Intensification of Ocean Fronts by Down-Front Winds

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ABSTRACT

Many ocean fronts experience strong local atmospheric forcing by down-front winds, that is, winds blowing in the direction of the frontal jet. An analytic theory and nonhydrostatic numerical simulations are used to demonstrate the mechanism by which down-front winds lead to frontogenesis. When a wind blows down a front, cross-front advection of density by Ekman flow results in a destabilizing wind-driven buoyancy flux (WDBF) equal to the product of the Ekman transport with the surface lateral buoyancy gradient. Destabilization of the water column results in convection that is localized to the front and that has a buoyancy flux that is scaled by the WDBF. Mixing of buoyancy by convection, and Ekman pumping/suction resulting from the cross-front contrast in vertical vorticity of the frontal jet, drive frontogenetic ageostrophic secondary circulations (ASCs). For mixed layers with negative potential vorticity, the most frontogenetic ASCs select a preferred cross-front width and do not translate with the Ekman transport, but instead remain stationary in space. Frontal intensification occurs within several inertial periods and is faster the stronger the wind stress. Vertical circulation is characterized by subduction on the dense side of the front and upwelling along the frontal interface and scales with the Ekman pumping and convective mixing of buoyancy. Cross-front sections of density, potential vorticity, and velocity at the subpolar front of the Japan/East Sea suggest that frontogenesis by down-front winds was active during cold-air outbreaks and could result in strong vertical circulation.

1. Introduction

A large portion of upper-ocean density fronts reside in the mixed layer, directly exposed to atmospheric forcing. Many fronts in the ocean, such as the subpolar fronts of the Gulf Stream and Kuroshio systems and fronts composing the Antarctic Circumpolar Current, are subject to powerful wind stress $\mathbf{\tau}_w$, blowing mostly in the direction of the frontal currents, yielding large, positive kinetic energy input $\mathbf{\tau}_w \cdot \mathbf{u}$ (Oort et al. 1994). Often the winds are accompanied by strong surface buoyancy fluxes. In the case of subpolar fronts in the Northern Hemisphere, these winds originate from the continent and in the winter bring cold dry air in contact with warm water of southern origin, extracting huge amounts of heat (up to 400 W m$^{-2}$) from the surface ocean (da Silva et al. 1994). The combination of the atmospheric forcing and out-cropping of the pycnocline at these fronts permits the formation and subduction of intermediate and mode waters, water masses that retain the heat, salinity, potential vorticity, and chemical properties that were set in the mixed layer. Upwelling and interaction with the atmosphere of these water masses at some point during their journey through the ocean gyres may play an important role in the decadal variability of the ocean–atmosphere climate system (e.g., Latif and Barnett 1994). The first step in understanding this important process is to determine the physical mechanisms responsible for subduction and the intensification of ocean fronts in the presence of strong atmospheric forcing.

Many theoretical and modeling studies on the dynamics of ocean fronts (e.g., MacVean and Woods 1980; Bleck et al. 1988; Wang 1993; Spall 1995, 1997) are founded in the inviscid, adiabatic frontogenesis theory of Hoskins and Bretherton (1972), and therefore neglect mixing by surface fluxes. The frontal model of Hoskins and Bretherton (1972) describes the manner in which a confluent geostrophic flow (which for an oceanic application might represent the collision of western boundary currents at gyre boundaries or, on the mesoscale, confluence by eddy circulations) intensifies an initially weak baroclinic zone via its hori-
zontal deformation field. This process involves the generation of an ageostrophic secondary circulation (ASC) whose convergent flow augments the confluen
cence and leads to the formation of an infinitely strong
front in a finite time. Not only is the ASC responsible
for frontogenesis, but its downwelling branch
determines the subduction rate. Therefore, understand-
ing the mechanics of the ASC is crucial to frontal
dynamics.

Ageostrophic secondary circulations arise at fronts
to keep the alongfront geostrophic flow in geostrophic
balance over subinertial time scales, as is required in
the semigeostrophic approximation (Hoskins 1982).
Advection of density and momentum by confluent
flow tends to push the frontal jet out of a thermal-
wind balance and hence induces an ASC whose spatial
structure is governed by the $\omega$ equation (Hoskins
et al. 1978). Like confluent flow, redistribution of
momentum or buoyancy by small-scale turbulent mix-
ing disrupts the geostrophic balance and, therefore,
likewise drive a geostrophy-restoring ASC (Ellissen
1951).

As mentioned previously, winds over many fronts in
the ocean are mostly down-front (i.e., they blow along
the frontal jet). Down-front winds destabilize the water
column as Ekman flow advects dense water over light
water for this wind orientation. Therefore, at these
fronts the winds as well as the destabilizing surface
buoyancy flux will lead to gravitational instability.
Mixing by gravitational instability redistributes bu-
yancy and can drive an ASC. If this ASC increases
the horizontal density contrast across the front, the poten-
tial for the down-front winds to destabilize the water
column is strengthened. This gives rise to the possibility
of the following frontogenetic scenario: down-front
winds drive convection, mixing buoyancy, and disrupt-
ing the geostrophic balance; the subsequent geostro-
phy-restoring ASC strengthens the front, further en-
hancing the wind-driven gravitational instability, bu-
yancy mixing, ASC, and frontal intensification. It will be
shown in this paper using an analytic theory and high-
resolution nonhydrostatic numerical simulations that
this frontogenetic mechanism does indeed occur and is
an efficient means for frontal intensification and sub-
duction.

The outline of the paper is as follows. First, the semi-
ageostrophic equations for a baroclinic zone forced by
wind stress and surface buoyancy flux will be formu-
lated. Next, the method and solution for the analytic
theory of the wind/buoyancy-driven ASC and its fron-
togenetic effects are presented in section 3. Following
this, the nonhydrostatic numerical simulations will be
detailed in section 4. In section 5 the implications of
wind-driven frontogenesis for the subpolar front of the
Japan/East Sea forced by cold-air outbreaks is touched
upon using observations. The paper is concluded in sec-
2. Semigeostrophic dynamics of a wind-forced
baroclinic zone

Consider a baroclinic zone with a geostrophic flow
$(u_x, v_x)$. As often assumed in frontal studies, the along-
front length scale is supposed to be much larger than the
cross-front length scale so that the geostrophic flow is
nearly two dimensional, dominated by the alongfront flow
(which is arbitrarily chosen to be oriented in the zonal
direction); that is, $u_x \gg v_x$. The properties of the
baroclinic zone are concisely described by three param-
eters: the square of the buoyancy frequency $N^2 = \partial b/\partial z$
[where $b = -(g/\rho_o)p$ is the buoyancy, $g$ is the gravita-
tional acceleration, $\rho_o$ is a reference density, and $p$ is
the density of the fluid]; a quantity proportional to the ver-
tical component of the absolute vorticity of the geo-
strrophic flow $F = f - \partial u_x/\partial y$, where $f$ is the Coriolis
parameter; and a quantity describing the baroclinicity
of the zone expressed through the thermal-wind bal-
ance

$$S^2 = f \frac{\partial u_x}{\partial z} = -\frac{\partial b}{\partial y},$$

(1)

Using the semigeostrophic approximation considering
flow evolution on subinertial time scales (Hoskins and
Draghici 1977), the dynamics of the front is governed by
the geostrophic momentum and buoyancy equations,

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + v_x \frac{\partial u_x}{\partial y} - \frac{f^2}{f} v + \frac{S^2}{f} w = X \tag{2}$$

$$\frac{\partial b}{\partial t} + u_x \frac{\partial b}{\partial x} + v_x \frac{\partial b}{\partial y} = S^2 v + N^2 w = Q, \tag{3}$$

where $(v, w)$ is the ageostrophic secondary circulation
in the $(y - z)$ plane, and $X$ and $Q$ are generic momen-
tum and buoyancy source terms, whose specific form
will be described below. A single equation for the streamfunction $\psi$ (where $v = \partial \psi/\partial z$ and $w = -\partial \psi/\partial y$)
describing the ASC can be obtained by using (1) to
eliminate the rate of change and geostrophic advection
terms of (2) and (3), yielding the time-independent
form of the Eliassen–Sawyer equation (Eliassen 1951;
Sawyer 1956),

$$\mathcal{L}(\psi) = -f \frac{\partial X}{\partial z} - \frac{\partial Q}{\partial y} + \mathcal{J}_x,$$

(4)

where the operator $\mathcal{L}$ is

$$\mathcal{L} = F^2 \frac{\partial^2}{\partial z^2} + 2S^2 \frac{\partial^2}{\partial y \partial z} + N^2 \frac{\partial^2}{\partial y^2} \tag{5}$$

and

$$\mathcal{J}_x = 2 \left( \frac{\partial b}{\partial y} \frac{\partial u_x}{\partial y} + \frac{\partial b}{\partial x} \frac{\partial u_x}{\partial y} \right) \tag{6}$$

is the “geostrophic forcing” reflecting the tendency of
the geostrophic flow to destroy itself (Hoskins et al. 1978). When momentum/buoyancy sources are neglected from the dynamics, (4) becomes the two-dimensional semigeostrophic ω equation (Hoskins 1982).

The nature of the solutions of (4) is determined by the sign of the discriminant of the operator L (5), which is equal to the two-dimensional potential vorticity (PV) of the geostrophic flow

\[ q_g = f \omega_z \cdot \nabla h = N^2 F^2 - S^4, \]

where \( \omega_z \) is the absolute vorticity vector in the \((y-z)\) plane (Hoskins 1982). The sign of the PV is determined by the relative slopes of surfaces of constant density to those of constant absolute momentum

\[ M = u_x - fy. \]

When the PV is positive, isopycnals are flatter than \( M \) surfaces and (4) is elliptic so that solutions for \( \psi \) decay away from regions of forcing. Solutions for the ASC take on a cellular nature when isopycnals are steeper than \( M \) surfaces, that is, when the PV is negative, as (4) is hyperbolic in this case.

Apart from momentum/buoyancy sources or geostrophic forcing, Ekman pumping also drives an ASC. The Ekman pumping and the Ekman transport

\[ M_e = -\frac{\tau_w}{\rho_0 (f + \xi_z)} \]

(\( \tau_w \) is the alongfront, zonal wind stress applied to the baroclinic zone) is modified by the surface vertical vorticity of the geostrophic flow \( \xi_z = -\partial u_y / \partial y \) at \( z = 0 \). This result, which was first derived by Stern (1965) and Niiler (1969), is a consequence of nonlinear advection of momentum by the meridional Ekman flow \( v_e \). In deriving (9) it has been assumed that the momentum source/sink \( X \) represents a wind-induced frictional force confined to the Ekman layer of thickness \( \delta_e \). It has also been assumed that the Ekman layer is much thinner than the vertical scale \( D \) of the geostrophic flow; that is,

\[ \delta_e \ll D. \]

This approximation greatly simplifies the theoretical analysis by allowing the ASC to be split into Ekman layer and interior solutions. In this paper, geostrophic flows with vertical scales comparable to the mixed layer depth \( h \) are considered. With these vertical scales in mind, approximation (10) becomes \( \delta_e \ll h \). Although it is conventionally assumed that the mixed layer and Ekman layer depths are equivalent, this need not be true for fronts with deep mixed layers, as is the case for fronts associated with the Antarctic Circumpolar Current, where the mixed layer depth can reach 500 m (Rintoul and England 2002). For mixed layers less than 100 m deep, direct observations of wind-derived ageostrophic transport indicate that wind-driven momentum can penetrate through the base of the mixed layer (Chereskin and Roemmich 1991; Wijffels et al. 1994; Lee and Eriksen 1996). Therefore, the theoretical analysis and use of approximation (10) may be more applicable to fronts with deep mixed layers. Having stated this caveat however, it should be pointed out that no approximation for the relative sizes of the mixed layer and Ekman layer depths is required for the numerical simulations of section 4 so that numerical experiments with Ekman depths of the same order as the mixed layer depth can and will be performed to see how critical approximation (10) is to the frontogenetic mechanism detailed in the theoretical analysis.

For the theoretical calculation, Ekman pumping/suction sets the upper boundary condition on the interior ASC \( \psi \),

\[ \frac{\partial \psi}{\partial y} \bigg|_{z=0} = \frac{1}{\rho_0 (f + \xi_z)} \frac{\partial \tau_y}{\partial y} - \frac{\tau_x}{\rho_0 (f + \xi_z)^2} \frac{\partial \xi_z}{\partial y}, \]

and is nonzero even for the case of spatially uniform winds owing to meridional variations in the surface vertical vorticity.

Meridional Ekman flow likewise advects buoyancy. As shown by Thomas and Rhines (2002), for thin Ekman layers for which (10) is true, vertical mixing of buoyancy within the Ekman layer dominates over local rate of change of buoyancy on subinertial timescales and hence balances this lateral advection. The resulting approximate buoyancy equation in the Ekman layer is

\[ \frac{\partial b}{\partial y} \bigg|_{z=0} = -\frac{\partial F^B_e}{\partial z}, \]

where \( F^B_e \) is the buoyancy flux associated with vertical mixing in the Ekman layer. In (12), the lateral buoyancy gradient has been evaluated at the surface owing to its negligible vertical variation across the Ekman layer in the limit where \( \delta_e \ll D \). Integrating (12) down from the surface through the Ekman layer, one obtains an expression for the flux of buoyancy exiting the base of the Ekman layer

\[ F^B_{\text{eff}} = F^B_{\text{atm}} + M_e \frac{\partial b}{\partial y} \bigg|_{z=0}, \]

where \( F^B_{\text{atm}} \) is the buoyancy loss/gain to the atmosphere. The second term in (13), a result which has been independently derived in the context of thermocline ventilation (Marshall and Nurser 1992), wind effects on convection (Straneo et al. 2002), and nonlinear stratified spinup (Thomas and Rhines 2002), is a wind-driven buoyancy flux (WDBF) representing the flux of buoyancy exiting (entering) the fluid beneath the Ekman layer that is required to balance the tendency of lateral advection of buoyancy by the flow in the Ekman layer to reduce (increase) the buoyancy when the wind stress is down-front (up-front).

As mentioned in the introduction, down-front winds with their positive WDBF destabilize the water column
and lead to gravitational instability. Gravitational instability evolves on fast time scales $O(N^{-1})$ and is characterized by overturning cells of small horizontal width of order the mixed layer depth. Therefore, the effects of gravitational instability on the larger-scale, subinertial ASC must be parameterized. Convection is parameterized by a buoyancy sink/source equal to the convergence of the convective buoyancy flux $F^B = w_b b_c$ (where $w$, $b$, are the vertical velocity and buoyancy perturbations of the convection, and the bar represents an ensemble average), that is,

$$Q_c = -\frac{\partial F^B}{\partial z}.$$  

In convective oceanic boundary layers, the convective buoyancy flux exhibits nonlocal behavior; that is, convective plumes cause an upward buoyancy flux whose strength depends more strongly on the surface flux and the thickness of the mixed layer rather than on the local strength of the vertical gradient in buoyancy. The so-called “K profile parameterization” (KPP) of Large et al. (1994) incorporates this nonlocal behavior into their mixing scheme for unstable conditions, parameterizing the turbulent buoyancy flux in the convective limit by the following form:

$$F^B_c = F^B G \left( \frac{z}{h} \right),$$

where $F^B_c$ is the surface buoyancy flux and $G$ is a function describing the vertical structure of the flux in the mixed layer of thickness $h$. In the limit that $\delta < h$, the surface buoyancy flux can be approximated by the effective buoyancy flux of (13); that is, $F^B_c \rightarrow F^B_{\text{eff}}$.

The novelty of the present study is the incorporation of nonlinear Ekman effects [see (9), (11), (12), and (13)] into the dynamics of the ASC. In doing this, processes that drive an ASC, that is, lateral gradients in vertical mixing of buoyancy and Ekman pumping, are no longer solely determined by the atmospheric forcing, but instead are functions of the lateral density contrast and vertical vorticity of the evolving front. In this way, Ekman pumping and the buoyancy source/sink term of (4) are analogous to the geostrophic forcing, and like the geostrophic forcing, it will be shown that they too drive a frontogenetic ASC.

3. Analytic theory

a. Simple model of a baroclinic zone

To illustrate the frontogenetic capability of the WDBF using theory, an analytically tractable, yet physically realizable configuration for a baroclinic zone is adopted. The configuration consists of a laterally homogeneous baroclinic flow $\vec{u}(z)$ in a thermal wind balance with a buoyancy field $\vec{b}(y,z)$, that is, $\vec{S} = fdlH/dz = -\partial \vec{b}/\partial y$ (where $\vec{S}$ is a constant for simplicity; i.e., $\vec{u}$ has uniform vertical shear), embedded in a mixed layer of thickness $h$, bounded below by a pycnocline (see Fig. 1). For the subinertial time scales appropriate to semigeostrophic dynamics, the vertical stratification in the mixed layer is nonzero because of restratification by geostrophic adjustment (Tandon and Garrett 1994). The degree to which small-scale turbulent processes mix zonal momentum determines the mixed layer vertical stratification $N_{\text{ml}}$, yielding values ranging from $N_{\text{ml}} = \frac{S}{f} > 0$ for complete mixing of momentum, to $N_{\text{ml}} = 0$ if the geostrophic flow is unaffected. With this range of stratification, the potential vorticity (PV) in the mixed layer $\eta_{\text{ml}} = N_{\text{ml}} f - \vec{S}$ is less than or equal to zero, that is, $-\frac{S}{f} \leq \eta_{\text{ml}} \leq 0$, owing to the geostrophic shear and lateral density gradient. Within the mixed layer $\vec{S} = \delta \eta_{\text{ml}}/\partial z$ is constant, while beneath the mixed layer, it is specified that $\vec{S}$ grows linearly with depth, increasing by an amount $\frac{S^2}{f} dz$ over a depth $\delta$; that is,

$$\vec{S} = \begin{cases} N_{\text{ml}}^2, & z > -h \\ N_{\text{ml}}^2 - \frac{S^2}{f} \delta (z + H), & z < -h, \end{cases}$$

where $H$ is the thickness of the mixed layer at $t = 0$. This is meant to be a simple representation of a pycnocline whose stiffness is reflected in the smallness of $\delta$. For simplicity, the limit of a rigid pycnocline, that is, $\delta/H \rightarrow 0$ will be considered. In this case, $h = H$ for all times, and the vertical velocity must vanish at the base of the mixed layer. Solutions for flexible pycnoclines can be found in Thomas (2003) and differ only in the
details from solutions for the rigid pycnocline limit discussed below.

Added to this laterally homogeneous flow, which will be referred to as the “basic state,” is a meridionally varying geostrophic disturbance with zonal velocity $u(y, z, t)$ and buoyancy $b(y, z, t)$ meant to represent an incipient frontal jet. To make the analysis amenable to analytical solutions, the disturbance is considered weak relative to the basic state, i.e., $\ddot{u} \ll \pi$ and $\dot{b} \ll \overline{b}$. The objective of the analysis is to determine if the frontal jet grows when exposed to a spatially uniform down-front wind stress $\dot{\tau}_w = \tau_w > 0$. So as to isolate the effects of atmospheric forcing, no geostrophic confinement is imposed; that is, $v_\infty = 0$.

At $t = 0$ the atmospheric forcing is initiated, driving Ekman pumping/suction (11) and an effective surface buoyancy flux (13), which for weak disturbances take the following linearized forms:

$$
\frac{\partial \tilde{\psi}}{\partial y} \bigg|_{z=0} = -M' \frac{\partial \tilde{\psi}}{\partial y} \bigg|_{\tilde{z}=0} \quad \text{and} \quad (17)
$$

$$
F_{\text{eff}} = M' \tilde{\psi}^2 + F_{\text{dyn}}, \quad (18)
$$

where

$$
F_{\text{dyn}} = M' \tilde{\psi}^2 \left( \frac{\tilde{S}_\zeta}{\tilde{S}_\zeta^2} - \tilde{\xi}_s \right). \quad (19)
$$

$M' = \tau / \rho_0 f$ is the magnitude of the classical Ekman transport, $\tilde{\xi}_s = -\ddot{u} / \partial y |_{z=0}$, and $\tilde{S}_\zeta^2 = f \ddot{u} / \partial z |_{z=0} = -\dot{b} / \partial y |_{z=0}$. The effective surface buoyancy flux (18) is mostly spatially uniform but is modulated by the “dynamic cooling” (19), that is, the part of the WDBF that evolves with the disturbance’s surface lateral buoyancy gradient and vertical vorticity, the latter functionality arising from the dependence of the WDBF on the nonlinear Ekman transport (9), which decreases in magnitude in regions of positive vertical vorticity.

As $F_{\text{eff}}$ is positive—that is, destabilizing—convection is induced. The convective buoyancy flux is parameterized using (15), yet a choice must be made on its vertical structure function $G$. In the mixed layer, $F_c$ typically decreases linearly with depth, turning negative if the convection is penetrative as it entrains fluid from the pycnocline (Large et al. 1994). For simplicity, the convection is approximated to be nonpenetrative—that is, its buoyancy flux is zero at the base of the mixed layer:

$$
F_c = \begin{cases} 
F_c^B(1 + \frac{z}{h}), & z > -h \\
0, & z < -h.
\end{cases} \quad (20)
$$

b. Governing equations for weak disturbances

The equations for the evolution of the zonal velocity and buoyancy of a weak disturbance in the mixed layer are obtained by linearizing (2) and (3) about the basic state, assuming that buoyancy sinks/sources are due solely to convection—that is, $Q = Q_c$—and that momentum sources/sinks are confined to the Ekman layer (hence their effects are captured in the Ekman dynamics), yielding

$$
\frac{\partial \tilde{u}}{\partial t} - f \frac{\partial \tilde{\psi}}{\partial z} - \tilde{S}_\zeta^2 \frac{\partial \tilde{\psi}}{\partial y} = 0 \quad \text{and} \quad (21)
$$

$$
\frac{\partial \tilde{b}}{\partial t} - \tilde{\psi} \frac{\partial \tilde{\psi}}{\partial z} - N_m^2 \frac{\partial \tilde{\psi}}{\partial y} = - \frac{F_{\text{dyn}}}{H}. \quad (22)
$$

The streamfunction of the disturbance’s ASC is governed by the linearized form of (4):

$$
f^2 \frac{\partial^3 \tilde{\psi}}{\partial z^3} + 2S_f \frac{\partial^3 \tilde{\psi}}{\partial y \partial z} + N_m^2 \frac{\partial^2 \tilde{\psi}}{\partial y^2} = \frac{1}{H} \frac{\partial F_{\text{dyn}}}{\partial y}, \quad (23)
$$

and is subject to the following boundary conditions: $w$ is equal to the Ekman pumping/suction at the surface [see (17)] and $\tilde{w} = \tilde{\psi} = 0$ for $z \leq -H$ (a consequence of the rigid pycnocline).

c. Solution for the ageostrophic secondary circulation

The ASC is forced by the Ekman pumping/suction (17) and convective mixing of buoyancy, which are functions of the disturbance’s surface vertical vorticity $\tilde{\xi}_s$ and the dynamic cooling $F_{\text{dyn}}$. To obtain a solution for the ASC, a meridional structure for $\tilde{\xi}_s$ and $F_{\text{dyn}}$ must be chosen. The meridional structure of the disturbance is chosen to be sinusoidal with a wavelength $L$:

$$
\tilde{\xi}_s = 2 \pi / \rho_0 f, \quad F_{\text{dyn}} = 2 \pi / \rho_0 f, \quad (24)
$$

Within the mixed layer, the solution for the ASC is decomposed into components driven by convective mixing $\psi_c$ and Ekman pumping $\psi_{cp}$, that is,

$$
\psi = \psi_c + \psi_{cp}, \quad z \geq -H, \quad (25)
$$

while beneath the mixed layer $\psi = 0$. The Ekman pumping driven ASC is a boundary driven homogeneous solution to (23) having the form

$$
\psi_{cp} \sim \exp \left[ \frac{H}{2} \left( y - \frac{S_f^2}{f} z \right) \right] \tilde{g}(z), \quad (26)
$$

reflecting the manner in which the circulation tilts slantwise along the absolute momentum surfaces of the basic state with slope $\gamma = f / S_f^2$. The ansatz (26) transforms the homogeneous form of (23) into an ordinary differential equation for the vertical structure function $\tilde{g}$

$$
\left( \frac{d^2}{dz^2} - \frac{q_{ml}^2}{f^2} \right) \tilde{g} = 0. \quad (27)
$$

Equation (23) written in this form illuminates the crucial role of the PV in determining the vertical structure of the solution. When $q_{ml} > 0$, (27) is elliptic, yielding
solutions which decay away from the surface. In this case, the depth over which the ASC penetrates is the semi-geostrophic Prandtl depth $\delta_{SG} = L/f(2\pi/N_{ml} - S^2/f^2)$. When $q_{ml} < 0$, (27) is hyperbolic and solutions are cellular so that the ASC does not necessarily decay away from the surface. Mathematically summarizing the Ekman pumping part of the solution:

$$\psi_{ep} = \Re\left\{-M_c^{f} \frac{\tilde{\zeta}}{f} \sin[m(z + H)] e^{i\theta y - z/y}\right\},$$

(28)

with $m = \pi L_o/(2HL)$, where

$$L_o = \frac{4H\sqrt{-q_{ml}}}{f^2}$$

(29)

is a length scale of dynamical importance.

The solution for the convective mixing driven ASC $\psi_c$ solves (23) subject to an upper boundary condition of $w = 0$ and is as follows:

$$\psi_c = \Re\left\{\Psi_p \left[1 - \frac{\sin[m(z + H)]}{\sin(mH)} e^{-i\theta y} + \csc(mH) \sin(mz)e^{-i\theta y} \right]e^{i\theta y}\right\},$$

(30)

where the scale factor $\Psi_p$ satisfies the following relationship

$$-il\Psi_p N_{ml}^2 = -\frac{\hat{F}}{H},$$

(31)

which demonstrates how $\psi_c$ is an ASC that balances convective mixing through vertical advection of buoyancy.

d. Evolution equations and stability analysis

Although the spatial structure of the secondary circulation is known, its temporal evolution is as yet unknown. The solution for $\psi$ depends only parametrically on time through the surface vorticity and dynamic cooling, therefore it is the evolution equations of $\tilde{\zeta}$ and $F_{dyn}$ that determine the time-dependence of $\psi$ and the disturbance.

The vertical vorticity is generated by stretching of planetary vorticity (STR) and by tilting to the vertical of the meridional vorticity associated with the basic state geostrophic shear (TILT):

$$\frac{\partial \tilde{\zeta}}{\partial t} = \Re\left\{f \frac{\partial w}{\partial z} \bigg|_{z=0} + \frac{S^2}{f} \frac{\partial w}{\partial y} \bigg|_{z=0}\right\}. \quad (32)$$

The dynamic cooling is affected by both $\tilde{\zeta}$ and $S^2$, the latter of which is governed by the following equation:

$$\frac{\partial S^2}{\partial t} = -S_z \frac{\partial v}{\partial y} \bigg|_{z=0} + \frac{N^2}{f} \frac{\partial w}{\partial y} \bigg|_{z=0} + \frac{1}{H} \frac{\partial F_{dyn}}{\partial y}. \quad (33)$$

Subtracting $M_{ml} S^2 f \times (32)$ from $M_{ml} \times (33)$ yields an equation for the evolution of $F_{dyn}$

$$\frac{\partial F_{dyn}}{\partial t} = \Re\left\{d_{ml} \frac{M_c^{f} \partial w}{f^2} \bigg|_{z=0} + \frac{M_c^{f} \partial F_{dyn}}{H \partial y} \right\}. \quad (34)$$

While the squeezing together of isopycnals by the meridional flow (SQZ) and STR play dominant roles in the dynamics of the surface lateral buoyancy gradient and vertical vorticity, they do not directly affect the dynamic cooling. Only the difference between differential vertical advection of density (DVADV) and TILT, the term labeled DMT in (34), can potentially lead to feedback and growth of the dynamic cooling since differential dynamic cooling (DDC) is in quadrature with $F_{dyn}$.

Equations (32) and (34) constitute a set of coupled differential equations that govern the evolution of the disturbance. Using (24), (25), (28), and (30) to evaluate the terms in (32) and (34) generates a system of coupled ordinary differential equations in time for $\tilde{\zeta}$ and $F$:

$$\frac{d}{dt} \begin{pmatrix} \tilde{\zeta} \\ F \end{pmatrix} = \begin{pmatrix} \text{ilm} M_c^{f} \cot(mH) \frac{f m}{N_{ml}^2 H} & \text{cot}(mH) - \frac{\text{il}}{m \gamma} - \csc(mH)e^{-i(\theta y)} \\ -\frac{\hat{F} d_{ml} (M_c^{f})^2}{f^3} & \frac{\text{il} M_c^{f}}{H} \end{pmatrix} \begin{pmatrix} \tilde{\zeta} \\ F \end{pmatrix}. \quad (35)$$

Substituting a solution of the form $(\tilde{\zeta}_n, F) = (\tilde{\zeta}_n, F^n) e^{\omega t}$ into (35) generates an eigenvalue problem for the growth rate $\sigma$ of the disturbance. In this way, a stability analysis can be performed that determines whether or not a geostrophic disturbance of a particular wavelength introduced to the baroclinic zone grows or decays.

The dependence on $L$ of the growth rate of the fast-growing eigenvector for a basic state with the following parameters: $S^2 = 8.8 \times 10^{-5}$ s$^{-2}$, $f = 9.3 \times 10^{-5}$ s$^{-1}$, $H = 50$ m, $q_{ml} = 0.01 S^2$, and $\tau_o = 0.1$ N m$^{-2}$ was calculated and is plotted in Fig. 2. The growth rate is complex, $\sigma = \sigma_r + i\sigma_i$, and varies greatly with the wavelength of the disturbance and the PV of the mixed layer. For $q_{ml} < 0$, disturbances with wavelengths $L = L_o/(2n + 1)$, with $n$ being an integer greater than or equal to...
zero, are the fastest growing and are quasi-stationary, that is, $|\sigma|^8 \gg |\sigma|$, while propagating disturbances travel with or against the Ekman transport and grow more slowly. Rapidly propagating disturbances have wavelengths $L \sim L_o/(2n)$ and small $\sigma$. Unlike the $q_{ml} < 0$ case, all disturbances in positive PV mixed layers propagate in the direction of the Ekman transport (southward), that is, $\sigma < 0$, and the fastest growing disturbances are those with the shortest wavelengths—that is, there is no scale selection.

When the ratio $q_{ml}S^4$ is fixed, the shape of the growth rate spectrum is unaffected by changes in $S^2, H$, and $\tau_{\omega}$, however, its scale is. Increasing the wind stress $\tau_{\omega}$, thinning the mixed layer thickness $H$, or reducing the background lateral buoyancy gradient $S^2$ all amplify the growth rate by an amount consistent with an advective time scale scaling (that is, $\tau = H/L^2$) for the evolution of the disturbance: that is,

$$\frac{|\sigma|}{f} \sim \frac{1}{\sqrt{\tau}} = \frac{\tau_{\omega}}{\rho_o H^2 S^2 \sqrt{-q_{ml} S^4}} \left( \frac{L_o}{L} \right).$$

(36)

### e. Implications for frontogenesis

The objective of this study was to determine if destabilization of the water column by down-front winds could drive frontogenetic ASCs. It has been shown in the previous section that geostrophic frontal jets introduced into a baroclinic zone forced by down-front winds can propagate and grow. In this section, the physical mechanisms responsible for the propagation and growth of the frontal jets and the implication of these results for frontogenesis are summarized.

Propagation of the frontal jets is caused by vortex stretching associated with the Ekman pumping driven ASC $\psi_{cp}$. Vortex stretching forced by Ekman pumping (17) is in quadrature with the surface vertical vorticity and hence results in the propagation of $\zeta_{ep}$. As demonstrated in Fig. 3 the vertical structure of $\psi_{cp}$, which is set by the PV of the mixed layer and the wavelength of the disturbance, determines the propagation speed and direction. For mixed layers with positive PV, $\psi_{cp}$ decays in the vertical over a Prandtl depth, inducing vortex stretching that lags $\zeta_{ep}$ by $90^\circ$ in $y$ and forces southward propagation, in the direction of the Ekman transport. Reducing $L$ for $q_{ml} > 0$ decreases the Prandtl depth, makes vortex stretching stronger, and hence results in faster propagation (i.e., larger $\sigma$, see Fig. 2). Secondary circulations in mixed layers with negative PV form cellular structures and can decay, remain unchanged, or increase with depth depending on $L$. For $q_{ml} < 0$ and $L = L_o$ [more generally, $L = L_o/(2n + 1)$], surface flow is parallel to $M$ surfaces. Flow parallel to $M$ surfaces neither accelerates zonal velocity nor generates vorticity, so that the disturbance does not propagate. For $q_{ml} < 0$ and $L = L_o/1.75$, $\psi_{cp}$ increases with depth making vortex stretching lead $\zeta_{ep}$, driving northward propagation (i.e., $\sigma < 0$), counter to the Ekman transport.

The components of the analytical solution that are key to the growth of the frontal jets are shown in Fig. 4 using the solution for a disturbance with $L = L_o$ and initial surface vorticity of magnitude $0.0025f$ introduced into a mixed layer with negative PV. Seven inertial periods after the introduction of the disturbance, frontal interfaces have developed in the buoyancy field as a result of the convergent flow field of the ASC. Convective mixing is most intense beneath the fronts and drives an ASC $\psi_{c}$ characterized by downwelling on the dense side of the front, upwelling along the frontal interface, and northward surface flow coincident with the eastward flow of the frontal jets. The vertical vorticity of the frontal jets modifies the Ekman transport and results in Ekman pumping on the dense side of the front and suction/upwelling along the frontal interface. The spatial structure of the zonal velocity reflects the vertical circulation, with high momentum fluid subducted on the dense side of the front and low momentum fluid upwelled along the frontal interface, resulting in a reduction of the vertical vorticity at the front.

A schematic diagram illustrating the role of each key component of the analytical solution in the steps that lead to intensification of ocean fronts by down-front winds is shown in Fig. 5. These steps to frontogenesis are as follows. First, Ekman flux induces a wind-driven buoyancy flux and convective mixing of buoyancy that is concentrated at the front where the lateral buoyancy gradient is largest. Second, localized mixing drives an ASC with northward surface flow at the frontal outcrop, northward flow that accelerates down-wind frontal jets via the Coriolis force. Third, spinup of the frontal jets and associated surface vertical vorticity
strengthens the Ekman pumping/suction, forcing water to downwell to the north of the fronts and upwell along the frontal interface. Last, this differential vertical motion tilts the meridional component of the vorticity downward, reducing the vertical vorticity, and consequently enhancing the Ekman transport at the front. The differential vertical motion also reduces the lateral density gradient; however, for mixed layers with negative potential vorticity, the enhancement of the Ekman transport outweighs the slight reduction in the lateral density gradient so that the wind-driven buoyancy flux and convective mixing experiences a net increase at the front. As a result, the mixing driven ASC is strengthened, the spinup of the frontal jets is accelerated, the Ekman pumping/suction is magnified, and the fronto-genetic convergent flow of the total ASC is intensified. Repetition of these steps leads to the exponential growth of the disturbance and frontogenesis.

4. Nonhydrostatic numerical simulations

To test and extend the results of the theory outlined above, two-dimensional, nonhydrostatic, high-resolution numerical simulations run with the same basic state configuration of section 3a and forced by a spatially uniform down-front wind stress and atmospheric cooling were performed. A series of five experiments designed to cover a range of values of $L_0$, through variation of $S^2$, $H$, $f$, and $N_2^2$ were performed (see Table 1). All experiments were forced by the same wind stress $\tau_0 = 0.1 \text{ N m}^{-2}$ and atmospheric buoyancy flux $F_o = 6.3 \times 10^{-8} \text{ m}^2 \text{s}^{-3}$. The exact configuration of the numerical model is detailed in the appendix.

Listed in Table 1 is the thickness of the Ekman layer $\delta_0$ for each of the experiments. Notice that for none of the experiments is $\delta_0$ very much smaller than the initial depth of the mixed layer and hence the experiments do not strictly satisfy approximation (10) used in the theory. Therefore, a comparison of the numerical and analytical solutions can be used to determine if (10) is critical to the frontogenesis mechanism outlined in the theory.

A Hovmöller plot of $-\psi$ evaluated at the base of the Ekman layer for experiment E is shown in Fig. 6 and illustrates the typical $y-t$ structure of the near-surface

![Fig. 3. Wind stress blowing out of the page and surface vertical vorticity $\zeta_s$ (curves in the top of each panel) induce Ekman pumping/suction and an ageostrophic secondary circulation $\psi_{ep}$ (contours). The ageostrophic secondary circulation $\psi_{ep}$ for mixed layers with (a),(b) positive and (c),(d) negative PV are contrasted. Disturbances with $L = L_0$ and $L = L_0/1.75$ are plotted in the left and right panels, respectively. Arrows indicate the direction of the flow and dashed contours are surfaces of constant absolute momentum.](image-url)
ASC of the numerical simulations. Small-scale convective overturning cells of width the depth of the mixed layer \( H \) fill the domain after a half an inertial period. By two inertial periods, out of this chaos emerge three large-scale secondary circulations characterized by intense downwelling centered at fronts that do not travel with the Ekman flow, but remain stationary.

Meridional sections of the density and streamfunction for experiment E reveal that frontal interfaces form, convection is concentrated beneath and to the north of the fronts, and Ekman transport (southward flow in the Ekman layer, i.e., upper 10 m) is channeled down the frontal interface (Fig. 7a). The zonal velocity of the numerical solution, like the analytical solution (cf. Figs. 4b and 7b) reflects the vertical circulation, with high momentum subducted down the dense side of the front and low momentum upwelled at the base of the frontal interface. Down-wind surface jets develop just to the south of the front in a region where the Ekman flow stagnates or reverses. The lateral structure of the surface expression of the fronts is illustrated in Fig. 8. So as to highlight the robust features of the fronts of experiment E, a composite front has been constructed and is used in the figure. The method of constructing this composite front is as follows: all fronts in a particular meridional section are aligned relative to their maximum lateral density gradient and the particular variable of interest is then averaged both spatially over the number of fronts in the section and temporally over the last two inertial periods of the experiment. The intensity of this composite front is evidenced by the precipitous drop in buoyancy and zonal velocity (which has been averaged in the vertical over the Ekman layer) crossing the front from south to north (Figs. 8a and 8b). The tremendous lateral shear in the zonal velocity gives rise to positive vertical vorticity at the front that greatly exceeds \( f \). With such large vorticity, nonlinear Ekman theory [see (9)] predicts that the front should effectively form a barrier to the Ekman transport. In Fig. 8c, the Ekman transport derived from (9) is compared with \(-\psi\) evaluated at the base of the Ekman layer. The Ekman transport estimated using the vertical vorticity calculated from a smoothed version of the zonal velocity agrees well with \(-\psi\) north of the front. On account of the singularity of (9) at \( \zeta = -f \), the Ekman transport calculated from the zonal velocity without smoothing is scattered, yet drops to zero at the center of the front, similar to \(-\psi\). This sharp shutdown of the Ekman transport generates powerful Ekman pumping and leads to the downturn of \( \psi \) at the fronts shown in Fig. 7a.

South of the front the Ekman transport is lower than \(-\psi\) by 0.5–1.0 m\(^2\) s\(^{-1}\). To account for this discrepancy, there must be a northward transport added to the Ekman flow. This presumed northward flow is coincident with the down-wind surface jets. The Coriolis force associated with the northward flow is in the right sense to accelerate the frontal jet. Hence, in analogy with the analytical theory of section 3e, this flow may be attributable to the convective mixing driven ASC \( \psi_c \) that accomplishes this task in the theory. If this is the case, the downwelling of the northward flow at the front (where it must vanish as the discrepancy between Ekman transport and \(-\psi\) is small) should be a location of enhanced convective mixing. To test this, the convective buoyancy flux \( F^B = \bar{w}_c \bar{b}_c \) was calculated and compared with the wind-driven buoyancy flux of (13).

To calculate the convective buoyancy flux, \( w \) and \( b \) were high-pass filtered in \( y \) to separate \( w_c \) and \( b_c \) from the larger-scale flows associated with the ASC. The product of \( w_c \) and \( b_c \) was then averaged using the compositing method outlined above to obtain \( F^B \). As illus-
trated in Fig. 9a the wind-driven buoyancy flux and $F^B_{\text{eff}}$, evaluated at $z = -7.8$ m, near the base of the Ekman layer, scale well with each other within $\sim 700$ m of the front. Both buoyancy fluxes are concentrated sharply at the front where $\tau^x$ is largest. (b) The horizontal gradient in mixing induces an ASC $\psi$ (gray curves) that accelerates down-wind frontal jets $u_*$ via the Coriolis force associated with the ASC’s cross-front flow. (c) The jets’ vertical vorticity $\zeta_*$ modifies $M_z$, generating Ekman pumping/suction and an ASC $\psi_p$ (gray curves) with downwelling on the dense side of the front and upwelling along the frontal interface. (d) Differential vertical motion by Ekman pumping/suction reduces $\zeta_*$ at the front through tilting of the cross-front component of the vorticity ($\omega_z = i\alpha z/\eta$) associated with the jets’ vertical shear. Reducing $\zeta_*$ enhances the Ekman transport, which strengthens $F^B_{\text{eff}}$ and intensifies the mixing at the front for mixed layers with negative potential vorticity $q_{\text{ml}}$. Repetition of these steps leads to frontogenesis. The most frontogenetic ASCs have cross-front widths $L_\omega = 4H^2 \sqrt{-q_{\text{ml}}}/f^2$ and do not translate with the Ekman transport.

![Fig. 5. Steps to frontogenesis.](image)

- Wind stress $\tau^x$ directed down a front (thick black curves denote isopycnals) drives Ekman flow, with transport $M_z$, that advects dense water over less dense water and results in a destabilizing wind-driven buoyancy flux $F^B_{\text{eff}}$. Convection and turbulent mixing of buoyancy (shaded regions) are concentrated at the front where $F^B_{\text{eff}}$ is largest.
- The horizontal gradient in mixing induces an ASC $\psi$ (gray curves) that accelerates down-wind frontal jets $u_*$ via the Coriolis force associated with the ASC’s cross-front flow.
- The jets’ vertical vorticity $\zeta_*$ modifies $M_z$, generating Ekman pumping/suction and an ASC $\psi_p$ (gray curves) with downwelling on the dense side of the front and upwelling along the frontal interface.
- Differential vertical motion by Ekman pumping/suction reduces $\zeta_*$ at the front through tilting of the cross-front component of the vorticity ($\omega_z = i\alpha z/\eta$) associated with the jets’ vertical shear. Reducing $\zeta_*$ enhances the Ekman transport, which strengthens $F^B_{\text{eff}}$ and intensifies the mixing at the front for mixed layers with negative potential vorticity $q_{\text{ml}}$. Repetition of these steps leads to frontogenesis. The most frontogenetic ASCs have cross-front widths $L_\omega = 4H^2 \sqrt{-q_{\text{ml}}}/f^2$ and do not translate with the Ekman transport.

**Table 1.** Experimental parameters for numerical simulations.

<table>
<thead>
<tr>
<th>Expt</th>
<th>$S^2$ (s$^{-2}$)</th>
<th>$H$ (m)</th>
<th>$f$ (s$^{-1}$)</th>
<th>$\delta$ (m)</th>
<th>$N^2_{\text{ms}}$ (s$^{-2}$)</th>
<th>$\delta$ (m)</th>
<th>$L_\omega$ (m)</th>
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<tbody>
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<td>$1.0 \times 10^{-4}$</td>
<td>10</td>
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<td>2000</td>
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<td>50.0</td>
<td>$1.0 \times 10^{-4}$</td>
<td>10</td>
<td>$3.6 \times 10^{-5}$</td>
<td>5.00</td>
<td>4000</td>
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<tr>
<td>C</td>
<td>$6.3 \times 10^{-7}$</td>
<td>50.0</td>
<td>$1.0 \times 10^{-4}$</td>
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<td>$3.3 \times 10^{-5}$</td>
<td>5.00</td>
<td>5200</td>
</tr>
<tr>
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<td>$1.0 \times 10^{-4}$</td>
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<td>6000</td>
</tr>
<tr>
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<td>$7.9 \times 10^{-5}$</td>
<td>11.3</td>
<td>$5.8 \times 10^{-5}$</td>
<td>5.00</td>
<td>6400</td>
</tr>
</tbody>
</table>
reveals that gravitational instabilities both grow and propagate to the south, with the Ekman flow. The combination of growth and southward propagation of convection leads to the exponential increase of $F_B$ with decreasing $y$. The distance it takes the instability to reach finite amplitude depends on the strength of noise in the buoyancy and flow fields which sets the initial amplitude of the disturbance: the weaker the noise the longer the distance. When convection reaches finite amplitude, $F_B$ scales with the wind-driven buoyancy flux and density inversions are rapidly mixed away. Therefore, the finite distance over which density inversions persist in the experiment, evident in Figs. 7 and 9b, is a consequence of the weak noise level in the numerical model.

A key quantity predicted by the theory is the wavelength of the fastest growing disturbance. As shown in Fig. 6, there is a clear wavenumber selection for the ASC in experiment E. This was true for all the experiments listed in Table 1. To assess whether the distance of separation between fronts and wavelength of the ASC scales with the predicted length scale $L_\alpha$, spectra in $y$ of the streamfunction at the base of the Ekman layer were calculated, and are shown in Fig. 10. For all the experiments, the spacing of the fronts is proportional to $L_\alpha$, with a multiplier of 2/3. It was found that for all the experiments, the mixed layer depth shoaled from its initial value. An example of this can be seen in the depth of penetration of the convective buoyancy flux (see Fig. 9c), which is 30 m versus the initial mixed layer depth of 50 m. Since $L_\alpha$ is proportional to the mixed layer depth, it is conceivable that the reduced wavelengths of the ASCs in the numerical experiments are associated with the thinner mixed layers. The shoaling of the mixed layer was due to the large vertical diffusivity of buoyancy, which resulted in an upward diffusion of the pycnocline.

5. Implications for the subpolar front of the Japan/East Sea during cold-air outbreaks

Formed by confluence of cold and warm western boundary currents, the subpolar front of the Japan/East Sea is forced strongly by the atmosphere during wintertime outbreaks of cold, dry, Siberian air. These cold-air outbreaks are characterized by intense heat loss ($\sim 500 \text{ W m}^{-2}$) and powerful, northwesterly wind stress (up to 0.5 N m$^{-2}$) with a significant down-front component, making the front a prime testing ground for the theory. During January and February 2000, as part of the Japan Sea Initiative of the Office of Naval Research, a series of high-resolution SeaSoar (towed profiler instrumented to measure temperature, salinity, depth, and chlorophyll fluorescence) and ship-board acoustic Doppler current profiler (ADCP) surveys sampled the oceanic response to three cold-air outbreaks. During the outbreaks, the two-dimensional PV of the flow

$$q = f \left( f - \frac{\partial u}{\partial y} \right) N^2 + f \frac{\partial u}{\partial z} \frac{\partial b}{\partial y},$$
calculated from sections of density and alongfront velocity $u$, was negative at the front owing to the strong vertical shear and horizontal density gradient (see Fig. 11a). Beneath the front pycnostads were found with tracers of northern, surface waters: low PV, freshwater, and high chlorophyll fluorescence (not shown), a sign of active subduction. The alongfront velocity (Fig. 11b) was characterized by a high Rossby number ($|\zeta|/H < 0.7$), downwind jet at the surface, and an upwind flow at the base of the frontal interface, a flow pattern reminiscent of the upwelled low-momentum fluid of the analytical and numerical solutions (see Figs. 4b and 7b). The Ekman transport (9) estimated using the surface vertical vorticity and the zonal component of the wind stress derived from shipboard meteorological measurements differs dramatically from its linear counterpart, with strong divergence at the center of the front midway between the cyclonic and anticyclonic vorticity of the jet (Fig. 11d). Utilizing the Ekman transport to calculate the wind-driven buoyancy flux of (13), shows that the destabilizing buoyancy flux is concentrated at the front, reaching a peak value of $2.5 \times 10^{-6}$ m$^2$ s$^{-3}$ (Fig. 11c). In units of heat flux, this wind-driven buoyancy flux would be equivalent to a heat loss of 4900 W m$^{-2}$, nearly an order of magnitude larger than the atmospheric heat loss. The negative PV, evidence for subduction, subsurface upward flow, Ekman suction up the frontal interface, and strong, concentrated wind-driven buoyancy flux all suggest that wind-driven secondary circulation and frontal intensification are active at the front during cold-air outbreaks.

A comparison between the observed width of the subpolar front of the Japan/East Sea to $L_o$, the length scale of the fastest growing frontogenetic ASC, can be used to qualitatively evaluate the applicability of the theory to the observations. Substituting an average value for the negative PV in the core of the front based on the observations $\psi_{ml} \sim -2 \times 10^{-13}$ s$^{-4}$, a mixed layer depth $H \sim 50$ m, and the Coriolis parameter $f = 9.35 \times 10^{-5}$ s$^{-1}$ into (29) yields $L_o \approx 10$ km, a value comparable to the observed width of the front.

Because the Japan/East Sea subpolar front exhibits energetic meanders and eddies, two-dimensional theories cannot fully characterize its dynamics. Nonetheless, the theoretical model developed in this study can provide an upper bound on the strength of vertical circulation at the front. For wind-driven frontogenesis, the frontal vertical velocity scales with the Ekman pumping and downwelling balancing convective mixing. As shown in Fig. 11d, the Ekman transport changes by about 5 m$^2$ s$^{-1}$ in 5 km so that the Ekman pumping/suction would be 0.1 cm s$^{-1}$. Using the scaling for $\psi_{ml}$, and

![Fig. 7. Meridional section of (a) $\psi$ (shades) and (b) $u$ with isopycnals contoured in white, at $t = 4$ inertial periods for experiment E. The ragged regions of $\psi$ concentrated about the fronts are zones of active finescale convection.](image-url)
Fig. 8. Characteristics of the composite front for experiment E: (a) buoyancy; (b) zonal velocity averaged over the Ekman layer, smoothed (gray) and raw (dots); and (c) $-\nu$ at the base of the Ekman layer, $z = -11.78$ m, (black) as well as the Ekman transport, i.e., (9), estimated from the vertical vorticity of the smoothed (gray) and raw (dots) zonal velocity plotted in (b).

(31) to estimate the strength of the downwelling balancing convective mixing, that is, $w \approx F_{\text{eff}} / (N^2 H)$, with $F_{\text{eff}} \approx 2 \times 10^{-6}$ m$^2$ s$^{-3}$, $N^2 \sim 2 \times 10^{-5}$ s$^{-2}$ (based on an average value for the vertical stratification at the core of the front), and $H \sim 50$ m, yields a value of 0.2 cm s$^{-1}$. Such large vertical velocities of 0.1–0.2 cm s$^{-1}$ (86–170 m day$^{-1}$) will not necessarily cause the mixed layer to deepen by $\sim 100$ m in a day because vertical motions are not purely perpendicular to isopycnals on account of the slanted nature of the frontal interface. If the ASC at the subpolar front were similar to the analytical and numerical solutions plotted in Figs. 4 and 7, isopycnals would be distorted by the ASC in the following manner. Upwelling centered at the front by the Ekman divergence lifts the frontal interface, while beneath the Ekman layer convergence intensifies the lateral buoyancy gradient of the front. On the dense side of the front, water downwells on a slanted path, tucking surface fluid from the north under the frontal interface. This “lift-and-tuck” flow configuration of the ASC is conducive to subduction and might explain the existence of the boluses of low PV freshwater found underneath the front.

6. Conclusions

A spatially uniform wind blowing over a baroclinic zone in the direction of the surface currents leads to the formation of strong frontogenetic ageostrophic secondary circulations and multiple fronts within the zone.

Using an analytic theory, it has been shown that for mixed layers with negative potential vorticity, a particular length scale $L_o$ [see (29)] is selected by the most frontogenetic, stationary ASCs and determines the separation distance between fronts. Frontogenesis is ultimately a consequence of wind-driven gravitational instability and nonlinear Ekman pumping. Ekman flux of down-front winds advects dense water over light triggering convection centralize to the front. Mixing of buoyancy by this convection drives an ASC that accelerates the frontal jet. The vorticity contrast of the jet induces Ekman pumping/suction that enhances the Ekman flux at the front, hence strengthening the destabilizing density advection, subsequent convective mixing, and jet-accelerating ASC. Repetition of this process leads to frontal intensification within several inertial periods, with stronger winds producing faster frontogenesis. This frontogenesis mechanism does not require that the wind stress have negative curl (or any curl for that matter) as is the case with classic Ekman frontogenesis (e.g., Roden 1980; de Szoeke 1980; Cushman-Roisin 1981). Nor does it require a horizontal deformation field and thus is distinct from frontogenesis mechanisms based on the frontal model of Hoskins and Bretherton (1972).

The vertical circulation associated with the ASCs is characterized by subduction on the dense side of the front and upwelling along the frontal interface. Vertical velocity is scaled by both nonlinear Ekman pumping and downwelling balancing wind-driven convective mixing. The magnitude of the latter is set by a wind-driven buoyancy flux proportional to the product of the Ekman transport with the lateral buoyancy gradient at the front, a quantity that can be much larger than the atmospheric buoyancy loss owing to the large frontal density contrast.

Nonhydrostatic numerical simulations of a baroclinic zone forced by down-front winds capture the process of frontal intensification by the formation of frontogenetic ASCs in a manner consistent with the mechanism presented in the analytic theory. The experiments verify several key predictions of the theory: 1) fronts form that do not propagate with the Ekman flow, but remain stationary; 2) the stationary fronts are separated by a distance that scales with $L_o$; 3) convection is concentrated in the proximity of the front and leads to a convective buoyancy flux that scales with the wind-driven buoyancy flux; and 4) frontal jets develop with strong positive vertical vorticity that effectively form a barrier to the Ekman transport, causing the Ekman transport to be channeled down the dense side of the front. This
agreement between theory and numerical experiments, experiments with Ekman and mixed layer depths of comparable size, indicates that assumption (10) used in the theory that the Ekman layer is much thinner than the mixed layer is not crucial to frontal intensification by down-front winds.

High-resolution cross-front sections of density, horizontal velocity, and PV made at the subpolar front of the Japan/East Sea during strong atmospheric forcing by northwesterly winds with a significant down-front component associated with cold-air outbreaks suggest that wind-driven frontogenesis is active at the front. Evidence to support this hypothesis was: the occurrence of negative PV at the front; the discovery of waters beneath the frontal interface of northern origin apparently having been recently subducted; the estimation of a wind-driven buoyancy flux focused at the front peaking at a value equivalent to a heat loss of 4900 W m$^{-2}$, an order of magnitude larger than the atmospheric heat flux; and the calculation of significant cross-front variation of the Ekman transport owing to the vertical vorticity contrast of the frontal jet. The width of the front was found to scale with $Lo$. Upper bound estimates of the vertical velocity based on the theory (86–170 m day$^{-1}$) were quite large in magnitude, indicating that neglecting wind-forcing and using the quasigeostrophic equation to infer vertical velocity at wind-forced fronts could lead to significant errors. However, assuming that the front is purely two-dimensional by neglecting the effects of geostrophic forcing resulting from meanders and eddies could also lead to errors in the estimation of the vertical velocity. The Eliassen–Sawyer equation (4) accounts for both geostrophic and wind forcing and thus provides an appropriate theoretical model for describing vertical frontal circulations at wind-driven fronts with low to negative PV. In a future study, the complete set of data taken at the subpolar front of the Japan/East Sea, including both cross and alongfront measurements, will be used to obtain solutions of the Eliassen–Sawyer equation. These solutions will be utilized to both estimate the ASC at the front and to determine the relative importance of geostrophic versus wind forcing in the driving of the ASC.

The dominance of the destabilizing wind-driven buoyancy flux over the atmospheric buoyancy flux in the analytical and numerical solutions as well as in the estimate from the subpolar front of the Japan/East Sea suggests that Ekman advection of buoyancy could play a major role in the formation of mode waters at fronts.
forced by down-front winds. This is especially true for fronts in the Southern Ocean where there are no cold-air outbreaks on account of the lack of continents. Evidence pointing to the importance of Ekman dynamics in the formation of mode waters in the Southern Ocean is given by Rintoul and England (2002) who show that the magnitude of temperature and salinity variation in the Subantarctic Mode Water cannot be explained by air–sea fluxes of heat and freshwater, but can be accounted for by advection of cold, freshwater across the subantarctic front by northward Ekman transport. As shown in the example of the subpolar front of the Japan/East Sea forced by cold-air outbreaks, it is not necessary that the atmospheric buoyancy flux be weak for the wind-driven buoyancy flux to dominate. Like the subpolar front of the Japan/East Sea, the Kuroshio is forced by cold dry air during the winter. The formation of mode waters associated with the Kuroshio has been

Fig. 10. Time dependence of the spectra in $y$ of $\phi$ measured at the base of the Ekman layer for the five numerical experiments listed in Table 1.
attributed to wintertime convection driven by heat loss. However, Yasuda and Hanawa (1997) conclude that decadal variations in the temperature of the North Pacific central mode water are due to both decadal changes in the air–sea heat flux and Ekman advection of heat, with the latter contributing twice as much to the decadal variability. Both of these examples suggest that wind-driven frontogenesis may be active at these large-scale fronts of global importance. The connection with decadal variability. Both of these examples suggest that wind-driven frontogenesis may be active at these large-scale fronts of global importance. The connection with

care should be taken in designing global circulation models so that in frontal
regions the wind-driven buoyancy flux, ensuing gravitational instability, and frontogenetic ASCs are well parameterized.

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APPENDIX

Numerical Model Equations and Configuration

The nonhydrostatic numerical model integrates the following equations forward in time:

\[ \frac{\partial u'}{\partial t} - J(\psi, u') = \frac{f}{f} \frac{\partial \psi}{\partial z} + \frac{\psi}{f} \frac{\partial u'}{\partial y} + \frac{\kappa_v}{\partial z^2} + \frac{\kappa_H}{\partial y^2} \]

\[ \frac{\partial b'}{\partial t} - J(\psi, b') = \frac{\psi}{f} \frac{\partial b'}{\partial z} + \frac{\kappa_v}{\partial z^2} + \frac{\kappa_H}{\partial y^2} \]

\[ \frac{\partial \xi}{\partial t} - J(\psi, \xi) = \frac{\partial u'}{\partial z} + \frac{\partial b'}{\partial y} + \frac{\kappa_v}{\partial z^2} + \frac{\kappa_H}{\partial y^2} \]

where \( \psi \) is a streamfunction; that is, \( v = \psi \partial \psi / \partial z \) and \( w = -\psi \partial \psi / \partial y \), \( \xi = -\nabla^2 \psi \) is the zonal component of the vorticity, \( J(A, B) = A_x B_z - A_z B_x \) is the Jacobian, \( \kappa_v = 0.005 \text{ m}^2 \text{ s}^{-1} \) (\( \kappa_H = 0.1 \text{ m}^2 \text{ s}^{-1} \)) is the vertical (horizontal) diffusivity of momentum and buoyancy, and the total zonal velocity and buoyancy fields have been decomposed as \( u = \langle S^2 \rangle f \) + \( u'(y, z, t) \) and \( b = -\langle S^2 \rangle y + b'(y, z, t) \), where \( S^2 \) is a constant. The vertical stratification of the basic state (16) is incorporated into the initial condition for \( b' \)

\[ \frac{\partial b'}{\partial z} \bigg|_{z=0} = \begin{cases} N^2_{\text{ml}} \frac{z}{f} \sqrt{f} & z > -H \\ N^2_{\text{ml}} - \frac{\langle S^2 \rangle}{f} (z + H) & z < -H. \end{cases} \]

All other variables—that is, \( u' \), \( \xi \), and \( \psi \)—are set to zero at \( t = 0 \).

The model uses a staggered grid with the buoyancy
defined at the center of the grid, and \( u', \xi, \) and \( \psi \) at the corners of the grid. Vertical and horizontal grid spacings of the model \((\Delta z = 2 \text{ m} \text{ and } \Delta y = 10 \text{ m})\) are designed to be fine enough so that the Ekman layer (which unlike the idealized Ekman layer used in the theory is of finite thickness) and narrow convective plumes are well resolved. An Adams–Bashforth time-stepping scheme is used with a 6-s time step. Lateral boundary conditions at \( y = 0 \) and \( y = 12 \text{ km} \) are periodic. On the top and bottom boundaries of the domain a no normal flow condition is used: \( \psi = 0 \text{ at } z = 0, -D, \) where the total depth of the fluid \( D = 250 \text{ m} \) is much deeper than the Prandtl depth calculated from the anticipated length scale \( L_s \) of the ASCs. For the buoyancy and zonal velocity, flux and stress boundary conditions are specified

\[
\left. \frac{\partial b'}{\partial z} \right|_{z=0} = F_o, \quad \left. \frac{\partial b'}{\partial z} \right|_{z=-D} = 0,
\]

\[
\left. \frac{\partial u'}{\partial z} \right|_{z=0} = 0, \quad \text{and} \quad \left. \frac{\partial u'}{\partial z} \right|_{z=-D} = 0,
\]

where \( F_o \) is the wind stress and \( F_o \) is the atmospheric buoyancy flux. At \( t = 0 \), the wind stress and buoyancy flux is turned on impulsively and the buoyancy in the top grid is seeded with a weak perturbation of random noise to trigger gravitational instability.

**REFERENCES**


