Ventilation of Black Sea pycnocline by the Mediterranean plume

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Abstract

We present here results of numerical simulations with reduced gravity model of the Mediterranean plume intruding into the Black Sea. The model has horizontal resolution of 600 m. The scenarios analyzed in the paper aim at quantifying the sensitivity of the plume to the ambient stratification and the fluxes of mass, momentum and buoyancy through the Bosphorus Straits. The simulated plume characteristics are compared against observations. It is found that the mixing of Mediterranean and Black Sea water, as well as the termination depth of the plume, are very sensitive to specific combinations of the governing parameters. The behavior of gravity currents on the shelf and on the continental slope is also studied and the role of topographic control is demonstrated. The relatively large entrainment rate \( \sim 10^{-12} \) compared to the one in the Atlantic Ocean \( \sim 2-3 \), shallow penetration and small deflection to the right caused by the earth rotation are explained as a result of the specific combination of governing parameters, topography routing and ambient stratification. A simple two-component chemical model for the interaction between \( \text{H}_2\text{S} \) and \( \text{O}_2 \) is coupled with the dynamical model in order to investigate the impact of the Bosphorus plume (rich in \( \text{O}_2 \)) on the oxidation of anoxic water. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Gravity currents; Entrainment; Oxidation of hydrogen sulphide

1. Introduction

Gravity currents provide an important mechanism contributing to the ocean water mass formation/transport (Price and Baringer, 1994). Most of the recent studies are focused on the Mediterranean outflow (Price et al., 1993; Baringer and Price, 1997a,b; Jungclaus and Mellor, 2000) and on the outflows through the North Atlantic straits (Jungclaus and Backhaus, 1994). In the case of some semi-enclosed basins (characterized by limited exchange with the ocean), the conditions for deep water formation are too different from the oceanic case, and the corresponding processes themselves have much smaller space-scales but are of paramount importance. This motivates us to address the evolution of a buoyancy plume originating from a strait separating ocean basins with very different thermohaline characteristics. We focus on the Black Sea, which gives a unique test case, characterized by extremely sharp salinity difference with the neighboring basin (the Mediterranean Sea surface salinity is \( \sim 36 \) vs. \( \sim 18 \) in the Black Sea), very strong vertical stratification (salinity of \( \sim 18 \) at sea surface vs. 21.5 at \( \sim 300 \) m), and variable topography in the outflow area (shallow shelf of \( \sim 50 \) m and abrupt change of the depth up to 2000 m in 10–50 km only, Fig. 1).
The Black Sea is a typical example of an estuarine basin where the large river discharge and the limited exchange with the Mediterranean Sea creates an extremely stable vertical stratification, controlled by the density differences between the two seas. The observations do not indicate substantial ventilation of the deep layers, supporting speculations that the deep water has been formed a long time ago, and that the deep layers are currently unaffected by the sinking plume. One major focus in the present paper is the dependence of the termination depth and the entrainment in the Black Sea on the external forcing and stratification.

It is known from the theory of geostrophic currents flowing on a sloped bottom that after exiting the strait, the stream deflects to the right in the northern hemisphere, closely following the isobaths. Most observations have failed to trace the Mediterranean plume to the east of the Bosphorus Straits exit (that is to the right looking in the direction of the outflow), but there is clear evidence that it persistently takes a northwest track within the shelf (Tolmazin, 1985). Recent observations (Yuce, 1990; Latif et al., 1991; Oguz and Rozman, 1991; Di Iorio and Yuce, 1999) explain the westward deflection of the plume as due to the guidance by a narrow underwater channel. It is worth noting here that in the presence of bottom friction (or any drag), the flow tends to cross the isobaths, thus descending into the deeper layers. Also important is the fact that the width of the plume, when it exits the strait (~ 2 km), is much smaller that the deformation radius; therefore, its subsequent adjustment and spreading on the shelf is significantly ageostrophic. However, the fundamental issue of why there is: (1) no pronounced turn to the right after the plume reaches the continental slope, and (2) no further propagation along isobaths (as this is the case with the Mediterranean plume leaving the Gibraltar Straits) is still not resolved. In the present study, we answer this question by showing that after the shelf break, the plume dynamics is dominated by entrainment. In the extreme case of very strong drag (e.g., the entrainment occurring beyond the shelf break in our model), this descent will dominate and there will be little propagation along the isobaths.

Another fundamental issue concerning the mixing mechanisms arises from the fact that the vertical diffusion by itself could hardly explain the mixing in the stagnant intermediate layers (Murray et al., 1991; Özsoy et al., 1993). Now, it is widely accepted that the Mediterranean Sea Water (MSW) entrains cold surface or near-surface water and forms the Black Sea deep water. The importance of entrainment was proved in a very convincing way by Buesseler et al. (1991) and Staneva et al. (1999), who analyzed the observations and model simulations of Chernobyl tracers. The above findings give enough arguments to conclude that the entrainment and the lateral intrusions dominate the penetration of MSW into the interior basin and provide an important mechanism for internal mixing. However, there is still a large
uncertainty (see the beginning of Section 4) about the rates of entrainment and the depth of injections of MSW. The scales of the outflow (it spreads several tens of kilometers from the strait; see Fig. 1) dictates a separate study addressing the small-scale processes governing the gravity currents along the shelf and on the continental slope. The detection of the plume is difficult, particularly when the modified MSW approaches the properties of the Black Sea Water (BSW) on the continental slope (Oguz and Rozman, 1991; Gregg and Ozsoy, 1999; Di Iorio and Yuce, 1999).

The numerical simulation of gravity currents presents an important complement to observations, helping to quantify the dominating physical balances. Although stream tube models (Smith, 1975) are still applied to the outflows from marginal seas (Price and Baringer, 1994), reduced gravity models (Jungclaus and Backhaus, 1994; Simeonov et al., 1997) or fully 3D primitive equation models (Gawarkiewicz and Chapman, 1995; Chapman and Gawarkiewicz, 1995; Jungclaus and Mellor, 2000) give a more complete view of the dynamics of sinking plumes under realistic conditions. As demonstrated by Simeonov (1996) and Simeonov et al. (1997), the use of reduced gravity model in the Black Sea conditions gives a description of the processes over realistic topography, which is more adequate than the one obtained with stream tube models. At the same time, the reduced gravity model consumes much less computer resources than the fully 3D models, which make it possible to run it with very high resolution (600 m in our case). We will illustrate that our model provides a tool-to-trace MSW in the areas where existing observations are sparse and their accuracy is low. Our objectives here is to focus on: (1) the phenomenology of the plume as simulated by the model, and (2) the entrainment and ventilation depth as dependent on external conditions, such as the magnitude of the fluxes in the strait and the vertical stratification. The third objective is to extend the research beyond the pure physical approach by including chemical processes, which are fundamental in the Black Sea. There are indications that the Bosphorus plume and the signals originating from the strait can be clearly detected in the chemical composition of sea water; therefore, the analysis of the spatial distribution of chemical tracers like O₂ is very useful when studying the termination depth of gravity plumes and ventilation processes. This motivates us to address here the role of the Bosphorus inflow in the process of oxidation of anoxic water.

In Section 2, we describe the numerical model and the details of individual simulations. The spatial characteristics of the simulated plume are analyzed in Section 3. The mixing of MSW and BSW is addressed in Section 4, followed in Section 5 by the description of coupled dynamical–chemical model designed to study the spatial characteristics of the ventilation of deep water in the vicinity of the Bosphorus Straits. The paper ends with short conclusions.

2. The model and scenarios

2.1. The numerical model

The numerical model is essentially the same as the one described in the paper by Jungclaus and Backhaus (1994), where it has been used for studies on the outflow through the Denmark Strait. The governing equations of the model are given in Appendix A. The model consists of a turbulent lower layer of height $H$ the plume, underlying an upper layer at rest. The outflow is a density-driven bottom current accelerated by the local gradients of topography. Bottom friction tending to decelerate the flow obeys a quadratic drag law. The corresponding coefficient $r = 3 \times 10^{-2}$ (see Appendix A) has been chosen after carrying out a number of calibration experiments with friction coefficient ranging from $3 \times 10^{-3}$ to $3 \times 10^{-2}$ and is in the ranges determined from observations in the same region (Di Iorio and Yuce, 1999). The major difference between simulations with small and enhanced friction is that in the second case, the flow slightly slows down. However, the model sensitivity to this parameter seems to be much weaker than the sensitivity to the coefficient $c_L$, which controls the interfacial exchange. The model entrainment ensures that the exchange at the interface between the plume and the ambient water depends on the Richardson number, which becomes significant if the current is accelerated by the sloped topography. The coefficients of
turbulent exchange and diffusion are taken as $A_h = A_s = 50 \text{ m}^2 \text{s}^{-1}$, respectively.

The lateral extent of the plume is determined by the movable boundary technique, described by Jungclaus and Backhaus (1994). In the shelf regions of tidal seas, where the local water depth is smaller than the tidal range, some grid points may become “dry” in the ebb period. The same can be applied to the Bosphorus outflow—wherever it is absent, the interface depth and the sea bottom depth coincide. When the density currents reach some point, the interface leaves the bottom and this point is further considered as “wet” point. In the opposite case (a retreat of density current), the “wet” point is further considered as “dry” point. In the opposite case (a retreat of density current), the “wet” point is further considered as “dry” point.

For more details on the numerical formulation of this technique, we refer to the paper by Jungclaus and Backhaus (1994). The Black Sea set-up of the model, as well as the analysis of the dominating balances is described by Simeonov (1996) and Simeonov et al. (1997). The sensitivity of the model response to different topographies and bottom roughness is analyzed by Peneva and Stanie (2000).

2.2. The model domain and boundary conditions in the control run

The model domain (Fig. 2) is resolved with a horizontal grid interval of 600 m. The topography is taken from the bathymetric maps of Latiff et al. (1991), showing that the strait of Bosphorus has an underwater extension in the shelf area of ~10 km. The channel has the same orientation as the strait in the first 8 km. Then it turns to the north, and after that to the northwest, reaching the wide and flat shelf area (Fig. 2b) with a slope of ~1:500. The continental slope has orientation from northwest to southeast and an abrupt change of the depth between 100 and 1000 m occurs in ~10 km only. The model area has almost solid boundary to the south, with a small opening, where the characteristics of the Strait outflow are prescribed. The other three boundaries are open. The model area is extended towards the basin interior with a buffer zone (10 points in each direction), where the horizontal grid size increases linearly up to a maximum value of 27.7 km. This buffer zone is not shown in the figures and is also excluded in the presentation of model results.

At the open boundaries (the boundaries of the large model domain), we prescribe zero normal gradients,

$$\frac{\partial S}{\partial n}_{l} = 0, \quad \frac{\partial T}{\partial n}_{l} = 0, \quad \frac{\partial \sigma}{\partial n}_{l} = 0,$$

$$\frac{\partial \tau}{\partial n}_{l} = 0, \quad \frac{\partial \zeta}{\partial n}_{l} = 0,$$

where $L$ corresponds to eastern, northern and western boundaries, which allows free propagation of the plume (for the notations, see Appendix A).

At the solid boundary $M$ ($M$ corresponds to the southern boundary):

$$U|_M = 0, \quad V|_M = 0.$$

The initial width of the plume is limited by the straits opening (three grid elements), which is smaller than the baroclinic radius of deformation ($\sim 10–20$ km in the Black Sea). The thickness of the plume at the straits exit is specified such that the simulated transport was comparable to the observed one ($10^4 \text{ m}^2 \text{s}^{-1}$; Unluata et al., 1990). This value (corresponding to a net annual outflow of 310 km$^3$ year$^{-1}$) is reached in the model when the thickness of the plume at the boundary is prescribed as 26 m. The salinity in the inflow points is set to 37. This large value (double the ambient salinity) provides the negative buoyancy flux and maintains the slope convection in the model. The temperature in the inflow is 14 °C (Yuce, 1990; Murray et al., 1991; Oguz et al., 1990; Özsoy et al., 1993).

The heat and salt fluxes at the interface are defined as the product of the entrainment velocity and the temperature/salinity differences between the plume and its environment (see Appendix A). The evolution of these fluxes in time shows that the model reaches quasistationary state after ~20 days of integration. Most of this adjustment to forcing and environmental conditions is reached in ~5 days.

The ambient stratification corresponds to the annual mean profiles of temperature and salinity, and is resolved with a vertical discretization of 10 m. These data are used for the entire model domain and are kept constant in time during the integration. The two major topographic features in the outflow zone, the underwater channel and the shelf break are well pronounced in Fig. 2b. Practically, the ambient fluid
acts as a motionless sponge, where the intrusions from the plume terminate. Thus, our results provide diagnostics of different types of penetration of MSW under predetermined stratification. This one-way exchange is an important model simplification.

2.3. Model scenarios

Our study addresses the question of how the characteristics of outflowing water depend upon source water properties and oceanic conditions. We
vary boundary conditions (temperature, salinity and thickness of the outflow) in some ranges corresponding to present-day conditions, as well as to conditions that could have dominated the water mass formation in the past. Thus, we create different combinations of governing parameters by varying MSW temperature and salinity, the thickness of the plume, as well as the ambient stratification, assigning to them realistic (R) and test (T) values. Every individual experiment (Table 1) is identified by the combination of three such letters in the following order: ambient stratification, initial thickness of the plume, and MSW salinity. The smallest number of experiments needed to study the individual and collective impact of boundary conditions on the behavior of the plume is $2^3$. We will refer to the experiment RRR as to control run (CR).

The increase of salinity contrast (between the MSW and the Black Sea ambiance) and the thickness of the outflow at the same time leads to unrealistically large transport. Normally, these parameters are negatively correlated; therefore, in the T experiments, we decided to use salinity values that are two times smaller than the ones in the control run. The R and T stratifications are shown in Fig. 3a,b. To create T-profile, we just decrease the salinity difference between each level and surface level two times. This case of decreased stability of stratification could roughly represent the oceanographic conditions in the paleo-Black Sea, which were established under different (from present-day) freshwater flux and strait exchange. The extreme of this experiment is the one with homogeneous vertical stratification and is denoted as HRR (Homogeneous ambient fluid, R-thickness of the plume and R-salinity contrast in the outflow location). This experiment aims at simulating the bottom boundary currents in the transition period after the reestablishment of the connection with the Mediterranean Sea. At this time, the Black Sea was almost a freshwater lake with oxic conditions and deep ventilation. The weak stratification did not present an obstacle for the gravity currents

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R, real; T, test; H, homogeneous.

Fig. 3. Ambient fluid stratification: (a) salinity (solid) and temperature (dashed line) in RRR experiment; (b) density in R-experiments (full line) and in T-experiments (dashed line).
originating from the strait, and the MSW reached the deepest levels. We admit that the processes in the real sea could have developed quite differently (with very slow increase of the undercurrent); however, there are recent speculations about possible catastrophic scenarios (Ryan et al., 1997), and the experiment HRR roughly corresponds to such a situation.

We calibrate the model parameters in the control run (see Section 2.1 and Appendix A) in a way to obtain the best agreement with observations (the data from observations are given with stars in Fig. 2a). These results (taken from literature sources) are probably not enough for precise model tuning, but they could give us at least a first-order estimate for such important parameters of the plume as width, thickness, extension, salinity contrast. Thus, the simulations in RRR are supposed to be the “perfect” ones, to which we will compare the results of our sensitivity experiments (Table 1). The model parameters (friction and mixing coefficients) are kept the same in all experiments.

3. Spatial characteristics of the plume

The plume has the form of a thin wedge with well defined boundaries along the underwater canyon and on the continental shelf. The thickness ranges from 1 m at the edge of the plume to 17 m in the channel (Fig. 4c). These values might seem slightly different from the estimates based on observations (10 m in the channel; Latif et al., 1991), but we must keep in mind that in the reduced gravity model, the temperature and salinity are homogeneous in the vertical and are bounded by material interface, whereas in the observed data, there is a continuous transition between the dense bottom layer and the ambient fluid. Thus, the calculation of the interface depth is subject to interpretation.

To give an idea about the agreement between observations and simulations, we refer to Fig. 2a where some bottom salinity values measured by Latif et al. (1991) are given, as well as their estimates about the most probable path of MSW. Both independent estimates show good agreement, indicating that the model is well tuned to the observations. The plume turns to the right on the flat shelf, which is an indication that the Coriolis force starts to dominate the solution in this region (compare also with Fig. 11 of Yuce, 1990, Fig. 4 of Latif et al., 1991, and Fig. 4 of Di Iorio and Yuce, 1999). The curvature of the right turn is comparable to the one corresponding to the Rossby radius of deformation. The effluent is thin in this area, reaching ~ 3 m in the simulations (Fig. 4c) vs. 2 m in the observations reported by Latif et al. (1991). Taking into account the number of model simplifications, this agreement seems rather good.

As the current flows onto the continental slope (steepness ~ 1:10–20), its topographic acceleration increases (Fig. 4d), which is accompanied by strong entrainment. The result is a further reduction of the plume salinity (Fig. 4a), but an increase in its thickness (Fig. 4c). The loss of density contrast on the slope is followed by a deceleration of the plume. However, unlike the case with the Gibraltar outflow, we do not distinguish in the RRR experiment (and also in the observations) a situation where the bottom current follows the isobaths as a quasi-geostrophic flow.

The model plume does not reach the model boundaries (Fig. 4), supporting the observational evidence that it cannot be found to the west of 28°50’E (Oguz and Rozman, 1991). According to these authors, the plume is almost undetectable on the continental slope, which is due to the very small differences between the plume and ambient water properties. In the deeper levels, lateral intrusions dominate the penetration of the MSW, but these processes are not well represented in our model since the plume is arrested to the bottom. For vanishingly small values of density contrast (between the plume and ambient fluid), the plume looses its identity and simulations are not representative for a gravity current. A similar problem exists in the stream tube models: if the current vanishes, the layer thickness must become infinite in order to conserve the flux of water. The loss of identity below some small value of the density difference (Δρcrit = 0.3 density units; Fig. 4e) might serve as a criterion to trace the boundary beyond which the results of the model lose credibility. This number was chosen after analyzing the sensitivity of results against this parameter, and it roughly corresponds to values, beyond which the plume is difficult to be observed in the natural conditions (Gregg and Ozsoy, 1999).
The simulated entrainment velocity (see Eqs. (A6)–(A9) and (A11)–(A13)) and heat/salinity flux show clearly that there are two regions where the mixing is well pronounced (Fig. 5). The first region is in the underwater channel and just after it, and is due to the large contrast between MSW and BSW (Fig. 5d). The second (and major) one is close to the shelf edge/continental slope. Thus, three characteristic areas are noteworthy: the two areas where the salinity fluxes, and hence the effects on plume dilution by entrainment is increased (Fig. 5a,d), and the one situated on the flat shelf, where both salt and water exchange with the ambient fluid are relatively small.

In the case of homogeneous stratification, the gravity plume penetrates much deeper (and farther from the strait). This result may serve as a proof that under specific (but unrealistic for the present day Black Sea) conditions, the model plume can behave similarly to other well known ocean plumes (e.g., the one exiting from the Strait of Gibraltar). Note that the temperature and salinity patterns in the outflow area are completely dominated by the plume dynamics (Fig. 4a), which is demonstrated by the fact that bottom patterns are completely different from the ones of ambiance stratification at bottom (Fig. 2a).

By integrating the heat and salinity exchange between the plume and ambiance over the interface (in the area of the validity of model), we obtain generalized information about the depth ranges where the mixture of Mediterranean and Black Sea surface water is formed (Fig. 5e,f). The dynamic control is extremely strong and both signals follow the same path. The ratio between the corresponding fluxes of buoyancy due to salinity and heat (dashed line in Fig. 5e) ranges between 70 in the upper layers and 15 below 100 m, demonstrating the dominating role of salinity. The main conclusion from Fig. 5e,f is that the exchange between the plume and ambiance constantly increases on the shelf, reaching a maximum at ~100 m. This is actually the shelf edge, which is plotted with the full-contour line in Fig. 5a,d. Below this depth, the interaction between the gravity plume and environment gradually weakens, but small fluxes are still observed down to ~350 m. It is noteworthy that the large net values of exchange on the shelf are mostly due to the large extension of area of the plume at these depths, while the large values beyond the shelf edge are mostly due to the large entrainment. Though the above results show some sensitivity to the parameter $\Delta \rho_{crit}$ (see the dashed and dotted line in Fig. 5f), the major conclusions about the regime of interaction between the Mediterranean and Black Sea waters do not change when different values of $\Delta \rho_{crit}$ are used, particularly in the upper 250 m, where most of the exchange between the plume and ambiance occurs.

In the context of further analyses on the model entrainment, it is noteworthy that the plume can be well identified over a large part of the continental slope by its density contrast with surrounding water. The latter is presented in Fig. 4e, where the area bounded by the isoline $\Delta \rho_{crit} = 0.3$ (the density contrast, which is physically well resolved by the model) and isolath 100 m coincides with the area of enhanced mixing on the continental slope (Fig. 5). In this area of relatively large contrasts, the thickness of the plume is still relatively small (less than 15 m, as seen in Fig. 4c), and the model produces physically correct results. However, the simulations are subject to some caveats since in the deep sea (the real basin), the plume is not necessarily arrested at the bottom (as it is the case in the model) but mixes with the ambiance water at smaller depths. Thus, the limitations introduced by the $\Delta \rho_{crit}$ value actually exclude from the analyses of simulations the depths, below which the real plume cannot be observed on the bottom. In the experiments listed in Table 1, the patterns are similar to those in the control run, but the horizontal gradients, or the contrast at the plume

Fig. 4. Simulations in experiment RRR after 6 days of integration. Dash curves are the 60-, 80-, 100- and 500-m isobaths. (a) Bottom salinity, the contours are 18–36 by 2 salinity units; (b) bottom temperature, the contours are 8–14.5 by 0.5 °C; (c) thickness of the plume, the contours are 2–26 by 2 m; (d) gravity currents, the scale is 0.4 m s$^{-1}$; (e) difference between plume ambient density, the contours are 1–13 × 1 $\sigma$-unit. Masking is done where the density difference is less than 0.3. The area here (and in all following horizontal plots) coincides with the area in Fig. 2; therefore, the geographic coordinates are not plotted.
interface change in different experiments. We will not analyze the individual patterns in all T-cases, but in the next section, we will rather focus on some general characteristics that are specific for each experiment.

4. Mixing and ventilation depth

The idea that the MSW mixes on the shelf significantly with surface water and does not sink to the bottom but to some intermediate depths (Bogdanova,
1961) was widely used to estimate the entrainment rates, defined as the entrained water versus the Bosphorus inflow. The estimates of different authors vary in large ranges—4:1 in Østlund (1974), 1:4 in Boudreaut and Leblond (1989), 3.3:1 according to Murray et al. (1991), 10:1 in the work of Buesseler et al. (1991). Most of them give much larger values than what is currently agreed about the entrainment of MSW in the Atlantic ocean (2–3, Baringer and Price, 1997a). This suggests that the mechanisms of the spreading of dense MSW into the two basins (Atlantic ocean and Black Sea) might differ substantially. One of the aims of the present paper is to analyze the dependency of model entrainment on the forcing and environmental conditions, and to give better understanding on the processes controlling the ventilation of Black Sea anoxic waters.

If we assume that \( Q_b \) km\(^3\) s\(^{-1}\) Mediterranean water with salinity \( S_h \) entrains \( Q_c \) km\(^3\) s\(^{-1}\) cold intermediate layer (CIL) water with salinity \( S_c \) and forms a mixture with salinity

\[
S_p = \frac{\sum S(i,j) H(i,j)}{\sum H(i,j)},
\]

where \( \sum \) is a sum over the plume points (see also Appendix A), then

\[
Q_b S_h + Q_c S_c = (Q_b + Q_c) S_p.
\]  
(3)

The ratio between \( Q_b \) and \( Q_c \) can be used to quantify the entrainment. If we take for the Mediterranean salinity \( S_h = 37.0 \) for the salinity in the CIL \( S_c = 19 \) and for the salinity of the mixture \( S_p = 21.5 \) (we assume that this is the salinity value, below which we cannot detect the plume; see Fig. 4a), then \( Q_c/Q_b \) is \( \sim 6 \). If we assume that \( S_c = 20 \), which would also include mixing with deeper waters (see Fig. 3a), this factor would increase to 10.3. These numbers obtained with the classical methods used in oceanography give us only rough estimates of the entrainment rate. However, they are fundamental for the interpretation of deep ocean mixing. With decreasing the density contrast between the simulated plume and ambient, the simulated entrainment rate tends to increase because mixing with deeper water is included, which is similar to the case in the above bulk estimates.

Our sensitivity analysis demonstrated that by integrating the entrainment velocity over the plume interface, we obtain more precise idea about the model entrainment than by using simple bulk approaches. The evolution of entrainment rate in time (Fig. 6) shows that 20 days are sufficient for the plume to reach quasistationary state (this is achieved usually much earlier).

In the following, we will analyze mixing properties of the plume and their dependence on forcing parameters and ambient stratification. We define the global entrainment rate as the area integrated water flux entrained at the interface between the plume and the ambient fluid normalized by the outflow magnitude:

\[
E(k) = \frac{\sum \sum w_c(i) \Delta s \Delta y}{Q_b},
\]  
(4)

where the integration is taken over the interface between the sill depth and the model level \( z(k) \).

The increase of \( E(k) \) with depth takes place mainly in the upper 110–300 m (Fig. 7); thus, the depth reached by the sinking plume can be roughly estimated as the depth below which the global en-

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Fig. 5. Entrainment velocity \( w_c \) at the interface of the plume (in m h\(^{-1}\), see Eq. (A8)–(A11)). (a) Experiment RRR. (b) Experiment HRR. (c) Heat flux in experiment RRR \( [J \ m^{-2} \ s^{-1} \times 10^2] \). (d) Salt flux in RRR experiment \([kg \ m^{-2} \ s^{-1} \times 10^{-4}]\). Dashed contours represent 60-, 80- and 500-m isobaths. The 100-m isobath is shown with a solid line. Averaged in the different depth intervals over the plume interface area fluxes of (e) heat \([J \ m^{-1} \times 10^2]\) and (f) salt \([kg \ m^{-1}]\). The dashed line in (e) gives the ratio between buoyancy fluxes caused by salinity and temperature \( R_{12} = -g B Q_h / g \alpha T_{c}(\rho_c - \rho_w) \), where \( \alpha = 1.3 \times 10^{-4} \ \degree C^{-1} \) and \( \beta = 7.5 \times 10^{-4} \ \text{psu}^{-1} \) are the thermal expansion and saline contraction coefficients, \( Q_h \) and \( Q_c \) are the heat and salt fluxes (for convenience, the ratio is shown with opposite sign on graph). The numbers for the ratio (x-axis) are read as non-dimensional. The dashed and dotted lines in (f) give an idea about the sensitivity of presentation of results against parameter \( \Delta \rho_{crit} \). These lines (non-dimensional number) give the ratio between entrained water and the inflowing from the strait Mediterranean water (further in the text, we call that ratio entrainment number). Long-dashed line corresponds to \( \Delta \rho_{crit} = 0.5 \), short-dashed line to \( \Delta \rho_{crit} = 0.3 \), dotted line to \( \Delta \rho_{crit} = 0.1 \).
Fig. 6. The establishment of quasistationary state in (a) Rxx experiments and (b) in Txx experiments.

entrainment does not increase substantially anymore (see also the results in Fig. 5f). For brevity, we will refer further to this depth as “termination depth”. Note that the increase of entrainment is small on the flat part of the shelf (~ 2 in the first 100 m in RRR). The maximum salt flux in Fig. 5f occurs above the depth below which the validity of model could become questionable, which ensures the credibility of the major conclusions regarding the Black Sea mixing characteristics.

The model simulates a trend of decreasing depth reached by gravity currents under stronger stratifica-
tion (Fig. 7). Though the entrainment values in three of the experiments plotted in Fig. 7b are higher than what is commonly agreed nowadays about the entrainment in Black Sea, these results are instructive about the consequences which are to be expected under different (from the present day) stratification and outflow conditions. We could roughly say that if the salinity and the outflow thickness were equal to or greater than 37 and 26 m, respectively (the combination $S = 37$ and $h = 27$ m provides strong forcing in the straits exit), and if the stratification were at least half the present stratification (experiment TRR), the plume would reach the very deep bottom. In the

Fig. 7. Global entrainment rate (see Eq. (4)) in: (a) Rxx experiments and (b) Txx experiments (x is R, or T).
following text, we will analyze the global entrainment rate in the individual Txx and Rxx experiments, where the index ‘x’ denotes either T or R.

In all Rxx experiments, the termination depth is smaller than in the Txx experiments (stratification weaker than in the present); however, this dependency changes in a complicated way from experiment to experiment. Similar clearly defined dependence of termination depth on the inflow characteristics also exists: (1) the depth in all xTxx experiments (initial thickness smaller than in the present) is smaller than the one in the corresponding xRx experiments; (2) this depth is smaller in all xT experiments (initial salinity smaller than in the present) relative to the one in the xRxx experiments. The above results could be expected from general theoretical considerations since the deepening of the plume increases as: (1) the stability of stratification decreases, (2) the salinity of the outflow increases, and (3) the thickness of the outflow increases (Fig. 8). However, this dependency is far from linear, as seen from the results of our eight experiments. This is partially explained by the fact that the physics of sinking plumes is governed by the fluxes at the strait exit. The rate of increase in termination depth depends in a complex way on the ambient stratification and the thickness of plume in xTxx and xRx experiments (Fig. 8). The sensitivity is strongest for weaker stratification and thicker outflows (changing depth from 300 m in TRT to bottom in TRR). This dependence is much weaker for conditions that are closer to a contemporary stratification regime (the depth changes from 150 m in RRT to 300 m in RRR and from 125 m in RTT to 180 m in RTR). This demonstrates that without changing substantially the vertical stratification, one cannot expect substantial change of the depth of penetration of gravity currents in the Black Sea.

The above values of the depth of penetration of gravity currents (see also Table 2) approach in some

Fig. 8. Schematic representation of the dependency of entrainment and termination depth on the stratification and outflow parameters. The eight corners of the cube represent the eight combinations of the governing parameters (the first eight experiments listed in Table 1). Stratification changes to the “east” (increase of stability), the thickness of the outflow changes to the “south” (decrease of thickness), the salinity contrast between the outflow and ambient water changes “upwards” (decrease of contrast). The arrows on the axes give the direction in which the depth of sinking increases. At each corner of the cube, the values of entrainment and sinking depth are also given.
Table 2
Simulated plume characteristics in different experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$Q$ [m$^3$ s$^{-1}$]</th>
<th>$E$</th>
<th>$z$ [m]</th>
<th>$Q\Delta S$ [$10^3$ kg s$^{-1}$]</th>
<th>$QU$ [m$^3$ s$^{-2}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RRR</td>
<td>9976</td>
<td>11.2 (2.3)</td>
<td>300</td>
<td>190</td>
<td>2126</td>
</tr>
<tr>
<td>RRT</td>
<td>6590</td>
<td>4.2 (2.3)</td>
<td>150</td>
<td>59</td>
<td>928</td>
</tr>
<tr>
<td>RTR</td>
<td>2825</td>
<td>10 (3.9)</td>
<td>180</td>
<td>53</td>
<td>344</td>
</tr>
<tr>
<td>RTT</td>
<td>1838</td>
<td>5 (3)</td>
<td>125</td>
<td>16</td>
<td>144</td>
</tr>
<tr>
<td>TRR</td>
<td>10,013</td>
<td>25 (2)</td>
<td>$&gt;500$</td>
<td>90</td>
<td>2142</td>
</tr>
<tr>
<td>TRT</td>
<td>6646</td>
<td>11 (1.9)</td>
<td>300</td>
<td>60</td>
<td>943</td>
</tr>
<tr>
<td>TTR</td>
<td>2842</td>
<td>20 (3.5)</td>
<td>288</td>
<td>54</td>
<td>345</td>
</tr>
<tr>
<td>TTT</td>
<td>1862</td>
<td>8 (3.5)</td>
<td>160</td>
<td>17</td>
<td>148</td>
</tr>
<tr>
<td>HRR</td>
<td>13,092</td>
<td>40 (2)</td>
<td>bottom</td>
<td>406</td>
<td>3662</td>
</tr>
</tbody>
</table>

Entrainment numbers in parentheses are for the upper 100-m water column.

experiments the depth beyond which the model is not applicable for present conditions. In the case of RRR experiment, the error is not large, as seen from Fig. 5f. Unfortunately, there is no observational evidence that could serve as criteria for model validity under a quite wide range of parameters examined here. This could motivate further studies with more adequate models. Therefore, the analysis below gives the general trends of sensitivity to forcing, the precise numbers are subject to further investigation.

The increase of termination depth in $xTx$ relative to $xRx$ experiments (the transition from the front to the back side of the cube in Fig. 8) shows a strong dependency on the stratification and salinity of the outflow. The sensitivity is stronger under weak stratification and when the plume is thicker at the outflow location. This is illustrated in Fig. 8 by the arrow on the initial height axis (the direction of strongest increase of the termination depths is shown by the arrows on the main axes).

Now, we will examine the change of termination depth in $Rxx$ relative to $Txx$ experiments. This rate decreases for thinner outflows and when the contrast between outflow and ambient salinity is smaller. The most dramatic sinking (down to the bottom) is observed when switching from RRR to TRR. On the contrary, the stratification does not affect substantially the transition RTT (125 m)–TTT (160 m). This indicates the following very important result. The weak outflows (for the magnitudes, see Table 2) simulated when thin plumes with small salinity contrast intrude the Black Sea shelf are diluted at the very shelf. They rapidly lose potential energy, and do not reach deep levels, even under much weaker stratification than what exists nowadays.

In the following, we will analyze the dependence of model entrainment on the governing parameters (Table 2). In vertically homogeneous fluid (experiment HRR), as well as under weak stratification (experiment TRR), the total amount of entrained water increases monotonically with depth (Fig. 7). This increase is almost linear in the two experiments, but larger in HRR. In most of the experiments listed in Table 2, there is a good correlation between the depth reached by density currents and the entrainment. In all experiments, the entrainment number usually decreases with decreasing thickness of the plume and its salinity at the exit of the strait, and increasing the stratification. The effect of outflow salinity and ambience stratification almost compensate each other in TRT experiment, resulting in entrainment, which is almost equal to the one in RRR. Note, however, that the transports are $\sim 50\%$ larger in RRR (Table 2) than in the relatively less energetic experiment TRT.

The entrainment does not always increase with increasing the initial thickness of the plume as can be seen in the transition between RRT and RTT. Unlike most of the experiments where the entrainment increases when passing from the front side of the cube to the back one, the transition between RRT and RTT experiments has inverse direction (Fig. 8). The bottom boundary currents reach shallower depth in RTT, while the entrainment in this experiment is larger than in the RRT. Recall that the potential energy provided in RTT is much smaller than in RRT and the transports in the two experiments differ considerably. This trend changes the balances in the main driving terms and, consequently, the major physical characteristics of the plume.
The compensation of mutually opposing factors is well demonstrated by comparing the results of RTR and TTT experiments, as well as the results of RRR and TRT experiments (Table 2). The major difference between these experiments is their energy level (the outflow at the exit in the first two experiments is several times smaller than that in the second pair; Table 2). Increased stratification in the RTR experiment compensates for the effect of strong salinity signature of the outflow, thus seen from the viewpoint of transport/entrainment these experiments are similar. The same is valid if the plume in the strait exit is thicker (RRR and TRT experiments). In the latter case, however, the transport is much larger and the plume reaches larger depths (Fig. 8).

5. Oxidation of anoxic water in the outflow region

The formation and evolution of Black Sea anoxic zone is governed by complicated biochemical reactions. However, the problem of their representation in the models is not fully resolved at present. For simplicity, we address here only the process of $H_2S$ oxidation. According to Cline and Richards (1969), the interaction between $H_2S$ and $O_2$ could be parameterized with a kinetic reaction of second order. Thus, a two-component chemical model for the evolution of $H_2S$ and $O_2$ is coupled here with the dynamical model. The corresponding equations are similar to those of temperature and salinity (see Appendix A) and are identical to ones used in the basin-wide model of Stanev (1989):

$$\frac{\partial [H_2S]}{\partial t} + \vec{u} \cdot \nabla [H_2S] + \vec{v} \cdot \nabla [H_2S] + \frac{w_e}{H} [H_2S] - [H_2S]_o = \frac{A_I}{H} \nabla (H \nabla [H_2S]) - K_{H_2S} [H_2S] [O_2].$$

(5)

$$\frac{\partial [O_2]}{\partial t} + \vec{u} \cdot \nabla [O_2] + \vec{v} \cdot \nabla [O_2] + \frac{w_e}{H} [O_2] - [O_2]_o = \frac{A_I}{H} \nabla (H \nabla [O_2]) - K_{O_2} [O_2] [H_2S].$$

(6)

where $[H_2S]$ and $[O_2]$ are concentrations of hydrogen sulphide and oxygen in the plume and $[H_2S]_o$ and $[O_2]_o$ are the ambient properties (Fig. 9a). The

![Image](image-url)
Fig. 10. Tracer patterns in CR: (a) bottom $\text{H}_2\text{S}$ [µM]; (b) bottom $\text{O}_2$ [µM]; (c) difference between ambient and plume $\text{H}_2\text{S}$ [µM]; (d) difference between ambient and plume $\text{O}_2$ [µM]; (e) the product of ambient $\text{H}_2\text{S}$ and $\text{O}_2$ concentrations [µM$^2$]; (f) the product of plume $\text{H}_2\text{S}$ and $\text{O}_2$ concentrations [µM$^2$]. The results are shown for the area of model validity ($\Delta \rho > \Delta P_{\text{crit}}$). The isobaths are plotted as in Fig. 5.
H$_2$S and O$_2$ ambient profiles present adequately the onset of anoxic zone. The corresponding bottom values calculated from these profiles show clearly the decoupling between surface (oxygenated) and deep (anoxic) water, which occurs on the steep continental slope. The decay term in the right-hand side parameterizes the oxidation in the model. Following Cline and Richards (1969) and the simulations of Stanev (1989), the constants in the decay terms are set to $K_{H_2S} = 0.5 \times 10^{-3}$ $\mu$M$^{-1}$ h$^{-1}$ and $K_{O_2} = 3K_{H_2S}$.

The boundary conditions are similar to those for the temperature and salinity, i.e. we assume zero normal gradients at the open boundary (Eq. (1)). The plume concentration of H$_2$S and O$_2$ at the strait opening is prescribed as 0 and 300 $\mu$M, respectively.

The bottom patterns of H$_2$S and O$_2$ after the 6th day of integration (by this time the stationary state is reached; Fig. 6) are presented in Fig. 10a,b. The differences between ambient properties (Fig. 9b,c) and plume concentration of H$_2$S and O$_2$ are also given. Obviously, there are similarities between chemical and dynamical properties (compare with Fig. 4), in particular for the oxygen. The largest differences between ambience and plume values are observed on the continental slope where the mixing intensifies the process of oxidation, thereby “flushing” the H$_2$S below 100–200 m. Note that the patterns in Fig. 10b,d are very similar to the entrainment patterns (Fig. 5), proving that the chemical tracers could be used to detect important physical processes. This is demonstrated by the product of oxygen and hydrogen sulfide concentrations (Fig. 10e,f), showing the area of most intense oxidation. It is clear that the flow of rich oxygen surface water, originating from the strait, displaces the zone of maximum oxidation on the continental slope to deeper levels.

In the context of studies on the ventilation of Black Sea anoxic waters, it is instructive to analyze the fluxes of O$_2$ at the plume interface, calculated as the entrainment velocity multiplied by the difference between concentration of tracers in the plume and ambience. The integrated value of oxygen flux in the whole water column is $2.9 \times 10^{11}$ M year$^{-1}$. This value is $\approx 3$ times larger than the amount of oxygen penetrating into the Black Sea with the Bosphorus underflow (see the formulation of boundary conditions for oxygen and Table 2 for the outflow rate). Our vertical profiles (Fig. 11c) are in a qualitative agreement with the profiles presented in the study of

Fig. 11. Fluxes of (a) H$_2$S and (b) O$_2$ [M m$^{-2}$ s$^{-1}$ $\times 10^{-4}$]. The isobaths are plotted as in Fig. 5. (c) Fluxes of O$_2$ [M s$^{-1}$] integrated in the different depth intervals over the plume interface area.
Konvalov et al. (2001); the latter are based on calculations using a much simpler 1D inverse model. However, the profiles obtained by our simulations are sharper and the flux associated with plume dynamics does not affect depths larger than 350 m. The difference between the estimates based on two different approaches is indicative for the impact of local processes (physics governed by gravity currents over realistic topography). Thus, we could speculate that the mixing in the deep layers simulated in the 1D models might not be (only) directly linked to the signals originating from the strait, but (also) to the diapycnal mixing associated with the circulation, which shapes the vertical stratification basin wide.

The results presented above demonstrate the potential of simulations as a valuable complement to observations when addressing the mechanisms of mixing between Mediterranean and Black Sea waters. The local maxima below 250 m could indicate an intermittence of mixing, which is caused by the rugged topography of the continental slope. The small extensions of the area where most of the exchange of plume water and anoxic water from the strata occur (Fig. 11a) indicate the need to increase the horizontal resolution in further simulations when addressing the details of chemical interactions in the region of the Bosphorus outflow (see also Fig. 10e).

6. Conclusions

The formation of deep water is a fundamental issue in the physical oceanography of the Black Sea, having important implications for internal mixing, circulation mechanisms and biogeochemical fluxes. This issue is crucial for the understanding of the sensitivity of water masses to external climatic and anthropogenic impacts. The changing termination depth of MSW could give an illustration of the evolution of Black Sea physical system from a state typical for a freshwater lake to its present stagnant state. We have analyzed the dependence of this depth on the ambient stratification and parameters of the Mediterranean outflow. Our model simulations show an encouraging agreement with observations. The comparison with other well known oceanic outflows demonstrates the unique behavior of the Mediterranean plume in the Black Sea. Though this outflow is characterized by much denser source water than the Gibraltar one, it sinks to a much shallower depth. With the results of a number of sensitivity experiments, we explained the shallow sinking by the fact that the outflow entrains substantial amount of CIW. This entrainment reduces the salinity of the plume and its density, so that the neutral buoyancy depth is reached at 200–400 m.

The model simulations could help in elucidating the regional physical balances, and to quantify the number of subtle characteristics of the plume. The best agreement with observations (thickness of the plume, bottom temperature and salinity, pathways) are simulated when the entrainment of CIW by the MSW is \( \sim 1 \times (10^{-12}) \). This entrainment rate correlates with other independent estimates (Buesseler et al., 1991), proving that the model is adequately calibrated, and ensures a good agreement between the simulated plume phenomenology and the results from observations. The agreement is better on the shelf and on the shelf edge, where most of the transformation of MSW occurs.

The fundamental difference between the behavior of the Mediterranean plumes in the Black Sea and in the Atlantic ocean is that in the former case, the absence of a pronounced turn to the right after the plume reaches the continental slope and further propagation along isobaths is due to the extremely large entrainment, which causes a rapid loss of potential energy of the plume on the shelf.

We showed that, without substantially changing the vertical stratification in the Black Sea, one cannot expect substantial change of the depth of penetration of gravity currents. Changing the salinity difference between the Black Sea and Mediterranean Sea two times (under present day stratification) can only lead to changing about two times the depth of penetration of the signal from the strait. Thus, it is hard to believe that oscillations in the source salinity may nowadays reach the abyssal plane. Another important result is that when weak outflows (thin plumes) are diluted at the Black Sea shelf, they rapidly lose potential energy and cannot reach deep levels, even under much weaker stratification than the one which exists at present.

Some important properties of gravity currents are well illustrated by the simulations of the oxidation of
H₂S by oxygen-rich surface water. The active dynamics (intense mixing on the continental slope caused by the accelerated plume) results in a displacement of the zone of maximum oxidation to larger depths (Fig. 10e) relative to the case of no outflow dynamics (Fig. 10f). The simulations seem to be a useful complement to other independent studies on the chemical ventilation of Black Sea. The ratio between the entrained flux of oxygen and the direct oxygen flux from the Strait of Bosphorus is several times smaller than the corresponding ratio for water fluxes, thus illustrating the pronounced differences in the efficiency of penetration of different tracers into the deep layers. We remind here that the maximum oxygen flux occurs 50 m deeper than the maximum of salt flux, which might be due to the differences in the ambient stratification of physical and chemical tracers.

Admittedly, there are simplifications in the present study. Some of them are related to the model, e.g., (1) “arresting the plume at the bottom”, or considering it as fully mixed in the vertical, (2) assuming that the upperlying layers are at rest, and (3) using very simple chemical model. Other assumptions are related to the formulation of model driving (we have chosen the simplest driving configuration here). Further model development has to deal with improving the resolution, which is still insufficient in the strait exit area, and combining strait and shelf physics in one model.

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Appendix A. Model equations

We define first the integrated transports:

\[ \vec{V} = (U, V), \quad U = \int_{-D}^{-\xi} \rho d \zeta, \quad V = \int_{-D}^{-\xi} \omega d \zeta, \quad (A1) \]

and averaged velocities:

\[ \bar{u} = \frac{U}{H}, \quad \bar{v} = \frac{V}{H}, \quad H = D - \zeta. \quad (A2) \]

The model equations in vertically integrated form read:

\[ \frac{\partial U}{\partial t} + \nabla \cdot \left[ \frac{U}{H} - A_b H \nabla \frac{U}{H} \right] = f \bar{V} \]

\[ = -g' H \frac{\partial \zeta}{\partial x} \frac{\partial \rho}{\partial x} - \frac{\tau_{by}}{\rho_b}, \quad (A3) \]

\[ \frac{\partial V}{\partial t} + \nabla \cdot \left[ \frac{V}{H} - A_b H \nabla \frac{V}{H} \right] = f \bar{U} \]

\[ = -g' H \frac{\partial \zeta}{\partial y} \frac{\partial \rho}{\partial y} - \frac{\tau_{by}}{\rho_b}, \quad (A4) \]

where \( A_b \) is the coefficient of horizontal turbulent exchange,

\[ g' = \frac{\rho - \rho_b}{\rho_b} g \]

is the “reduced gravity”, subscript ‘a’ denote ambient quantities, and \( \rho_b \) is the reference density. The continuity equation reads:

\[ \frac{\partial \zeta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = w_e, \quad (A6) \]

where \( w_e \) is the entrainment velocity, parameterizing the entrainment of ambient water into the turbulent flow. Bottom stress in the model obeys a quadratic drag law:

\[ \frac{\tau_b}{\rho_b} = r|\nabla|\nabla, \quad (A7) \]

where \( r \) is the dimensionless friction coefficient. Integrated conservation equations for heat and salt read:

\[ \frac{\partial T}{\partial t} + \bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} + w_e \frac{T - T_a}{H} = \frac{A_b^T}{H} \nabla (H \nabla T), \quad (A8) \]

\[ \frac{\partial S}{\partial t} + \bar{u} \frac{\partial S}{\partial x} + \bar{v} \frac{\partial S}{\partial y} + w_e \frac{S - S_a}{H} = \frac{A_b^S}{H} \nabla (H \nabla S), \quad (A9) \]
where $A_h^2$ is the coefficient of horizontal turbulent diffusion. The density,

$$\rho = f(T, S),$$  \hspace{1cm} (A10)

is calculated using a linear equation of state. For the entrainment velocity $w_e$, we use the parameterization given by Jungclaus and Backhaus (1994):

$$w_e = \frac{c_L^2}{S_m} \sqrt{\bar{u}^2 + \bar{v}^2 + \frac{g' H}{S_m}},$$  \hspace{1cm} (A11)

where, $c_L = 0.086$ is a proportionality constant and $S_m$ is the turbulent Schmidt number. The last is defined by the formula given by Mellor and Durbin (1975):

$$S_m = \frac{Ri}{0.725 \left( Ri + 0.186 - \sqrt{Ri^2 - 0.316 Ri + 0.0346} \right)},$$  \hspace{1cm} (A12)

$$Ri = \frac{g' H}{\bar{u}^2 + \bar{v}^2}$$  \hspace{1cm} (A13)

is the Richardson number.

References


