Estimating the thickness of ridged sea ice from ship observations in the Ross Sea

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Abstract: Ship-based observations of sea ice thickness using the Antarctic Sea Ice Processes and Climate (ASPeCt) protocol provide information on ice thickness distribution at relatively low cost. This protocol uses a simple formula to calculate the mass of ice in ridges based on surface observations. We present two new formulae and compare these with results from the “Original” formula using data obtained in the Ross Sea in autumn and winter. The new “r-star” formula uses a more realistic ratio of sail and keel areas to transform dimensions of sails to estimates of mean keel areas. As a result, estimates of “equivalent thickness” (i.e. mean thickness of ice in ridged areas) increased by over 200%. The new “Probability” formula goes one step further, by incorporating the probability that a sail is associated with a keel underwater, and the probability that keels may be found under level surfaces. This resulted in estimates of equivalent thickness comparable with the Original formula. Estimates of equivalent thickness at one or two degree latitude resolution are sufficiently accurate for validating sea ice models. Although ridges are small features in the Ross Sea, we have shown that they constitute a significant fraction of the total ice mass.

Key words: Antarctica, ASPeCt, pressure ridges, sea ice thickness distribution

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Introduction

A thorough understanding of the sea ice thickness distribution around Antarctica is essential for validating climate models and for detecting any responses of sea ice to climate change. Field expeditions dedicated to measuring sea ice thickness in the Southern Ocean are expensive and relatively infrequent, but observations of ice thickness can be made from any vessel travelling through the ice pack. These can provide information on the ice thickness distribution at relatively low cost, and give potentially comprehensive spatial and temporal coverage. With the goal of defining a climatology of the sea ice thickness distribution in the Southern Ocean, the Antarctic Sea Ice Processes and Climate (ASPeCt) program of the Scientific Committee on Antarctic Research (SCAR) Global Change and the Antarctic Program (GLOCHANT) developed a standard protocol for recording snow and ice thickness observations from ships (Worby 1999, Worby & Allison 1999). The modifications are based on previous unavailable information on pressure ridge morphology. The new formulae use a more realistic ratio of sail and keel areas to transform dimensions of sails to estimates of mean keel areas, and also incorporate the probability that a sail is associated with a keel underwater, and the probability that keels may be found under level surfaces. We then apply the Original and the new formulae to ship observations data and compare the estimate of equivalent thickness from the different formulae to examine the effects of incorporating the new information on ridge morphology on equivalent thickness.

Previous work and existing methodology

Previous work

Estimating the amount of sea ice underwater from surface observations has been a long-standing problem. In studies where concurrent measurements of freeboard and ice thickness were not available, it was common practice to assume that each ridge sail was hydrostatically balanced by a keel and then estimate the volume of the keel from the volume of the sail (e.g. Zubov 1944, Kirillov 1957, Hibler et al. 1974, Leppäranta 1981). Snow and ice densities and sail and keel porosities are key parameters in these calculations but field-derived values were not always available in these studies.
In studies where concurrent measurements of freeboard and ice thickness were available, the mean state of hydrostatic equilibrium is represented by the ratio between mean draft and freeboard obtained from field data, removing the need for parameterization of densities and porosities. These ratios were first quantified for 20 and 50 km sections of coincident airborne laser/lidar and submarine sonar data for Arctic ice (Comiso et al. 1991, Wadhams et al. 1992). Subsequently they have been derived from drill data obtained around ridges only and applied to geometrical models of individual ridge sails to estimate equivalent thickness from laser profiling data (Dierking 1995) and ship observations data (Worby & Allison 1999).

Sails, as recorded by surface observations, are features on the ice surface in the absence of snow, and are features on the snow surface in the presence of snow. In all previous studies, the presence of a sail on the top surface (called here the “snow sail”) was taken to imply the presence of a sail on the ice surface (called here the “ice sail”) and the presence of a keel. In a few studies, the volume of snow in the snow sail was explicitly calculated and removed when estimating the amount of ice in the ridge (e.g. Leppäranta 1981) but in most studies, the volume of snow was assumed to be negligible (e.g. Wadhams et al. 1992).

ASPeCt protocols and the Original formula for estimating equivalent thickness

According to the ASPeCt protocol, a standard set of observations is made hourly by an observer on the ship’s bridge. These include the ship’s position and total ice concentration, and an estimate of the areal coverage, thickness, floe size, topography (i.e. areal coverage and mean height of ridge sails) and snow cover of the three thickest ice types within a radius of approximately 1 km around the ship. The mean thickness of ice in ridged areas is estimated from the areal coverage and mean height of snow sails, and the surrounding level ice thickness. The geometric model that is used is shown in Fig. 1. Each unit area of a ridged floe comprises three distinct components: a base layer of level ice thickness, $z_u$, a sail represented by a triangle with a height, $S$, and areal coverage, $R$; and a keel of no prescribed shape. The cross-sectional area of the snow sail is $0.5RS$ and the cross-sectional area of the keel is $r'$ times that of the snow sail. The average, or equivalent, thickness, $z_r$, can then be calculated according to

$$z_r = 0.5RS(r'^4+1) + z_u$$

The amount of ice in the snow sail is exactly $0.5RS$ when there is no snow. The parameter $r'$ is the ratio of ice below and above the sea level in ridged areas, as derived from drill hole measurements around ridges on ice floes, and is used as a proxy for the ratio of ice below and above the base layer (Worby & Allison 1999). It is a representation of the average state of hydrostatic equilibrium in ridged areas and has been calculated to be 4.3 for the East Antarctic sea ice cover (Worby & Allison 1999).

New aspects of ridge morphology

Limitations of hydrostatic equilibrium

Field studies have shown that although local hydrostatic imbalance is common (e.g. Weeks et al. 1971), hydrostatic equilibrium must be maintained on all floating ice floes (Bowen & Topham 1996). Since keels are wider than snow sails (e.g. Kovacs 1972, Timco & Burden 1995, Kankaanpää 1997), hydrostatic equilibrium across a keel is maintained between the keel, its sail and the surrounding level ice above the keel. If the amount of ice underwater is estimated to be the volume that is needed to hydrostatically balance the sail alone, the amount of ice underwater tends to be underestimated, because the hydrostatic effect of the
surrounding level ice has been ignored. This is illustrated in Fig. 2, which compares keel cross-sectional areas calculated from the dimensions of snow sails assuming hydrostatic equilibrium, with keel areas obtained by summing the amount of ice below sea level across the extent of each keel. The majority of the calculated keel areas are less than the measured keel areas. Therefore, in order to estimate the volume of keels from the height and areal coverage of snow sails, we need a ratio between snow sail and keel volume which is scaled for unit surface area covered by sails.

Evidence for snow sail/keel dissociation

Previous studies all assumed that the presence of a snow sail implied the presence of an ice sail and a keel (Fig. 3a) (Kirillov 1957, Worby & Allison 1999). However, drill data from the Ross Sea show evidence that not every ice sail is associated with a keel (Fig. 3b), not every keel is associated with an ice sail (Fig. 3c), and not every snow sail is associated with an ice sail (Fig. 3d & e). Keels without ice sails have also been reproduced in laboratory experiments and results showed that these keels tended to form from ice sheets with heterogeneous internal structure (Tuhkuri & Lensu in press). On the other hand, observers on ships may not be able to distinguish between snowdrift and deformation features, and so some of the observed sails may be formed from wind redistribution and not related to the ridging process which produces keels. Therefore, in order to estimate keel cross-sectional area from dimensions of the observed sail, it is necessary to account for the relationship between the presence of a snow sail and the presence of a keel.

Data collection and analysis

Data

Two different types of data were used in this study. The first was obtained by drilling on Antarctic first year sea ice floes at 1 m intervals along transects 50 to 150 m long. At each drill hole, snow thickness, \( z_s \), ice thickness, \( z_i \), and freeboard, \( z_f \), were measured. The mean values for each variable were used to derive values for the parameters in the formulae for estimating equivalent thickness, i.e. the probability of snow sail/keel dissociation and the ratio between snow sail and keel cross-sectional areas per unit snow sail width. The data were obtained on three cruises in the Ross Sea in autumn and winter 1995 (cruises NBP 95-3 and NBP 95-5a) and autumn 1998 (cruise NBP 98-3) (Fig. 4).

Snow sails and keels were identified from the snow and ice bottom surfaces respectively, following the identification scheme presented in Tin & Jeffries (2001).
Since the snow sails are features which must be observable from ships, the identification scheme characterizes snow sails as features which protrude significantly \((\geq 0.3 \text{ m})\) above the surrounding level snow surface. Keels are identified as features which are significantly more voluminous than \((\geq 2.25 \text{ times thicker than})\) the surrounding level ice.

The second type of data was obtained from ship observations following the ASPeCt protocol on the same cruises. The formulae for estimating equivalent thickness were applied to observations made along the southbound legs of each cruise, which closely followed longitude \(180^\circ\) (Fig. 4). Limiting the study to data obtained along \(180^\circ\) simplifies the comparison of the results obtained with each of the formulae. The data were edited to exclude observations within 11.1 km of the previous observations to prevent biasing in areas of heavy ice where the ship’s speed was reduced (Worby & Allison 1999).

**Analysis**

The Original formula is modified in two steps. In the r-star formula, the parameter of \(r’\) is replaced by \(r*\) which is the ratio between the cross-sectional area of the snow sail and the cross-sectional area of the keel per unit snow sail width. The r-star formula is then modified to include the probability that a snow sail is associated with a keel underwater, and the probability that keels may be found under level surfaces to form the Probability formula.

The r-star formula

Assuming the same geometrical model as the Original formula (Fig. 1), the r-star formula is

\[
z_r = 0.5RS(r^*+1) + z_u \tag{2}\]

where

\[
r^* = \frac{\sum A_k}{\sum A_{s}} \tag{3}\]

and \(A_k\) is the cross-sectional area of a manifest keel (a keel associated with a snow sail); \(A_s\) is the cross-sectional area of the associated snow sail calculated from its width and height, assuming a triangular cross-section as in the geometrical model of Fig. 1; and \(m\) is the number of manifest keels.

The Probability formula

Building on the r-star formula, we develop the Probability formula to include the effects of snow sail/keel dissociation. The equivalent thickness, \(z_r\), comprises the equivalent thickness of observed features, \(Oz_r\), plus the equivalent thickness of inferred features, \(Iz_r\).

\[
z_r = Oz_r + Iz_r \tag{4}\]

Observed features are those that can be seen from a ship, i.e. the snow sail and the base layer.

\[
Oz_r = 0.5RS + z_u \tag{5}\]

Inferred features are those that can only be inferred from the observations. The equivalent thickness of inferred features comprises the equivalent thickness of manifest keels, \(Mz_r\), plus the equivalent thickness of hidden keels (keels not associated with snow sails), \(Hz_r\).

\[
Iz_r = Mz_r + Hz_r \tag{6}\]

Each manifest keel has an area \(r^*\) times that of the snow sail above. However, not every snow sail is associated with a keel. Therefore, we factor in \(p_R\), the probability that a snow sail is associated with a keel, to obtain a mean estimate of the equivalent thickness of a manifest keel.

\[
Mz_r = p_R r^* 0.5RS \tag{7}\]

The probability \(p_R\) is derived from drill data as

\[
p_R = \frac{\sum n_j W_j}{\sum W_j} \tag{8}\]

where \(W_j\) is the entire width of a snow sail which fully or partially overlaps with a keel; \(W_j\) is the width of a snow sail; \(n_j\) is the number of snow sails which fully or partially overlaps with a keel; and \(n_j\) is the total number of snow sails. The probability \(p_R\) is a function of the characteristics and conditions of the ice floes, such as their strength, the atmospheric and oceanic forcing acting on them, and the aeolian distribution of snow on their surfaces.

The mean estimate of the equivalent thickness of a hidden keel can be obtained by multiplying the average thickness of such a keel, \(K_l\), by the probability that a keel is below a level snow surface, \(p_L\):

\[
Hz_r = X[p_L(1 - R) K_l] \tag{9}\]

where \(X = 1\) for \(R > 0.05\); \(X = 0\) for \(R \leq 0.05\)
The average thickness of a hidden keel, $K_L$, is derived from the drill data as

$$K_L = \frac{\sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} W_{ki}},$$

where $A_i$ is the keel area of a hidden keel, $W_{ki}$ is the width of the keel and $q$ is the number of hidden keels.

The probability of $p_L$ is defined as

$$p_L = \frac{\sum_{i=1}^{n} W_{ki}}{\sum_{i=1}^{n} W_k},$$

where $W_k$ is the entire width of a contiguous section of the snow surface which is not part of any snow sail and does not overlap with any manifest keel but which overlaps with hidden keels; $W_{ki}$ is the width of a contiguous section of the surface which is not part of any snow sail and does not overlap with any manifest keel; $t_i$ is the number of contiguous sections of the snow surface which is not part of any snow sail and does not overlap with any manifest keel but which overlaps fully or partially with hidden keels; and $t_j$ is the number of contiguous sections of the surface which is not part of any snow sail and does not overlap with any manifest keel. The probability $p_L$ is a function of the homogeneity of the ice floes and the atmospheric and oceanic forcing acting on them. It is the probability that a unit area of level surface overlies part of a hidden keel, and contains information on the number of hidden keels per unit distance, i.e. the frequency of hidden keels. Only long transects of concurrent top and bottom surface profiling can provide accurate information on the frequency of snow sails and keels on a large scale. This type of data is not available in our study, so we derive an approximate value of $p_L$ from the short transects of drill data and use it to demonstrate its application in the Probability formula.

The probability that there is a keel underneath a level surface is related to the amount of deformation in the region, which is most closely parameterized in terms of the areal coverage of sails, $R$. The relationship between $p_L$ and $R$ cannot be derived from the existing data. Therefore, in order to avoid overestimating the equivalent thickness in young ice regions, $p_L$ is only considered when $R$ is greater than 5%, as a first approximation of the relationship between $p_L$ and $R$.

The resultant Probability formula is thus

$$z_r = 0.5 R S + z_u + p_R^*0.5 R S + X [p_L (1 - R) K_L]$$

where $X = 1$ for $R > 0.05$; $X = 0$ for $R \leq 0.05$

### Statistical methods

**Propagation of errors**

To assess the equivalence of estimates of equivalent thickness from different formulae, it is necessary to quantify variances of the estimates linked to measurement and observation errors. For a function $V$ where,

$$V = V(x, y)$$

the variance in $V$, $s^2_v$, is defined as a function of the independent and uncorrelated errors in $x$, $s_x$, and $y$, $s_y$, as:

$$s_v^2 = \left(\frac{\partial V}{\partial x}s_x\right)^2 + \left(\frac{\partial V}{\partial y}s_y\right)^2$$

(Beers 1957, Neuilly 1999). Errors in estimates of equivalent thickness arise from errors in the measurements of $z_r$, $z_u$, and $z_l$ on the drill transects, and from errors in the visual estimates of $R$, $S$, $z_u$, and ice concentration, $c$, from the ship observations. Table I lists the mean errors for observations and drill measurements, estimated from data bin sizes and accuracy of measurement instruments respectively. The errors of $r^*$, $r^*$, and $K_L$ were estimated from the propagation of measurement and observation errors following Eq. 8. The variance of the probabilities $p_L$ and $p_R$ were calculated from

$$s_p^2 = \frac{p(1 - n)}{n}$$

where $s_p^2$ is the variance of the probability $p$ and $n$ is the number of samples (Lapin 1983). These errors were propagated through the Original, r-star and Probability formulae following Eq. 14 to obtain the errors of the estimates of equivalent thickness for the different formulae.

### Statistical tests

Two-sample Student’s $t$-tests were used to quantify equivalence of the estimates of equivalent thickness within the variance linked to measurement and observation errors (Moore & McCabe 1993). A minimum sample size of five is required to ensure the robustness of the two-sample $t$-test; therefore observations were analysed in groups of two

<table>
<thead>
<tr>
<th>Table I. Mean estimated errors of ship observations and drill measurements.</th>
</tr>
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<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Observed quantities</td>
</tr>
<tr>
<td>Areal coverage of snow sails, $R$</td>
</tr>
<tr>
<td>Mean height of snow sails, $S$</td>
</tr>
<tr>
<td>Level ice thickness, $z_u$</td>
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<tr>
<td>Snow thickness, $z_l$</td>
</tr>
<tr>
<td>Concentration or areal coverage of ice category, $C$</td>
</tr>
<tr>
<td>Measured quantities</td>
</tr>
<tr>
<td>Ice thickness, $z_r$</td>
</tr>
<tr>
<td>Freeboard, $z_f$</td>
</tr>
<tr>
<td>Snow thickness, $z_s$</td>
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</table>
degrees of latitude, which contained six to sixteen observations. For each group, estimates from the r-star and Probability formulae were tested against estimates from the Original formula. F-tests were first used to assess the equivalence of variances. In the cases where variances were statistically different, the Welch statistic was calculated to modify the degrees of freedom of the group before the t-test was performed (Swan & Sandilands 1995).

Results

Drill data from NBP 95-3, NBP 98-3 and NBP 95-5a amounted to 8926 drill holes along 101 transects. A total of 24 keels were completely sampled by the transects, and 5 of them overlapped with snow sails. The derived values of $r^*$, $p_R$, $p_L$, $K_L$ and $r'$ are listed in Table II.

Figure 5 plots estimates of equivalent thickness by each formula for each two degree latitude band. The estimates from the Original formula are consistently the lowest in value and error, while those from the r-star formula are always the highest. T-tests did not show any statistical evidence to suggest that mean Probability estimates averaged over two degrees of latitude were different from mean Original estimates. On the other hand, r-star estimates are different from the Original estimates at the 95% significance level for all two degree latitude bands, except the northernmost band on each of the three cruises.

The differences between the three formulae arise from the estimate of the cross sectional area of the keel. In the Original formula, the mean keel area is calculated as $r'$, which is 3.58 times that of the snow sail area (Table II). In the r-star formula, $r'$ is replaced by $r^*$ and the mean keel area is then nearly 8 times that of the snow sail area, which was more than twice that of the Original formula. In the Probability formula, the mean keel area is calculated as $r^*$ times that of the snow sail area but the probability that a snow sail is associated with a keel, $p_R$, is factored in. This lowers the estimate of the mean keel area significantly to 3.93 times that of the snow sail area, which is close to the $r'$ value used in the Original formula. Incorporating the low probability that keels may be found under level surfaces, $p_L$, raises the estimates of equivalent thickness only slightly, since $p_L$ is small (0.07) and less than 15% of $p_R$. As a result, the estimates from the Original formula are very similar to those from the Probability formula.

Discussion

Use of formulae

Of the three formulae, the Probability formula provides the most accurate physical model for the estimate of equivalent thickness, and should be used to derive estimates of equivalent thickness where there are sufficient data to derive values for the parameters $r^*$, $K_L$, $p_L$ and $p_R$. However, data on pressure ridges in the Southern Ocean are sparse and in regions where there are insufficient data to derive values for the parameters, the r-star formula, with its realistic ratio of snow sail and keel area, should be used.

Table II. Mean values and errors of derived parameters used in formulae for estimating equivalent thickness. Drill holes are 1 m apart.

<table>
<thead>
<tr>
<th>Derived parameters</th>
<th>Mean value</th>
<th>Error</th>
<th>No. of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^*$</td>
<td>7.93</td>
<td>0.068</td>
<td></td>
</tr>
<tr>
<td>$p_R$</td>
<td>0.48</td>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>$p_L$</td>
<td>0.07</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>$K_L$</td>
<td>0.35 m</td>
<td>0.007 m</td>
<td>481 drill holes from 19 hidden keels</td>
</tr>
<tr>
<td>$r'$</td>
<td>3.58</td>
<td>0.00004</td>
<td>850 drill holes</td>
</tr>
</tbody>
</table>

The differences between the three formulae arise from the estimate of the cross sectional area of the keel. In the Original formula, the mean keel area is calculated as $r'$, which is 3.85 times that of the snow sail area (Table II). In the r-star formula, $r'$ is replaced by $r^*$ and the mean keel area is then nearly 8 times that of the snow sail area, which was more than twice that of the Original formula. In the Probability formula, the mean keel area is calculated as $r^*$ times that of the snow sail area but the probability that a snow sail is associated with a keel, $p_R$, is factored in. This lowers the estimate of the mean keel area significantly to 3.93 times that of the snow sail area, which is close to the $r'$ value used in the Original formula. Incorporating the low probability that keels may be found under level surfaces, $p_L$, raises the estimates of equivalent thickness only slightly, since $p_L$ is small (0.07) and less than 15% of $p_R$. As a result, the estimates from the Original formula are very similar to those from the Probability formula.

Fig. 5. Estimates of equivalent thickness, $z_r$, from the Original, r-star and Probability formulae, together with mean observed areal fraction of ridge sails, $R$, and mean observed height of ridge sails, $S$, for 2 degree latitude band along the southbound leg of three cruises. Error bars around $z_r$ indicate the range enclosed by mean ± 2 times error of the estimate. *indicates no observations in latitude band. a. Cruise NBP 95-3, b. Cruise NBP 95-5a, c. Cruise NBP 98-3.
Nevertheless, the user must bear in mind that the results from the r-star formula tend to be overestimates of equivalent thickness, as illustrated in Fig. 5, since \( p_L \) and \( p_R \) have not been taken into account. As reflected by the numerically similar estimates of equivalent thickness from the Original and Probability formulae, it is possible that, under certain conditions, the more simple Original formula can be used as a proxy for the Probability formula where data on pressure ridges are not available. However, its validity for conditions other than early autumn and late winter in the Ross Sea needs to be tested.

**Biases introduced by sampling process and identification scheme**

The parameters \( r^*, p_L, p_R \) and \( K_e \) are essential inputs to the Probability formula but their values are highly influenced by the methods of data collection. Here, we try to quantify the biases that the sampling process of linear drill transects and the identification of sails and keels from drill profiles may introduce into the derived values of these parameters.

The process of sampling three-dimensional pressure ridges with linear transects may be a source of bias. A linear transect may not simultaneously sample a sail and a keel which are longitudinally separated, and might lead to false transect may not simultaneously sample a sail and a keel ridges with linear transects may be a source of bias. A linear and the identification of sails and keels from drill profiles may introduce into the derived values of these parameters.

The effect of missing small sails and keels on the estimate of equivalent thickness is minimal, since sails less than 0.3 m above the surrounding snow surface are unlikely to be observed from the ship and recorded in the ASpeCt data, and keels less than 2.25 times as thick as the surrounding snow give little to total ice volume. In the case where little level ice is sampled by a drill transect which is 50 m to 150 m in length, we estimate that approximately 10% of the total number of keels have been missed by the identification scheme.

The parameter \( p_L \) represents information on the frequency of hidden keels which can only be accurately obtained from long transsects of concurrent top and bottom surface profiling. It is possible that the value of \( p_L \) used in this study differs from the true value, since it has been derived from short drill transects. Equivalent thickness as estimated from the Probability formula is balanced between \( p_L \), which prevents underestimation of the amount of ice underneath level surfaces, and \( p_R \) which prevents the overestimation of the amount of ice underneath sails. How would a realistic value of \( p_L \), obtained from long transsects of concurrent top and bottom surface profiling, affect the relative effects of \( p_L \) and \( p_R \) on estimates of equivalent thickness? In order to address this question we make use of an additional 103 drill profiles obtained on five other cruises in the Ross, Amundsen, Bellingshausen and Weddell seas to provide a dataset which spans 204 profiles and nearly 19 km. Approximately 20 snow sails and keels were identified in this dataset. This is equivalent to snow sail and keel frequencies of the order of \( 10^3 \text{ m}^{-1} \), which is of the same order of magnitude as the average snow sail frequency measured in the Ross Sea by an airborne laser profilometer along transects of approximately 100 km in length \( (2.5 \text{ snow sails km}^{-1}, \text{ Weeks et al. 1989}) \). Therefore, we expect the effect of \( p_L \) on equivalent thickness to remain significantly weaker than that of \( p_R \), even with the application of a realistic value of \( p_L \).

**Accuracy of equivalent ice thickness**

The mean error of estimates of equivalent thickness from individual observations is 18%. When observations are summarized for two degree latitude bands, the error for each band drops to a mean of 6.3% or 3 cm. The accuracy of the estimate of equivalent thickness increases from averaging over increasing number of samples, i.e. with decreasing spatial resolution. Although the accuracy of individual observations is low, equivalent thickness estimates at one or two degree resolution have improved accuracy and can be used for validating sea ice models. However, in order to detect changes in equivalent thickness, data from multiple cruises are needed to first establish the baseline equivalent thickness with some confidence and then to accurately quantify any changes.

**Contribution of ridging to total ice mass**

From estimates of equivalent thickness, we can estimate the percentage underestimation, \( U \), of total ice mass if keels are ignored and the average thickness is approximated by only the thickness of the surrounding level ice,

\[
U = \frac{Z_e - Z_u}{Z_u}
\]  

where upper case letters represent the summation of \( z_e \) and \( z_u \) along the cruise track. This is also a measure of the
contribution of ridge snow sails and keels to the total ice mass. If every snow sail represents a keel underwater and no keels are underneath level surfaces, then the percentage underestimation by level approximation is 46%, as derived from the r-star formula, for the southbound leg of NBP 95-3 which was just over 1000 km long (Fig. 6a). By incorporating the probabilities of snow sail/keel dissociation into the r-star formula, the percentage underestimation by level approximation drops to 26% in the Probability formula, similar to the 24% derived from the Original formula (Fig. 6a). The percentage underestimation by level approximation is similar along the southbound leg of NBP 95-3a, which is close to 700 km in length, the percentage underestimation by level approximation ranges from about 40% according to the Original and Probability formulae to 77% according to the r-star formula (Fig. 6b). These results show that ridging is an important process in determining the ice thickness distribution in the Ross Sea, and although pressure ridges are small features, they constitute a significant fraction of the total ice mass in the area in the Ross Sea during the autumn/winter period.

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