removed between cross-sections 1 and 2 (Figure 3), De Young and Pond (1987) present the expression

\[
P = \rho g \frac{\eta_0 \omega S_1}{2} \sin(\epsilon)
\]

for the flux of the constituent with angular frequency \( \omega \) and amplitude \( \eta_0 \), where for small \( \epsilon \) and \( \phi_i \) one has

\[
\sin(\epsilon) \approx \frac{\phi_i}{1 + \frac{S_i - S_2}{S_1 + S_2}}
\]
STACEY (1984) and, if the width of the fjord is constant between the two sections, FREELAND and FARMER (1980).

If there is a location along the fjord where the cross-sectional area decreases dramatically, such as when the width of a fjord constricts near the sill, the amplitude and phase of a tidal constituent can change significantly across the sill so that the assumptions made above are no longer valid. To handle this situation Tinis (1995), who studied Sechelt Inlet, developed the expression

$$P = \frac{\rho g \eta^2 S \omega \sin(2\phi)}{2}$$

(5)

where \(\eta\) is the amplitude of the tidal constituent at section 1 and \(S\) is in this case the total surface area up-fjord of the sill. Tinis assumes that the surface height changes abruptly at the sill and that on either side of the sill the amplitude and phase do not vary. Expression (5) was also derived, in a different manner and by making different assumptions, by STIGEBRANDT (1999). Note that (5) shows that at most one half of the incoming power can be extracted.

**THE PARTITION OF TIDAL ENERGY**

An expression similar to (3) was used by FARMER and FREELAND (1980) to estimate the energy removed from the barotropic tide in Knight Inlet. They had time series of sea-surface height on either side of the sill spanning more than a year and they found that the \(M_2\) and \(S\) phase change across the sill varied as a function of time. They found that the rate at which energy was removed from the barotropic tide varied significantly with time over the course of a year. The phase change across the sill for the \(M_2\) constituent varied from about 0.75 to about 1.75 degrees, depending on the stratification. This result showed that processes other than boundary friction were playing an important role in removing energy from the barotropic tide since boundary processes would not cause the average rate at which energy was removed to vary. The conclusion was drawn that internal processes that depend on the degree to which the inlet is stratified must make a very significant contribution to how much energy is removed from the barotropic tide. Freeland and Farmer estimated that internal processes accounted for about 95% of the rate at which energy was removed from the barotropic tide.

Tidally-forced, internal processes take many forms and cover a wide range of space scales, but it is clear that it is the interaction of the tidal flow with the sill that provides much of the forcing for these processes. There are the hydraulic jumps, jets, lee waves etc. in the immediate vicinity of the sill, and there is also the internal tide. The jets, if the water over the sill is dense enough, may travel as a density current over the sill and down the lee slope, causing a tidally-modulated renewal which may depend on the spring-neap cycle. Regarding Knight Inlet, Freeland and Farmer did not conclude which of the various internal processes was most efficient at removing energy from the barotropic tide, but they did note that mixing caused by these processes would influence the mean circulation in the inlet.

Some light was shed on how energy is removed from the barotropic tide in Knight Inlet by STACEY (1985). He applied a simple model of the internal tide that relied heavily on the earlier work of STIGEBRANDT (1976, 1979, 1980). The sill was approximated as an infinitely thin, vertical strip and the bottom of the inlet on either side of the sill was assumed to be flat. The dynamic modes for the observed stratification, which could be different on either side of the sill, were determined using the normal mode equation

$$\frac{d^2Z_n(z)}{dz^2} + \left(\frac{N^2(z)}{\omega^2} - 1\right)k^2Z_n(z) = 0$$

(6)

where

$$\dot{Z}_n(0) = Z_n(H) = 0,$$

$$\tilde{W}_n(z) = i\tilde{Z}_n(z)$$

$$\dot{\alpha}_n(z) = -\frac{1}{k_n} \frac{d\tilde{Z}_n(z)}{dz}, \quad \text{and}$$

$$\tilde{\rho}_n(z) = -\frac{\rho g N^2(z)}{\omega g} \ddot{Z}_n(z)$$

\(i = \sqrt{-1}\), \(g\) is the acceleration due to gravity, \(\rho_0\) is a reference density for water (103 kg/m3), \(\omega\) is the angular frequency of the tidal constituent of interest, and \(k_n\) is the wave number of the \(n\)-th mode; \(\tilde{W}_n(z), \tilde{\alpha}_n(z)\) and \(\tilde{\rho}_n(z)\) are the amplitudes (each a function of depth and undetermined to within the same multiplicative constant) of the vertical and horizontal velocity, and the density fluctuation respectively [e.g., \(u_n(x, z, t) = A \alpha_n \exp(\exp(\alpha_n)\) is a complex constant]; \(N(z)\) is the Brunt-Vaisala frequency calculated from the observed density profile, and the inlet depth \(H\) can be different on either side of the sill. Note: (1) \(k_n\) can be positive or negative, depending on whether the wave is traveling in the positive or negative \(x\)-direction; (2) at tidal frequencies \(N^2(z)/\omega^2 \gg 1\) so strictly speaking the \((-1)\) within the brackets of (6) could be neglected; and (3) the transformation \(\tilde{W}_n(z) = i\tilde{Z}_n(z)\) is required only to emphasize that \(\tilde{\alpha}_n(z)\) and \(\tilde{\rho}_n(z)\) are real, the \(i\) phase shift being incorporated into the vertical velocity \(\tilde{W}_n(z)\).

The total tidal along-channel velocity field on either side of the sill was taken to be the superposition of the ten lowest modes and the barotropic velocity. The barotropic velocity was calculated from the inlet geometry and the tidal height...
energy flux away from the sill that could account for almost all of the energy removed from the barotropic tide. The relative importance of the two modes depended on the stratification, with both modes being important when there was a distinct surface layer due to increased fresh-water runoff during the summer. This result suggested that the internal tide in Knight Inlet is more important than the smaller scale processes in the immediate vicinity of the sill as an agent for removing energy from the barotropic tide. It also suggested that it would not be appropriate when there is a distinct surface layer to approximate the inlet as being composed of only two homogeneous layers, the thin surface layer and a much thicker underlying layer. Such an approximation would allow for the existence of only one internal mode and could result in the energy flux due to the internal tide being significantly underestimated.

Stacey (1984) used the same simple model to calculate the energy flux of the internal tide away from the sill of Observatory Inlet (Figure 1B), another fjord with a sill that is narrow relative to a tidal excursion. He also used an expression equivalent to (3) to calculate the rate at which energy was being removed from the barotropic tide. It was found that the rate at which energy was removed from the internal tide could vary from less than 10 MW to greater than 20 MW, depending on the stratification (Figure 4). The simple model of the internal tide could account for essentially all of this energy flux. Predictions of the along-channel velocity using the simple model were also in rough agreement with the available current observations, which had $M_2$ amplitudes almost twice as large as that which could be due to the surface tide alone (8 cm/s). The internal tide was therefore seemingly important even though images of acoustic backscatter near the sill [Figure 5; there are similar images for Knight Inlet (e.g., Farmer and Armi, 1999b)] showed intense small scale motions. Indeed, the influence of significant nonlinear effects was noticeable in the current meter observations also. At the MSF frequency (period = 14.7 days, the beat frequency of the $S_0$ and $M_2$ tides), an amplitude greater than 10 cm/s was determined from the current observations at 10 m depth and 4

![Figure 4](image-url)  
Figure 4. (A) Monthly freshwater runoff from the Nass River for the year 1982. (B) Power removed from the barotropic tide. The upper curve is the power removed from the five most important tidal constituents. The lower curve is the power removed from the $M_2$ tide alone. The solid squares are the predicted energy flux of the $M_2$ internal tide. [Adapted from Stacey (1984).]

![Figure 5](image-url)  
Figure 5. Acoustic backscatter of flow over the sill of Observatory Inlet, and predicted interface positions of a three-layer model. [Adapted from Stacey and Zedel (1986).]
km from the sill, even though, based on the surface height variation alone the MSF velocity would be no more than about 0.01 cm/s. Also, the phase of the MSF current changed by 180 degrees between the current meters at 10 and 100 m, which suggested the MSF current was internal in nature. It is very likely that nonlinear interaction of the $S_2$ and $M_4$ internal tide was causing most of the observed MSF current. The tides were obviously having a significant influence on the sub-tidal circulation.

Stacey and Zedel (1986) made an estimate of how much energy is removed by the small scale processes in the immediate vicinity of the sill of Observatory Inlet by developing a three-layer, hydraulic model of the flow over the sill. The model was hydrostatic so it could not reproduce the small scale internal waves which depend on nonhydrostatic effects for their existence, but it could produce the hydraulic jump in the lee of the sill. The velocity $u$, and thickness $h_i$, of each layer were calculated using the hydraulic equations

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i^2)}{\partial x} = -\frac{\partial p_i}{\partial x} - \frac{\partial p_{i-1}}{\partial x} - \frac{C_D}{h_i} \sum_{n=1}^{i+1} \frac{1}{x^2+n^2} |u_i - u_j| |u_i - u_j|$$

(7a)

$$\frac{\partial h_i}{\partial t} + \frac{\partial (u_i h_i)}{\partial x} = 0$$

(7b)

where $p_i$ and $p_{i-1}$ are the pressures due to the surface slope and the interface slope between layers $i$ and $i-1$ respectively (understanding that $p_{i,0} = 0$), and $C_D$ (set = 0.003) is an interfacial drag coefficient. The surface was assumed to be a 'rigid-lid' and the flow was forced at the $M_4$ frequency. During the acceleration of the tidal velocity over the sill, the lower interface of the model could reproduce the shape of the hydraulic flow in the lee of the sill (Figure 5), but when the flow started to decelerate the model could not simulate the observed transition that the flow was observed to go through. The dissipation rate due to the modeled hydraulic flow over the sill was calculated to be no more than about 10%, probably less, of the total rate at which energy is removed from the $M_4$ barotropic tide.

Webb and Pond (1986a) estimated the energy flux in the internal tide from observations of velocity and density in Knight Inlet. They deployed cyclesondes (profiling current meter CTD systems) at four locations in the inlet. The longest (shortest) time series produced were 32 (12) days long, and the cyclesondes profiled from within about 20 m of the surface (so no time series were obtained from within the fresh, surface layer) down to a maximum depth of about 170 m. For a time, two cyclesondes were deployed on opposite sides of the inlet at Tomakstum Island. From the available CTD profiles, Webb and Pond calculated $N(z)$ and the modal amplitudes $\hat{u}(z)$ and $\hat{p}(z)$ [see(6)] for the $M_4$ tide. The multiplicative constants for each up- and down-inlet traveling mode were determined from the observations by fitting in a least-squares sense the $M_4$ amplitude and phase profiles from a given location to the complex series $\hat{u}(y, z)$ and $\hat{p}(y, z)$ defined as

$$u(x, y, z, t) = \sum_{n=0}^{\infty} \hat{u}_n(z) \exp[-yf/c_n + i(\omega t - kx + \phi_n)]$$

$$+ b_n \exp[yf/c_n + i(\omega t + kx + \phi_n)]$$

(8a)

$$\rho(x, y, z, t) = \sum_{n=0}^{\infty} \hat{\rho}_n(z) \exp[-yf/c_n + i(\omega t - kx + \phi_n)]$$

$$- b_n \exp[yf/c_n + i(\omega t + kx + \phi_n)]$$

(8b)

where $u(x, y, z, t)$ and $\rho(x, y, z, t)$ are the along-channel velocity and density respectively at the fixed horizontal position $x$; $f$ is the Coriolis parameter, $c_n = \omega/k_n$ is the phase speed of the $n$-th mode, and $(a_n, \alpha_n = \phi_n - kx \omega)$ and $(b_n, \beta_n = \phi_n + kx \omega)$ are the (amplitude, phase) for the up- and down-inlet propagating waves respectively. The (-) sign appears before $b_n$ in the expression for $\hat{\rho}_n$ in (8b) because $c_n$ (and therefore $k_n$) has been taken to be positive even for the wave traveling in the minus x-direction, so [see(6)] an adjustment has to be made. Webb and Pond included the Coriolis parameter only when considering the two cyclesondes deployed on opposite sides of the inlet near Tomakstum Island. Also, the series $\hat{u}(y, z)$ and $\hat{p}(y, z)$ were truncated, with at most the lowest four modes ($n = 1$ to 4) being fitted to the observations, and with both the up- and down-inlet traveling modes not always being included in the fit. The barotropic $n = 0$ mode was calculated, and then removed, using knowledge of the tidal surface height variation and the inlet geometry. In principle, (8) can be used with the observations from each mooring to find the amplitude and phase of each mode of both the up- and down-inlet traveling waves at the mooring location. Webb and Pond found however that below about 20 m the shape of the mode 1 and 2 profiles were quite similar (Figure 6). They differed significantly only in the surface layer where they had no cyclesonde observations. Because of the similarity of modes 1 and 2, it was difficult for the fitting procedure to separate them, and anomalously large amplitudes were calculated for mode 1 when both its up- and down-inlet traveling component were included in the fit. To remove this 'singularity', Webb and Pond removed from the calculation the mode 1 component that was traveling toward the sill. In fact, five different fits to the observations, each fit including a different combination of modes and up- and down-inlet traveling wave components, were tried in order to obtain a reasonable estimate for the amplitudes and phases. It was found that up-inlet of the sill a reasonable fit to the observations could be obtained when only two up-inlet propagating
modes, modes 1 and 2, were used in the calculation. This result suggested to Webb and Pond that there was relatively little reflection, either at the head or at the bend of the inlet, of the internal tide that propagated up-inlet from the sill. Webb and Pond also found that the $M_2$ energy flux away from the sill at the mooring locations was significantly less than that predicted by the simple model of Stacey (1985). This suggested to them that either the simple model was overestimating the energy flux due to the internal tide or that the internal tide undergoes significant dissipation as it propagates away from the sill.

Webb and Pond (1986b) showed that for a wave of long wavelength (such as a tidal wave) propagating along a channel of constant width and depth, the amount of reflection the wave undergoes upon encountering a bend in the channel is directly proportional to the square of the ratio of the channel width and the incident wavelength. The constant of proportionality increases as the radius of the bend decreases. For a channel of the size of Knight Inlet, their calculation shows that the reflection coefficient for the internal tide at a 90° bend is less than 0.1%. They point out that this very small reflection coefficient is consistent with the finding of Webb and Pond (1986a) that a good fit of (8) to their observations, up-inlet of the sill, can be obtained if only waves propagating away from the sill are taken into account.

The results of Webb and Pond (1986a) are not consistent however with those of Farmer and Freeland (1983) or Freeland (1984) who analyzed data from two current meter moorings deployed on opposite sides of the inlet near Tomakstum Island. Each mooring had two currents meters. Farmer and Freeland (1983) calculated the total $M_2$ velocity (amplitude and phase) at each current meter and then removed the ‘known’ barotropic component to get the $M_2$ amplitude and phase for the internal tide at each current meter. The barotropic component was calculated in two ways: (1) from the sea-surface displacement and the inlet geometry, and (2) from the velocity observations from the four current meters, by using (8a) and by assuming the internal response was dominated by one mode only. They used a simple graphical technique to determine the barotropic tide from the observations at the four current meters. The two techniques yielded the same barotropic velocity. The rest of Freeland and Farmer’s calculation can be described referring to (8a) also. Considering the shallow current meters (at depth $z_n$) on both sides of the inlet (north $N$ at $y_N$ and South $S$ at $y_S$) and assuming only the $I$-th internal mode is important, we have from (8a):

$$\dot{S} = A_N \exp(i\psi_N) = \ddot{u}(z_n)[a_i q(-y_N)\exp(i\alpha_i) + b_i q(y_N)\exp(i\beta_i)]$$

$$\dot{N} = A_N \exp(i\psi_N) = \ddot{u}(z_n)[a_i q(-y_N)\exp(i\alpha_i) + b_i q(y_N)\exp(i\beta_i)]$$

(9)

where $(A_N, \psi_N)$ and $(A_N, \psi_S)$ are the known amplitude and phase of the $M_2$ internal tide at the shallow current meters and $q(y) = \exp(q(y_0)$). From (9) the reflection coefficient $R_{ref}$ becomes

$$R_{ref} = \frac{b_2}{a_1} = \frac{q^2(-y_N) + q^2(-y_S)R^2 - 2q(-y_N)q(-y_S)R\cos(\Delta \Psi)}{q^2(y_N) + q^2(y_S)R^2 - 2q(y_N)q(y_S)R\cos(\Delta \Psi)}$$

(10)

where $R = A_N/A_{ref}$ and $\Delta \Psi = \Psi_N - \Psi_S$. Equation (10) is equivalent to the expression given in Freeland (1984) although he did not use the same mathematical origin for the up- and down-inlet traveling waves. His exact expression is obtained from (10) by letting $q(-y_N) = q(y_S) = 1$ and $q = q(y_S) = q(-y_N)$. Note that one also obtains (10) if the deep current meters are used, although $R$ and $\Delta \Psi$ would then be determined from the deep observations. The advantage of having a deep and a shallow current meter at both moorings is that it allows for an independent determination of the barotropic tide. Note also that if the rotation rate of the earth is not taken into account (i.e., if it is assumed $f = 0$), then $q = R = 1$, and $\Delta \Psi = 0$, and $R_{ref}$ according to (10) becomes undetermined, so even though the inlet is narrow the influence of the earth’s rotation rate is crucial to this calculation. Farmer and Freeland (1983) assumed that the dominant internal mode was mode 1 and obtained a reflection coefficient of 0.93, while Freeland (1984) assumed that mode 2 dominated and obtained a reflection coefficient of 0.76. In either case, it was calculated that significant reflection takes place, in contrast to the conclusion of Webb and Pond (1986a,b).

Marsden and Greenwood (1994) used ADCP observations from Knight Inlet to estimate the energy flux, the dominant internal modes and the reflection coefficient. Their analysis differs qualitatively from the previous analyses in that they could make use of the fact that, although modes 1 and 2 may have similar vertical profiles below the surface layer, their horizontal scales are much different because their...
wavelengths can differ by more than a factor of two. They could take into account this difference in horizontal scale because their ADCP was ship-mounted and continuously collected data as the ship cruised up and down the inlet. There were also locations along the inlet where the ship would stop and where time series of velocity as a function of depth would be collected. And, although their analysis was less general than some previous analyses in that it did not take into account the Coriolis force, it did take into account variations in the width of the inlet, a property of the inlet that they found had a significant influence on the internal tide. Marsden and Greenwood developed the equation

\[
\frac{\partial^4 w}{\partial z^4} + \frac{\partial^2 B(x)}{\partial x^2} \frac{\partial w}{\partial x} + B(x) \frac{\partial^2 w}{\partial x^2} = 0 \quad (11)
\]

where \( w(x, z, t) \) is the vertical velocity and \( B(x) \) is the inlet width. This equation differs fundamentally from (6) only in that variations in the inlet width have been taken into account and that the \((-1)\) within the brackets of (6) has been neglected. They show, following from (11), that the horizontal velocity of a particular tidal constituent can be expressed as

\[
u(x, z, t) = \sum_{n=0} \alpha_n(x, z) \exp(-i\omega t)
\]

\[
= \sum_{n=0} A_n U_n(x) \hat{u}_n(x) \exp(-i\omega t) \quad (12)
\]

where \( U_n(x) = B^{-1/2}(x) \hat{u}_n(x) \exp(-i\omega t) \) and

\[
\frac{\partial^2 \hat{u}_n(x)}{\partial x^2} + \frac{\partial^2 \zeta_n(x)}{\partial x^2} = 0 \quad (13)
\]

where

\[
\gamma_n^2 = k_n^2 + \frac{1}{2} \left( \frac{\partial^2 B(x)}{\partial x^2} \right) \frac{3}{4} \left( \frac{\partial B(x)}{\partial x} \right)^2
\]

\[
\hat{u}_n(x) \quad (14)
\]

If \( \gamma_n^2 < 0 \) then \( \theta_n(x) \) is imaginary and the solution is dissipative.

Marsden and Greenwood fit their ADCP velocity measurements to the lowest five modes using an expression equivalent to

\[
u(x, z, t) = \sum_{n=1}^5 B^{-1/2}(x) \hat{u}_n(x, z) \alpha_n \exp[i(\omega t + \theta_n(x) + \alpha_n)]
\]

\[
+ b_n \exp[i(\omega t + \theta_n(x) + \beta_n)]
\]

(18)

If the Coriolis force is neglected and the inlet width is assumed to be constant, (18) becomes essentially equivalent to (8a). In (18) \( \hat{u}_n \) has been given a dependence on \( x \) because Marsden and Greenwood calculated the 'local' profile of \( \hat{u}_n \) at each position along the inlet. Marsden and Greenwood's calculation differs from that of Webb and Pond (1986a) and Freeland (1984) in that they did not fit the \( M_n \) amplitudes and phases calculated from harmonic analysis of the observations, but rather fit the observed time series of velocity directly to (18). The density of observations at given locations along the inlet was most often not great enough to make possible a determination of an amplitude and phase from the observations, although a mean profile could still be estimated and subtracted before doing the fit. Over the month that the observations were collected, the internal tide remained sufficiently stable to make such a fit to (18) possible. Also, since it was estimated that the \( M_n \) tidal velocity was about 9 (30) times more energetic than the \( S_n \) (\( K_p \)) velocity, the observations could be fit to (18) even though it includes only one frequency.

Marsden and Greenwood found that they could resolve modes 1 and 2, the two most important modes, unambiguously and that no a priori assumptions had to be made about whether the waves were up- and/or down-inlet traveling. They found that mode 1 was the most important in terms of energy flux on both sides of the sill and that there was a significant reflection coefficient, 0.8 (0.66) at Protection Point (Tomakstum Island). For mode 2, the reflection coefficient at Tomakstum Island was 0.65. At Protection Point, mode 2 was unimportant. The importance of mode 1 over mode 2, even in summer when there is a very distinct surface layer, is consistent with the simple model of Stacey (1985) who found that during the summer either mode 1 alone or that both modes 1 and mode 2 together could be important in terms of energy flux, depending on the details of the stratification. Marsden and Greenwood calculate a net energy flux away from either side of the sill that is in agreement with the results of the numerical model of Stacey and Pond (1992), to be described later in this paper. Two significant results of Marsden and Greenwood are (1) that the horizontal difference between modes 1 and 2, because of their different wavelengths, can be exploited with the appropriate observations, and (2) that variations in the width of an inlet can have a very significant influence on how the modes propagate. This result, that width variations in inlets can be important, combined with the calculation of Webb and Pond (1986b) that the bend in an inlet need not hinder propagation of the tide, points to the validity of using laterally-averaged, numerical...
models to simulate important aspects of the circulation in fjords.

The laterally-averaged equations can be expressed in cartesian coordinates as

$$\frac{\partial (Bu)}{\partial t} + \frac{\partial}{\partial x} (Bu^2) + \frac{\partial}{\partial z} (Bu w) = - \frac{B}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \frac{B A_H}{\rho} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{B A_V}{\rho} \frac{\partial u}{\partial z} \right) - C_J |u| u \tag{19a}$$

$$\frac{\partial (B\rho)}{\partial t} + \frac{\partial}{\partial x} (B\rho u) + \frac{\partial}{\partial z} (B\rho w) = \frac{\partial}{\partial x} (BK_H \frac{\partial \rho}{\partial x}) + \frac{\partial}{\partial z} (BK_V \frac{\partial \rho}{\partial z}) \tag{19b}$$

$$\frac{\partial}{\partial x} (Bu) + \frac{\partial}{\partial z} (Bu) = 0 \tag{19c}$$

$$\frac{\partial}{\partial x} \left( \frac{\rho |u|^2}{\rho} \right) - \frac{\partial}{\partial z} \left( \frac{\rho |u|^2}{\rho} \right) = -\rho g \tag{19d}$$

where $B(x, z)$ is the fjord width (a function of along-channel position $x$ and depth $z$), $u(x, z, t)$ and $w(x, z, t)$ are the along-channel and vertical velocities, $\rho(x, z, t)$ is the water density, $A_H$ and $A_V$ are the horizontal and vertical eddy viscosities, $K_H$ and $K_V$ are the horizontal and vertical diffusivities, and $C_J(x, z)$ is a drag coefficient. Note that neither the Coriolis force nor turns in the axis of the fjord are taken into account. These are the equations that STACEY et al. (1995) solved although they were manipulated to put them in a form more appropriate for numerical solution; the vertical coordinate was transformed so that very fine vertical resolution could be used in the numerical model even though the tidal range was large. The vertical diffusion coefficients $A_V$ and $K_V$ were calculated using the level 2.5 turbulence closure scheme of MELLOR and YAMADA (1982) (including a correction term of a kind the scheme is well known to require), and the horizontal coefficients were dominated by the typical Smagorinsky parameterization (Smagorinsky, 1963).

STACEY and POND (1992) broke the density up into a mean and a fluctuation [i.e., $\rho(x, z, t) = \bar{\rho}(x, z) + \rho'(x, z, t)$] and, assuming $|\omega \phi_p / \partial z| \gg |\omega \rho / \partial z|$, simplified (19b) to

$$\frac{\partial \rho'}{\partial t} + w \frac{\partial \rho'}{\partial z} = 0. \tag{20}$$

The grid in their numerical model was fixed in space so the vertical grid resolution could not be greater than the tidal range. STACEY and POND (1992) used (19a, c, d) and (20) to simulate the circulation below the surface layer, and the surface layer circulation was simulated using a laterally-integrated, single-layer version of (7). The two sets of equations were coupled through an interfacial shear stress and a pressure gradient. Because the mean density field did not vary with time there was no mixing in the model. $A_H$ was set proportional to the square of the horizontal grid spacing and $A_V$ was set equal to a constant.

STACEY and POND (1992) compared their simulation to the observations of WEBB and POND (1986a) and showed that the along-channel velocity and density fluctuations of the most important semi-diurnal, diurnal and shallow-water constituents, $M_2$, $K_1$, and $M_6$ could be accurately simulated. It was also found that when the advective terms in (19a) were dropped that $M_2$ and $K_1$ were still well simulated, but that $M_6$ became significantly underestimated. This result suggests that the nonlinear aspects of the flow are not crucial for accurately simulating the semi-diurnal and diurnal internal tide. It was also found that the energy flux of the most important constituent ($M_2$) along the inlet was not significantly influenced by the presence of the advective terms (Figure 7). It is also clear from Figure 7 that the energy flux due to the internal tide originates at the sill and that after increasing in magnitude for a short distance away from the sill begins to decrease in magnitude. The fluxes simulated by Stacey and Pond agree with those determined by MARSDEN and GREENWOOD (1994).

STACEY et al. (1995) allowed the mean density field to vary, so that the sub-tidal portion of the circulation caused by mixing could also be simulated. They compared their simulations to data that were collected during approximately month long experiments, during March and April 1988 (a period of relatively low runoff) and during June and July 1989 (a period of relatively high runoff). The comparison showed that the model could simulate a number of aspects of the sub-tidal as well as tidal component of the circulation. The winds were included in the simulation and had a significant influence on the sub-tidal circulation, but that topic is beyond the scope of this paper.

DE YOUNG and POND (1987) fit cyclesonde observations from near the sill and from in the basin of Indian Arm (Figure 1A) to (8), assuming $f = 0$, in order to estimate the energy flux due to the $M_2$ and $K_1$ internal tide. They, like WEBB and POND (1986a), also had to deal with the difficulty that below about 20 m the vertical structure of the lowest modes are
similar. They found that with few exceptions mode 1 was the most energetic and that their fits of (8) to the data were best when both up- and down-inlet and modes 1 and 2 internal waves were allowed for. They found that the energy flux near the sill (about 0.15 MW) was significantly larger than the flux about 8 km from the sill (about 0.03 MW) indicating to them that the tide undergoes significant dissipation as it propagates away from the sill. [Note that this is consistent with the results of STACEY and POND (1992) for Knight Inlet.] However, based on their fits at the basin station, the down-inlet energy flux was 20–30% of the up-inlet flux, so not all of the wave energy was dissipating before the wave could reflect. This reflection makes resonance of the internal tide possible. In fact, de Young and Pond found that the relative amounts of energy in the $M_2$ and $K_1$ internal tides changed dramatically during the course of their experiment (Figure 8a), in a way that was consistent with changes in the estimated resonant period in the inlet (Figure 8b). Note how the resonant period for the mode 1 wave changed from 12 h to 24 h between November 1983 and February 1984. Resonance in Indian Arm had been predicted earlier by DUNBAR (1985), and DUNBAR and BURLING (1987) who developed a numerical model of Indian Arm based on the equations (19). STACEY et al. (1991) confirmed the ability of their model to simulate the tides in Indian Arm.

The Burrard Inlet/Indian Arm system differs significantly from Knight and Observatory Inlets in that the sill for Indian Arm (basically all of Burrard Inlet) is long. Referring to their own work and the work of other’s, de Young and Pond (1989) note that about 18% of the barotropic $M_2$ tidal energy entering Burrard Inlet is not reflected back out of the system, while for Knight and Observatory Inlets the percentage is much less, never being more than about 2% and 4% respectively. The reason for this difference is that frictional dissipation at First and Second Narrows in Burrard Inlet (Figure 1A) is very important and removes much more energy from the barotropic tide than does the internal tide in Indian Arm. In Knight and Observatory Inlets, frictional dissipation is much less important and the internal tide dominates the rate at which energy is removed from the barotropic tide. Based on their modal fits, de Young and Pond estimate that the energy flux into Indian Arm is about 3% of the available barotropic energy, a percentage for the internal tide that is consistent with the results for Knight and Observatory Inlets.

De Young and Pond (1989) used cyclesonde observations to also estimate the energy flux due to high frequency internal waves in Knight and Observatory Inlets and in Burrard Inlet/Indian Arm. (A cyclesonde was deployed in Observatory Inlet for about one month during August 1982.) For all three inlets, the high frequency waves were estimated to account for less than 5% of the total energy flux. This percentage is consistent with that found for Observatory Inlet by Stacey and Zeedel (1986).

Sechelt Inlet (Figure 1D) was studied by Timis (1995), and Timis and Pond (2001). The phase shift in the $M_2$($K_1$) surface height across the sill was determined to be about 62° (42°), much higher than the values found for the other fjords. Energy was calculated using (5) to be removed from the $M_2$($K_1$) barotropic tide at a rate of about 29 (14) MW, which represents 84% (99%) of the total available power. Timis and Pond deployed two cyclesonde moorings, with Anderaa current meters (with temperature and conductivity) below them at 30 m intervals where necessary, and a surface mooring with five S4 vector averaging current meters (with temperature and conductivity) in the upper 12 m, so that the circulation in the surface layer was also observed. They collected two data sets of two months duration each (from late January until late March and late April until late June 1991) over the entire water column, so the orthogonality of the internal modes could be exploited when calculating their complex amplitudes [i.e., the least squares technique used by Webb and Pond (1986a), and De Young and Pond (1987) was not needed]. For example, for a given tidal constituent one has $u(z) = \sum_{m=1}^{\infty} A_m \hat{u}_m(z)$, where $\hat{u}_m(z)$ is given by (6), $A_m$ is the complex amplitude of the n-th mode, and $u(z) = a(z) \exp[i\omega(z)]$ is determined from harmonic analysis of the observations. From the orthogonality of the modes one has

$$A_n = \int_0^H \hat{u}_n(z)u(z) \, dz \quad (21)$$

since (assuming the modes are normalized)

$$\int_0^H \hat{u}_n(z)\hat{u}_m(z) = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m. \end{cases}$$

Once the $A_n$ and the corresponding complex amplitudes for the density have been determined, one can express each mode as the superposition of an up- and a down-inlet traveling wave, using expressions that are easily deduced from (8), and then calculate the energy flux for each mode. Timis (1995) found that this method and the least-squares method [see (8)] yielded very similar results. Because he had observations throughout the water column the modes were easy to differentiate and no assumptions about which modes were important and/or which were reflected had to be made when using the least-squares method. It was found that mode 1 of the $M_2$ ($K_1$) constituent had a reflection coefficient of about 0.4 to 0.8 (0.4 to 0.6) depending on the stratification. When only a sub-
set of the observations were used to do the least squares fit, all of the problems encountered by WEBB and POND (1986a) and DE YOUNG and POND (1987) re-appeared.

Tinis and Pond found that the energy flux due to the internal tide was never more than about 0.2 MW, much less than the barotropic flux determined using (5). The large phase change in surface height across the sill suggests that friction may be important and this is indeed what Tinis and Pond found. They had access to observations of currents made directly over the sill and used them to determine a drag term by balancing the pressure gradient across the sill with the frictional stress:

\[
\frac{1}{\rho} \frac{\partial p}{\partial x} = -\frac{g \Delta \eta}{L} = \frac{1}{H + \eta} (\delta + \gamma u + C_p u |u|)
\]  

(22)

where L is the channel length along the sill, \(\delta\) is an offset to account for the possibility that the two tide gauges used to determine \(\Delta \eta\) (the change in surface displacement along the sill) might not be leveled, and \(\gamma\) and \(C_p\) are coefficients in the linear-quadratic drag term. (Tinis and Pond also took into account the possibility that the local acceleration \(\partial u / \partial t\) can influence the dynamic balance when the velocity is small.) The three constants, \(\delta\), \(\gamma\) and \(C_p\), were determined by fitting (22) in a least squares sense to the available observations of \(\Delta \eta\) and \(u\) made during numerous flood tides. Because of the choking that occurs in the region of the sill, Tinis and Pond found that \(\Delta \eta = 1\) m was not uncommon. The power loss (dissipation) due to friction for each flood tide was calculated using

\[
P_{\text{loss}} = \rho W L (\gamma |u|^2 + C_p |u|^3)
\]  

(23)

where \(W\) is the channel width, and was plotted against the energy flux calculated using (5). The plot is shown in Figure 9 and one sees that the barotropic energy flux is balanced essentially completely by the power loss due to friction at the sill. Note that for individual events, which occurred during spring tides, energy was removed from the barotropic tide at a rate approaching 100 MW. This result is much different than that found for Knight and Observatory Inlets where much of the energy goes into the internal tide. Friction is even more important for Sechelt Inlet than for Indian Arm which has all of Burrard Inlet as an extended sill. The currents can be as large as 8 m/s over the sill of Sechelt Inlet, which has a mean depth of only 15 m, evidently resulting in very substantial frictional dissipation at the sill. The findings for Sechelt Inlet are consistent with those for the other fjords however in that the generation of small scale processes, such as high frequency internal waves and the tidal jet, account for no more than about 5% of the net energy flux of the barotropic tide. However in Sechelt Inlet only 0.5% is going into the internal tide which is one-tenth of the loss due to small scale processes.

STACEY and GRATTON (2001) simulated the circulation in the Saguenay Fjord (Figure 2) with the same model that STACEY et al. (1995) used to simulate Knight inlet, using for initialization and comparison current meter (CTD) data collected in 1979 (1992). The Saguenay study differs from the others in that the influence of more than one sill on the circulation was studied. Stacey and Gratton started with a representative but horizontally homogeneous density field, and simulated the circulation for a period of thirty days. The resulting mean density field, averaged over the thirty days of the simulation, showed that more mixing had occurred in the outer than in the inner basin. The reason for this extra mixing is that the tides are more vigorous in the outer basin (Figure 10). This extra mixing causes the density at depth in the outer basin to be less than in the inner basin, and the resulting horizontal pressure gradient causes a current across the inner sill from the inner to the outer basin. There is evidence of this flow in the current meter data collected in 1979, and also in the more recent data collected by BELANGER (2004). According to the model, energy is removed from the \(M_2\) barotropic tide at a rate of about 50–60 MW, depending on the initial density fields that are used in the simulations. Figure 11 shows the simulated net energy flux of \(M_2\) tidal energy.
along the fjord. (The inwards flux at the open boundary is somewhat less than the 50–60 MW cited above primarily because there is a counteracting flux due to the internal tide there.) Note that the energy flux tends to be concentrated in the outer basin. The energy fluxes in the outer basin are larger than those found even by Tinis and Pond (2001) for the $M_2$ tide. In this case however most of the energy is calculated to be fed into the internal tide, although about 25% of the net energy flux into the fjord is dissipated by friction within about 2 km of the outer sill. The diffusive flux of tidal energy away from the sill, which includes in a very crude way the contribution to the energy flux of high frequency waves etc., is never larger than about 0.4 MW.

**SUMMARY**

Studies on how tidal energy is partitioned in five Canadian fjords have been reviewed. For all of the fjords discussed, the tides are the dominant external energy source for driving the circulation and for mixing. The net flux of barotropic tidal energy into the fjords has been estimated primarily from the phase change in surface height across the sills and from numerical models. This phase change can vary from only a few degrees (Knight Inlet, Observatory Inlet, Saguenay Fjord) to more than 60° (Sechelt Inlet) depending on the stratification and the fjord’s geometry. This difference means that the percentage of the available barotropic tidal energy that stays (and eventually dissipates) in the fjord can vary from only a few percent to over 80%. Broadly speaking, there have been found to be two important sinks for the barotropic tidal energy; friction and the internal tide. Which of these two processes dominates can vary significantly between fjords. In the fjords for which the phase change is only a few degrees, the internal tide was found to be the most important, whereas frictional dissipation is by far the most important in Sechelt Inlet. In Burrard Inlet/Indian Arm the phase change is somewhat less than 20° and frictional dissipation in Burrard Inlet accounts for a larger energy flux than does the internal tide in Indian Arm. Even so, there is a noticeable internal tide in Indian Arm, and also in Sechelt Inlet. In the fjords for which the internal tide dominates, the rate at which energy is removed from the barotropic tide depends on the stratification. This was shown for Knight and Observatory Inlets and it probably holds for other similar fjords also.

The energy flux due to the internal tide was determined by doing modal fits to observations of along-channel velocity and density from cyclesondes, S4s and ADCPs. When only cyclesonde observations were available, the circulation in the surface layer could not be observed and, because the lowest and most important modes are similar in shape below the surface layer, it was therefore difficult to resolve the modes. Because of this limitation, subjective choices had to be made about which modes to include in a fit and about which modes were reflected back towards their source. For Knight Inlet it was found that a fit that allowed for no reflection produced results that were as acceptable as others based on other assumptions (Webb and Pond, 1986a). One possible cause of reflection up-fjord of the sill, the bend in the fjord, was shown not to cause significant reflection (Webb and Pond, 1986b). However, another analysis (e.g., Freeland, 1984) suggested significant reflection of the internal tide, in contradiction to the suggestion of Webb and Pond. This contradiction has been tentatively resolved by Marsden and Greenwood (1994) who, using ADCP observations of the along channel velocity field, resolved the important modes by taking into account their horizontal variability. Their calculations suggest that significant reflection occurs because of variability in the width of the inlet.

The energy fluxes calculated by Marsden and Greenwood were in agreement with those of the numerical model of Stacey and Pond (1992). The model simulated the tidal circulation quite accurately, as did the more sophisticated model of Stacey et al. (1995) which could also simulate the subtidal circulation. The observations to which the model of Stacey et al. (1995) was compared are much more complete than those used by Webb and Pond (1986a), but they have not yet been used to determine the tidal energy fluxes in the fjord.

De Young and Pond (1987) used cyclesonde observations to estimate the energy flux due to the internal tide in Indian Arm, and their modal fits suggest that, although the internal tide undergoes significant dissipation, there is reflection of the internal tide. There is evidence that the internal tide can at times undergo resonance when the stratification causes the natural frequency of Indian Arm to be similar to that of the diurnal or semi-diurnal tide.

The internal tide is estimated to undergo significant reflection in Sechelt Inlet also (Tinis, 1995). Tinis had access to observations within the surface layer (from $S_4$s) as well as from cyclesondes and Aanderas below the surface layer, so the important modes could be resolved. A priori assumptions about which modes are important did not need to be made.

For all of the fjords investigated in this paper, the small scale internal motions in the immediate region of the sill have been found to be responsible for no more than about 5–10% of the net tidal energy flux. These are still interesting and important motions however and they provide significant ob-
sorvational and modeling challenges. The small scale waves in the immediate vicinity of the sill are nonhydrostatic, so the hydrostatic models commonly used to simulate the internal tide are not appropriate for simulating the small scale motions. A thorough description of the research of this kind in Canadian fjords is beyond the scope of this paper, but it should be mentioned that Knight Inlet in particular has been heavily studied in this regard. Some recent papers include Farmer and Armr (1999a,b; 2001), Afanasiev and Peletier (2001a,b) and Cummins (2000). Isachsen and Pond (2000) is a recent study of mixing through constrictions in Burrard Inlet.

The sub-tidal circulation is also not thoroughly described in this paper, but it too is important. The estuarine circulation and deepwater renewal are important sub-tidal components of the circulation in fjords. Two relatively recent studies of the sub-tidal circulation include Baker and Pond (1995) (Knight Inlet) and Stacey, Pieters, and Pond (2002) (Indian Arm/Burrard Inlet). Pickard (1975) describes annual and longer term variability in a number of west coast fjords. To completely understand this component of the circulation, the tides, the winds and freshwater runoff must all be taken into account, and the role of mixing must be accounted for. How the tides influence the sub-tidal circulation in the Saguenay Fjord through mixing has been briefly described in this paper, but a general theoretical understanding about how to parameterize mixing in fjords does not presently exist and will be a topic of study for many years to come.

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