A two-dimensional coupled model for ice shelf–ocean interaction

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Abstract

A simplified coupled model of ice shelf–ocean interaction including an evolving ice shelf is presented. The model is well suited to climate simulation, as it is computationally inexpensive and capable of handling significant changes to the shape of the sub-ice shelf cavity as the shelf profile evolves. The ocean component uses a two-dimensional vertical overturning streamfunction to describe the thermohaline circulation. In order to smoothly accommodate evolution of the shelf, the equations have been converted to a time-dependent terrain-following (sigma) vertical coordinate. The shelf component is a model for the flow of a confined ice shelf of non-uniform thickness, which consists of equations for longitudinal spreading rate and velocity. The advection of ice thickness has been modified to include separate marine and meteoric layers. The ice shelf and ocean interact thermodynamically through a three-equation formulation for basal melting and freezing.

The model is applied to an idealized large-scale Antarctic ice shelf. Following a 600 year simulation, the shelf is found to have reached an equilibrium which represents a loss of approximately 10% of mass relative to its steady state when ocean interaction is not considered. Significant changes in the shelf morphology are also observed, notably an increase in basal slope near the grounding line. These changes are accompanied by shifts in the pattern of basal melting and freezing. Warming of the ocean produces a greater than linear increase in basal melting over several decades, gradually slowing to sublinear over approximately a century.

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1. Introduction

Ice shelves cover approximately 40% of the Antarctic continental shelf, an area of 1.5 × 10^6 km^2, effectively isolating the underlying ocean from any interaction with the atmosphere (Williams et al., 1998). The circulation in sub-ice shelf cavities is thus markedly different from that in the open ocean, consisting largely of a thermohaline circulation forced by melting and freezing processes at the ice shelf base. This circulation is of more than local importance, since it plays a key role in the production of Antarctic bottom water (AABW), a driver of the global thermohaline circulation. Sea ice formation over Antarctic coastal waters produces high salinity...
shelf water (HSSW) through the process of brine rejection during freezing; due to its high density, HSSW sinks and enters the sub-ice shelf cavity at depth. Although this water is near the surface freezing point (approximately $-1.9\, ^\circ$C), the decrease in the freezing point of seawater with increased pressure results in HSSW being sufficiently warm to cause melting at the ice shelf base. Cooled and freshened by the incorporation of meltwater, HSSW is converted into ice shelf water (ISW), which may be defined as water with potential temperature below its surface freezing point (Foldvik et al., 2004). ISW exits the cavity at mid-water depths and flows downwards over the continental shelf edge, mixing with circumpolar deep water (CDW) to finally form AABW. The properties of AABW are thus sensitive to thermodynamic interactions between ocean and ice shelf.

Much of the glaciological interest in ice shelves has been directed towards quantifying rates of basal ablation and accretion, with an eye towards determining the ice mass balance and its sensitivity to climate change. Interest is particularly strong for regions where high basal melting may threaten the stability of ice shelves; Pine Island Glacier in the Amundsen Sea is a notable example (Rignot, 1998). Although the melting of ice shelves would have little effect upon global sea level, the presence of ice shelves is believed to affect the flow of inland ice sheets by providing a compressive “backstress” which slows glaciers. Following the sudden collapse of the Larsen B ice shelf in 2002, for example, satellite-based observations indicated that glaciers flowing into the area formerly occupied by the shelf thinned rapidly and increased speed from two- to six-fold (Scambos et al., 2004).

The need for numerical modeling of ice shelf–ocean interaction is particularly acute due to a lack of extensive observational data, which results from the physical inaccessibility of the areas of interest. The most readily available observations are conductivity-temperature-depth (CTD) measurements made by ships approaching ice shelf fronts during summer months, as variable sea ice conditions in winter complicate the deployment and retrieval of moored instruments (Williams et al., 1998). Furthermore, the difficulty of drilling access holes through hundreds of meters of ice makes CTD measurements inside the cavities quite sparse (Williams et al., 1998), although instruments can be permanently deployed in this manner and thus gather seasonal and interannual data (e.g., Nicholls and Makinson (1998)). Airborne radio-echo sounding surveys can accurately determine the thickness profile of a shelf, and help to identify areas where marine ice has formed due to freezing at the ice shelf base (Fricker et al., 2001). Satellite-based altimetry and interferometry can also determine the thickness profile of ice shelves, while in addition measuring flow velocities (Rignot, 1998). However, the latter two methods provide no direct information about the sub-ice shelf circulation, leaving a significant role to be played by numerical models.

Earlier models of ice shelf–ocean interaction have typically assumed the ice shelf to be in a steady state, interacting with the ocean only through thermodynamics at the interface. For present-day simulations, this is generally considered to be a reasonable assumption, since calculated melt rates suggest minimal change to the ice shelf draft during the time simulated (Hellmer and Olbers, 1991). However, the effects of basal melting and freezing over the longer timescales at which ice shelves evolve have not been extensively studied. To date, the main effort in modeling a dynamic ice shelf has been by Grosfeld and Sandhager (2004), who couple a three-dimensional ocean general circulation model with a vertically integrated, two-dimensional ice shelf model using an asynchronous approach. The scale and complexity of this model place it in a different category from the simplified process model which will be developed here.

The primary goal of this study is to consider how basal melting and freezing affect the long-term mass balance of an ice shelf, including their effect upon its equilibrium shape. Investigating these questions requires simulations of both ocean and ice shelf for centuries of model time, making a computationally inexpensive reduced model an attractive option. In particular, the speed of a reduced model allows many experiments to be run in a reasonable time, so that extensive sensitivity studies become feasible. Furthermore, the physics and numerics of a reduced model can be made fairly transparent, which facilitates analysis of physical processes and model performance. (These comments are not intended to disparage general circulation models, but only to point out that process models have different, complementary goals.) In this study, we construct a reduced process model by modifying the vertical streamfunction model of Hellmer and Olbers (1989) so that it can easily be coupled with a simple model of ice shelf dynamics (Paterson, 1994). The new model is then applied to an idealized large-scale Antarctic ice shelf, and the resulting equilibrium is studied.
2. Model equations

2.1. Ocean

We begin with the standard hydrostatic, Boussinesq equations, which describe large-scale motion of an incompressible ocean:

\[ \begin{align*}
  u_t + uu_x + vu_y + wu_z - fv &= - \frac{1}{\rho_0} p_x + A_h (u_{xx} + u_{yy}) + A_v u_{zz}, \\
  v_t + uv_x + vv_y + wv_z + fu &= - \frac{1}{\rho_0} p_y + A_h (v_{xx} + v_{yy}) + A_v v_{zz}, \\
  0 &= -p_z - g\rho, \\
  u_x + v_y + w_z &= 0.
\end{align*} \]  

(1)

(2)

(3)

(4)

This notation is standard, with \( A_h \) and \( A_v \) respectively denoting horizontal and vertical eddy viscosities. The \( z \) axis is taken to be positive upwards, with \( z = 0 \) at the ocean surface. The \( y \) axis is taken to be perpendicular to the ice edge and positive towards the grounding line, with \( y = 0 \) at the ice edge.

We assume, following Robin (1979) and Hellmer and Olbers (1989), that the thermohaline circulation under an ice shelf is primarily transverse to the ice front, so that \( u \ll v \) and the flow lies in the \( yz \)-plane. Restriction to two-dimensionality forces us also to neglect Coriolis terms and \( x \)-gradients, which reduces the equations to

\[ \begin{align*}
  v_t + vv_y + wv_z &= - \frac{1}{\rho_0} p_y + A_h v_{yy} + A_v v_{zz}, \\
  0 &= -p_z - g\rho, \\
  v_y + w_z &= 0.
\end{align*} \]  

(5)

(6)

(7)

We note that while the neglect of Coriolis terms would be unjustified in a three-dimensional model, the term we have dropped from the remaining horizontal momentum equation is small when our assumption that \( u \ll v \) holds.

We now introduce a streamfunction \( \psi \) defined by \( v = \psi_z \) and \( w = -\psi_y \). Taking the curl of the remaining \( y \) and \( z \) momentum equations, and using the continuity equation to simplify the result, we obtain the vorticity equation:

\[ \begin{align*}
  \psi_{zt} + (v\psi_{zz})_y + (w\psi_{zz})_z &= \frac{g}{\rho_0} \rho_y + A_h \psi_{zzy} + A_v \psi_{zzz},
\end{align*} \]  

(8)

Defining the vorticity \( \zeta \) by \( \zeta = \psi_{zz} \), we write this equation in the form

\[ \begin{align*}
  \zeta_t + (v\zeta)_y + (w\zeta)_z &= \frac{g}{\rho_0} \rho_y + A_h \zeta_{yy} + A_v \zeta_{zz}.
\end{align*} \]  

(9)

The forcing is given by the density torque \( \frac{g}{\rho_0} \rho_y \), which couples the momentum equations to the advection of thermodynamic tracers. As a simplification, we will often assume that for the relatively narrow range of seawater properties found under ice shelves (cf. Figs. 2 and 3) the density \( \rho \) can be reasonably modeled by a linear equation in potential temperature (\( \theta \)) and salinity (\( S \)).

Tracer advection is described by the standard conservation equation

\[ \begin{align*}
  X_t + (vX)_y + (wX)_z &= K_h X_{yy} + K_v X_{zz},
\end{align*} \]  

(10)

where \( X \) represents either \( \theta \) or \( S \), and \( K_h \) and \( K_v \) are horizontal and vertical eddy diffusivities.

In order to facilitate coupling of the ocean to a one-dimensional ice shelf model, we will transform the governing equations into \( \sigma \) (terrain-following) coordinates. This will allow the ocean model to maintain a rectangular computational domain while modeling a physical domain with an irregular, time-dependent shape. We note that this approach will prevent our domain from extending to the grounding line, as \( \sigma \)-coordinates present difficulties with satisfying the Courant–Friedrichs–Levy stability criterion in very shallow waters, and...
become undefined for zero ocean depth. However, since we are not concerned with grounding line migration in this study, this approach is well suited to our present needs.

We define the new vertical coordinate by \( \sigma = \frac{T}{B} \), where \( T \) is the depth of the ice shelf base and \( B \) is the depth of the ocean floor. It follows that \( \sigma(z = T) = 0 \) at the top of the domain and \( \sigma(z = B) = -1 \) at the bottom. For the coordinate transformation which follows, we assume that \( T \) is a function of time (since the ice shelf evolves during the simulation) while \( B \) is time-independent. Note that \( \frac{\partial}{\partial \sigma} \) will now indicate a derivative taken with \( \sigma \) (rather than \( z \)) held constant.

The vorticity equation is then given by

\[
(D\zeta)_t + (Du\zeta)_y + (Dw\zeta)_z = \frac{g}{\rho_0} \mathcal{J}(\rho, z) + R(\zeta),
\]

where \( D \equiv T - B = z_\sigma \) is the total depth of the ocean (so that \( \zeta = D^{-2}\psi_{\sigma\sigma} \)), and \( \omega = \frac{\partial}{\partial \sigma}(\sigma) \) is the effective vertical velocity on the computational grid. The horizontal density gradient becomes \( \mathcal{J}(\rho, z) = z_\sigma \rho_v - z_\sigma \rho_a \), which is called the density-jacobian form, while the dissipation term is replaced by the corresponding operator from the Princeton Ocean Model,

\[
R(\zeta) = \frac{\partial}{\partial y} \left[ Dv_h \frac{\partial \zeta}{\partial y} \right] + \frac{\partial}{\partial \sigma} \left[ D \frac{\partial \zeta}{\partial \sigma} \right],
\]

which is already formulated in sigma coordinates (Mellor and Blumberg, 1985).

Similarly, the tracer equation becomes

\[
(DX)_t + (DuX)_y + (DwX)_z = R(X),
\]

where diffusion is likewise given by

\[
R(X) = \frac{\partial}{\partial y} \left[ DK_h \frac{\partial X}{\partial y} \right] + \frac{\partial}{\partial \sigma} \left[ D \frac{\partial X}{\partial \sigma} \right].
\]

Further details on the transformation to \( \sigma \)-coordinates may be found in Mellor (2004) and Walker (2006).

2.2. Ice shelf

We use a model proposed by Paterson (1994) for the flow of an ice shelf confined between parallel walls. Due to the extreme slowness of glacier flow (\( O(1 \text{ km yr}^{-1}) \)), acceleration terms may be neglected, and the equations of motion reduce to a balance of gravity against sidewall friction and hydrostatic pressure at the ice front. With the assumptions that ice is incompressible and that vertical shear is negligible, the flow becomes one-dimensional. The deviatoric stress \( s \) along the center-line of the shelf may be written as

\[
\bar{s}_y = \frac{1}{4} \rho_1 gh - \frac{\tau_0}{2H} \int_Y^Y \frac{H}{X} \, dy,
\]

where \( \rho_1 = 920 \text{ kg m}^{-3} \) is the density of ice, \( \tau_0 \) is the frictional stress (assumed constant), and \( 2X \) is the distance between the sidewalls. The height \( h \) above sea level of the shelf and its thickness \( H \) are related by the floating condition \( h = (1 - \frac{\rho_1}{\rho_W})H \), where \( \rho_W \) is the density of seawater. According to Glen’s law, the strain rate is given by

\[
\dot{e}_y = A\bar{s}_y^3,
\]

where \( A \) is a temperature-dependent parameter. Finally, since the flow is parallel to the \( y \)-axis and \( \dot{e}_y = \frac{\partial}{\partial y} \), we can integrate to find

\[
v_0(y) = v_0^I + \int_0^y \dot{e}_y \, dy,
\]
The total ice thickness follows the standard continuity equation
\[
\frac{\partial H}{\partial t} + \frac{\partial}{\partial y}(v^i H) = p - m, \tag{18}
\]
where \( p \) is the precipitation rate (assumed positive) at the ice shelf surface, and \( m \) is the melt rate (negative for freezing) at the ice shelf base. Following Holland (2002), the total thickness is divided into meteoric and marine ice layers. (Meteoric ice is formed from fallen snow and thus includes all of the glacial ice input by the ice stream; marine ice is formed when seawater freezes onto the base of an ice shelf.) Following the standard assumption of no vertical shear in an ice shelf, both layers are advected at the same velocity. We thus have
\[
\frac{\partial H_{\text{met}}}{\partial t} + \frac{\partial}{\partial y}(v^i H_{\text{met}}) = p, \tag{19}
\]
\[
\frac{\partial H_{\text{mar}}}{\partial t} + \frac{\partial}{\partial y}(v^i H_{\text{mar}}) = -m, \tag{20}
\]
where \( H_{\text{met}} \) and \( H_{\text{mar}} \) are the thicknesses of the meteoric and marine ice layers, respectively. The only exception to the above occurs when the marine ice thickness is zero and melting is indicated. In this case, the melting is applied to the meteoric ice layer.

2.3. Interface

The ice–ocean interface is subject to three constraints: the temperature must be at the local freezing point, and conservation of heat and salt are required in any phase change. These constraints lead to a system of three equations which may be solved for the interface temperature, salinity, and melt rate (Holland and Jenkins, 1999):
\[
T_B = aS_B + b + cp_B, \tag{21}
\]
\[
\{c_{pI}m(T_S - T_B)\} + c_{pw}\gamma_T(T_U - T_B) = mL_f, \tag{22}
\]
\[
\gamma_S(S_U - S_B) = mS_B, \tag{23}
\]
where the bracketed term appears only for the melting case. The first equation is a linearized expression for the freezing point, where \( T_B, S_B, \) and \( p_B \) are the temperature, salinity, and pressure at the base of the ice shelf. The coefficients are empirical constants with the values \( a = -5.73 \times 10^{-2} \degree C/\text{psu}, \ b = 9.39 \times 10^{-2} \degree C, \) and \( c = -7.53 \times 10^{-8} \degree C/\text{Pa}. \) The second equation is a heat balance, consisting of a sensible heat flux into the ice shelf, a diffusive flux between the interface and the upper layer of the ocean, and a latent heat term. The temperatures of the ice shelf and upper ocean are denoted by \( T_S \) and \( T_U, \) respectively, while \( c_{pI} \) and \( c_{pw} \) are the specific heats of ice and seawater, and \( L_f = 3.34 \times 10^5 \text{J/kg} \) is the latent heat of fusion for ice. The third equation, for salt conservation, is of the same form as the heat balance, except that the assumption of brine rejection means there is zero salt flux into the shelf. Diffusion of both heat and salt is parameterized by the exchange velocities
\[
\gamma_T = \frac{u_s}{2.12 \ln \left( \frac{u_s}{v} \right) + 12.5 Pr^2 - 9}, \tag{24}
\]
and
\[
\gamma_S = \frac{u_s}{2.12 \ln \left( \frac{u_s}{v} \right) + 12.5 Sc^2 - 9}, \tag{25}
\]
where the molecular Prandtl number \( (Pr) \) is the ratio of viscosity to thermal diffusivity and the molecular Schmidt number \( (Sc) \) is the ratio of viscosity to salinity diffusivity. The kinematic viscosity of seawater is denoted by \( v \) and taken as a constant, as is the length scale \( l \) over which mixing occurs. (The values used in this model are \( Pr = 13.8, Sc = 2432, v = 1.95 \times 10^{-6} \text{m}^2 \text{s}^{-1}, \) and \( l = 10 \text{m}. \) The friction velocity \( u_s \) at the interface is defined in terms of a dimensionless drag coefficient \( c_d \) and the upper layer velocity \( U_U \) by
\[
u_s^2 = c_d U_U^2. \tag{26}\]
This parameterization of turbulent processes follows Jenkins (1991), which is analogous to the more detailed treatment given by McPhee et al. (1987) but assumes that the ice shelf base is hydraulically smooth, unlike the bottom surface of sea ice. The model calculates the exchange velocities at each ocean time step using the horizontal velocity of the upper layer of the ocean. This dependence on ocean velocity will later be seen to strongly affect basal melting and freezing.

2.4. Boundary conditions

2.4.1. Vorticity equation

The model domain has one open boundary at the ice front, and three solid boundaries consisting of the ocean floor, the ice shelf base, and the vertical boundary near the grounding line. At the solid boundaries, the condition of no normal flow is imposed by setting \( \psi = 0 \), so that \( v = D^{-1}\psi_y = 0 \) at the vertical wall and \( \omega = \sigma_r - D^{-1}\psi_y = \sigma_t \) at the top and bottom of the domain. Note that at the ocean floor, this means \( \omega = 0 \), and that at the ice shelf base, the effective vertical velocity consists only of the motion of the interface itself, so there is no normal flow. A no-slip frictional boundary condition is imposed on \( v \) at the ocean floor and ice shelf base in order to reasonably simulate the bottom and top boundary layers. At the open boundary, a zero-gradient condition \( f_y = 0 \) is imposed on the vorticity (and thus on the streamfunction \( w \) as well, so that velocity contours follow \( \sigma \)-surfaces at this boundary). A related condition, \( \partial (\rho_z, z) = 0 \), is imposed on the density in order to prevent forcing. Although these are not formally exact radiation boundary conditions, they are sufficient (in combination with the well-posed boundary conditions of the advection–diffusion equation below) to prevent any significant wave reflection at the open boundary, while still allowing free inflow and outflow. We thus assume, following Holland et al. (2003), that the ice front is not a barrier to communication between the sub-ice shelf cavity and the open ocean.

2.4.2. Tracer advection

In order to solve the advection–diffusion equation

\[
(DX)_t + (DX\nu)_y + (DX\omega)_z = \frac{\partial}{\partial y} \left[ D\kappa_h \frac{\partial X}{\partial y} \right] + \frac{\partial}{\partial \sigma} \left[ \frac{\kappa_v}{D} \frac{\partial X}{\partial \sigma} \right],
\]  

we require boundary values of \( X \) for the advective terms and boundary fluxes of \( X \) for the diffusive terms.

At the open (ice front) boundary, boundary values of \( X \) are taken from an observed profile wherever inflow \( (v > 0) \) occurs; no boundary data is required for outflow, as the model uses an upstream-biased scheme for tracer advection. It is assumed that no diffusion takes place through the open boundary, and that no advection or diffusion occurs at the ocean floor or grounding line.

At the ice shelf base, boundary values are derived from the interface thermodynamics. The diffusive fluxes are given by

\[
\left. \frac{\kappa_v}{D} \frac{\partial X}{\partial \sigma} \right|_{\sigma=0} = \gamma_X (X_B - X_U),
\]  

where \( X_B \) is the calculated value at the interface and \( X_U \) is the upper layer value (i.e., the value at the center of the uppermost grid cell).

It is also important to consider the effect of meltwater advecting into the domain (Jenkins et al., 2001). In previous models, where the ice shelf base was stationary, this meltwater flux could not be handled as an advective boundary condition without creating an inconsistency in the modeled velocity field. Instead, a diffusive term

\[
\frac{\partial}{\partial \bar{z}} \left( \frac{\rho_f}{\rho_0} m (X_B - X_U) \right) = \frac{\partial}{\partial \bar{z}} (\bar{m} (X_B - X_U))
\]  

was added to the right hand side. If we consider forward-in-time differencing of this term, we have

\[
X^{n+1}_U = X^n_U + \frac{\Delta t}{\partial \bar{z}} (\bar{m} (X_B - X^n_U))
\]
as the new upper layer value. Recalling that \( \tilde{m} \) is a velocity, we observe that \( \alpha = \tilde{m} \delta t / \delta z \) is the proportion of the cell now filled by meltwater. The differencing then has the form

\[
X_U^{n+1} = \alpha X_B + (1 - \alpha) X_U^n,
\]

which is just a weighted average, so this procedure corrects.

Unlike earlier models, the present model explicitly includes a moving ice shelf–ocean interface, so meltwater advection is handled directly rather than by a correction to the diffusive term. The situation is complicated somewhat by the horizontal flux of ice shelf thickness, which changes the ocean thickness without any thermodynamic effect. The vertical velocity \( \omega \) at the upper boundary thus has thermodynamic and dynamic components, which must be handled separately. In order to avoid artificial forcing, the boundary values adverted by the dynamic component must equal the upper layer values; hence, the ocean interior conserves salinity and potential temperature rather than salt and heat content. The thermodynamic component is just meltwater advection and thus uses the interface values \( S_B \) and \( \theta_B \). It follows that the boundary value should be

\[
X|_{n=0} = \beta X_B + (1 - \beta) X_U,
\]

where

\[
\beta = \frac{\omega_{melt}}{\omega}
\]

and

\[
\omega_{melt} = -D^{-1} \frac{\partial t}{\partial \sigma} m = -D^{-1} \tilde{m}.
\]

Substituting this into the advection equation and applying forward-in-time differencing as before gives

\[
X_U^{n+1} = \frac{D^n}{D^{n+1}} \left[ X_U^n - \frac{\delta t \omega}{\delta \sigma} (\beta X_B + (1 - \beta) X_U^n) \right]
\]

at the top of the ocean. This may be simplified into the form

\[
X_U^{n+1} = (1 - \varepsilon) X_U^n + \varepsilon X_B,
\]

where

\[
\varepsilon = -\frac{\delta t \omega_{melt}}{\delta \sigma} \frac{D^n}{D^{n+1}} = \frac{\delta t \tilde{m}}{\delta \sigma D^{n+1}}.
\]

Noting that \( \delta \sigma D^{n+1} = \delta z^{n+1} \), we observe that \( \varepsilon \) is the proportion of the cell filled by incoming water at time \( n + 1 \), so we again have a weighted average of the tracer. This method is thus equivalent to the diffusive term of earlier models, with slightly greater accuracy since the changing volume of the cell is taken into account.

2.4.3. Ice shelf

Since the ice shelf model is one-dimensional (and the flow is unidirectional), the only boundary conditions necessary are values for the ice stream velocity \( v_I^0 \) and the ice stream flux \( v_I^0 H_0 \), where \( H_0 \) is the assumed thickness of the shelf at the grounding line. At the opposite boundary, ice thickness is allowed to advect freely out of the domain and may be assumed to be lost through calving; however, we make no attempt to explicitly model this process.

3. Numerical methods

The model is discretized on the Arakawa C grid (Arakawa and Lamb, 1977), with tracers located at cell centers, vorticity at cell corners, and velocities perpendicular to cell edges. The vorticity equation uses second-order centered differences in space and third-order Adams–Bashforth time stepping (Durran, 1991). The streamfunction is found by applying a standard tridiagonal solver to the system resulting from second-order centered differencing of the ordinary differential equation \( \zeta = D^{-2} \psi_{\sigma} \). The tracer advection equation is solved using the multidimensional positive definite advection transport algorithm (MPDATA), which
is second-order accurate in time and space (Smolarkiewicz and Margolin, 1998). Hydrostatic stability is maintained by applying the Rahmstorf algorithm for convective adjustment (Pacanowski and Griffies, 2000). The ice shelf equations, which are one-dimensional, are solved by trapezoidal integration. Full details of all numerical methods used may be found in Walker (2006).

4. Model application

4.1. Configuration

The standard configuration of the model is based upon the idealized Filchner ice shelf domain of Hellmer and Olbers (1989). In that study, the ice shelf base had a constant slope of 1/1000, with a draft of 330 m at the ice front, increasing to 930 m across the 600 km length of the domain. Because the present model simulates a flowing ice shelf, however, this profile is less than ideal for our purposes. While we will run the model until the ice shelf reaches equilibrium, any initial condition which could not have been produced by the shelf component may take centuries to advect out of the domain, delaying equilibrium and obscuring the effect of ocean thermodynamics. We instead will use an equilibrium state of the ice shelf component which is reasonably close to the linear profile of the original model; thus, all differences between the initial and final states of the shelf may be attributed to ocean interaction. (As the ice shelf component is one-dimensional, it can be run to equilibrium in seconds of computer time when there is no interaction with the ocean component.) We find that an ice stream velocity of \( u_0 = 0.5 \text{ km/yr} \) and thickness of \( H_0 = 1060 \text{ m} \) produce a reasonable initial state when a sidewall stress of \( \tau_0 = 90 \text{ kPa} \) is assumed. Since marine ice is by definition formed only by interaction with the ocean, the shelf is initially assumed to consist entirely of meteoric ice.

The bathymetry is a flat ocean floor at a depth of 1100 m. The ocean is forced by salinity and temperature restoring at the open (ice front) boundary, using CTD data from Station 292 (Rohardt, 2002) interpolated onto the \( \sigma \)-levels. In order to maintain accuracy as the depth of the ice front changes, the restoring data is re-interpolated on the ice shelf timescale of one year. Exchange velocities are calculated dynamically from the velocity of the uppermost layer of the ocean. Model parameters for this experiment are summarized in Table 1.

The model is run until the ice shelf reaches a steady state. For the control experiment, this occurs when the total ice shelf mass has been found to change by less than 0.5% between model years 500 and 600, with thickness changing by no more than 0.82% at any point during that time.

4.2. Initial ocean circulation

As in the earlier Hellmer and Olbers (1989) model, the ocean reaches an essentially steady state after less than 10 years of integration. The ice shelf thickness changes by less than 1% during this short period, so the effect of coupling is negligible. The circulation consists primarily of a large gyre (counterclockwise in the

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**Table 1**

Parameters for the control experiment

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
<th>Units</th>
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<td>s</td>
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<td>( \delta r )</td>
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<td>m(^2) s(^{-1})</td>
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<tr>
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<td>Vertical diffusivity</td>
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<td>m(^2) s(^{-1})</td>
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<td>Ice stream velocity</td>
<td>0.5</td>
<td>km yr(^{-1})</td>
</tr>
<tr>
<td>( H_0 )</td>
<td>Ice stream thickness</td>
<td>1060</td>
<td>m</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>Ice shelf sidewall stress</td>
<td>90</td>
<td>kPa</td>
</tr>
<tr>
<td>( a )</td>
<td>Precipitation rate</td>
<td>10</td>
<td>cm yr(^{-1})</td>
</tr>
</tbody>
</table>
figures) in which HSSW enters near the bottom of the cavity and a plume of ISW exits the cavity at a depth of approximately 500 m (Fig. 1). This plume reaches a peak speed of nearly 7 cm/s, while the remainder of the circulation is much slower. The meltwater signature is apparent in both salinity and potential temperature plots (Figs. 2 and 3). In particular, while the incoming HSSW has potential temperature $-1.94^\circ C$, the plume has potential temperatures as low as $-2.23^\circ C$; temperatures as low as $-2.36^\circ C$ occur inside the cavity. In

![Fig. 1. Streamfunction (m$^2$/s) of control experiment after 10 years of simulation. Ice shelf profile (shown in light blue) is vertically truncated.](image1)

![Fig. 2. Salinity (psu – 34.00) of control experiment after 10 years of simulation. White space is due to staggered computational grid.](image2)
contrast with the earlier model, the peak melt rate of 0.96 m/yr occurs 530 km from the ice front (70 km from the grounding line); Hellmer and Olbers (1989) reported a peak melt rate of 1.5 m/yr near the grounding line. This difference is primarily due to the use of velocity-dependent turbulent exchange velocities in this model. Over most of the domain, the calculated salinity exchange velocity is much higher than the relatively low constant value of $5.05 \times 10^{-7}$ m/s used in the earlier model; however, near the grounding line, where the circulation is slow, it falls below this constant. Since the melt rate is directly proportional to the salinity exchange velocity, the result is decreased melting near the grounding line. Near the ice front, where conditions favor marine ice formation, the higher salinity exchange velocity results in strongly increased freezing rates (a maximum of 0.29 m/yr, versus a maximum of 0.1 m/yr in Hellmer and Olbers (1989)).

4.3. Time evolution of the coupled system

Due to the coupling with the ice shelf, the initial steady state ocean circulation slowly evolves as the cavity shape responds to melting and freezing processes. The overall pattern of the circulation changes very little throughout the simulation (Fig. 4); the main effect is a mild decrease in its intensity as the combined effects of meltwater and reduced depth result in a higher freezing point at the shelf base, lessening the thermodynamic forcing. The base of the shelf shallows and flattens throughout most of the domain, but steepens considerably near the grounding line (Fig. 5). The most significant effect of this change in shape is a shift in the pattern of basal melting (Fig. 6). Since the ISW plume is gravitationally driven by the density gradient, a steeper basal slope allows meltwater to rise more rapidly. The increased ocean velocity beneath the steepest sections of the shelf leads to increased melting in these areas, which further increases the basal slope in a positive feedback which persists until finally balanced by horizontal flux of ice thickness in the continuity equation (18). This agrees with earlier studies, which found that increasing the basal slope of the shelf near the grounding line led to locally increased melt rate and plume velocity, but had little effect on the overall ocean circulation (Hellmer and Jacobs, 1992; Jenkins, 1991). The transition from melting to freezing occurs 140 km from the ice front throughout the entire simulation; however, the magnitude of the melt rate varies considerably. Between kilometers 150 and 530, melting decreases with time, falling by as much as 0.39 m/yr at kilometer 470 by year 600. Likely as a result, freezing near the ice front drops slightly, from 0.29 m/yr to 0.24 m/yr. Meanwhile, melting increases strongly near the grounding line, rising by as much as 1.29 m/yr at kilometer...
570 to reach a maximum of 2.17 m/yr by the end of the simulation. This result is affected by the no-normal-flow boundary condition imposed on the ocean at the grounding line, which reduces horizontal velocity in the upper layer under the shelf, thus decreasing the turbulent exchange velocities, and in turn the melt rate.

Fig. 4. Streamfunction (m² s⁻¹) of control experiment after 600 years of simulation. Marine ice (grey) is visible at the base of the shelf near the ice front.

Fig. 5. Evolution of ice shelf draft (m) for control experiment.
4.4. Final state of ice shelf

Over the course of the simulation, the ice shelf comes into a new equilibrium, with advection of meteoric ice balanced against ocean-driven melting and freezing at its base. In the process, 10.3% of the initial mass of the shelf is lost. The majority of this loss comes early in the simulation, with 4.5% of the initial mass lost in the first 100 years, and a total of 6.9% lost in the first 200 years. Marine ice thickness peaks at 20.5 m just inside the ice front, and changes little in the final 400 years of the run. Significant changes to the basal slope also occur. The initial shelf profile was close to the linear profile with slope $10^{-3}$ used in Hellmer and Olbers (1989), with a slope varying between $9 \times 10^{-4}$ and $1.3 \times 10^{-3}$. The final profile flattens throughout the majority of the domain, with slopes as low as $6.3 \times 10^{-4}$, but steepens sharply near the grounding line, reaching a peak slope of $3.6 \times 10^{-3}$. The consequences for shelf-ocean interaction of this change in shape have been previously discussed; however, there is also an effect on the flow of the ice shelf itself, since this flow is driven gravitationally by the horizontal thickness gradient. The steepening near the grounding line is outweighed by flattening in the rest of the domain, causing the velocity at the ice front to drop from 1.44 km/yr to 0.97 km/yr by the end of the simulation. This behavior is qualitatively correct according to the ice shelf model, which has a constant ice stream velocity imposed as a boundary condition; it remains to be seen whether such a loss of ice shelf mass would accelerate the ice stream in a more comprehensive model or in nature.

5. Model sensitivities

5.1. Uncertainty in physical parameters

Of the physical parameters affecting the behavior of the model, the two most important are the vertical viscosity $\nu_v$ and the vertical diffusivity $\kappa_v$. Unfortunately, due to a lack of velocity measurements for sub-ice shelf meltwater plumes, the proper values of these parameters remain uncertain. In this study, values consistent with most previous estimates have been used, but it remains necessary to examine the effects of varying these values.
The vertical viscosities considered here lie at the high end of the usual range; this is necessary due to the indirect manner in which the no-slip boundary condition at the shelf base is implemented on the Arakawa C grid (Walker, 2006). Increasing the vertical viscosity to $3 \times 10^{-3}$ m$^2$ s$^{-1}$ thus produces a relatively mild slowing of the circulation, with the maximum plume speed falling to 4.6 cm/s. As a consequence of this slowing, basal melting also decreases, so that only 7.2% of the initial ice shelf mass is lost during a 600 year simulation (versus a 10.3% loss in the control experiment). Decreasing the vertical viscosity to $1 \times 10^{-2}$ m$^2$ s$^{-1}$ produces stronger effects, with the maximum plume speed rising to 13.6 cm/s, and the mass loss to 20.4%.

Since the vertical diffusivity measures the strength of vertical mixing, we find that increasing $\kappa_v$ to $2 \times 10^{-4}$ m$^2$ s$^{-1}$ produces stronger overturning circulation, as should be expected. This effect is seen primarily in the streamfunction, which has a maximum value of 4.8 m$^2$ s$^{-1}$ after 10 years (versus 3.8 m$^2$ s$^{-1}$ for the control experiment) while the maximum plume speed is nearly unchanged. Mass loss also increases slightly, to 13.2% of initial following 600 years of simulation. Decreasing $\kappa_v$ to $5 \times 10^{-5}$ m$^2$ s$^{-1}$ results in a maximum streamfunction value of 3.1 m$^2$ s$^{-1}$ after 10 years and a final mass loss of 8.2%.

5.2. Bathymetry

To investigate the effects of bathymetry, the domain is modified so that the ocean floor slopes downwards (with a constant slope of $5 \times 10^{-4}$) from the original depth of 1100 m at the ice front to a new depth of 1400 m near the grounding line. The temperature and salinity profile from Station 292 is again imposed at the ice front. However, in order to avoid unphysical results when extrapolating, initial conditions for all tracer points below 1100 m are set to the 1100 m values. (Note that this varies somewhat from the sloping floor experiment in Hellmer and Olbers (1989) which reduces ocean depth at the ice front and shifts the observational data upwards.) The resulting flow after 600 years is shown in Fig. 7. As might be expected, the circulation intensifies when the inflowing HSSW is given a slope to descend, with the maximum plume speed reaching 10.2 cm/s after 10 years. Mass loss after 600 years increases to 16.2%, and the changes in morphology described for the control experiment are exaggerated, with the basal slope near the grounding line increasing to $5.6 \times 10^{-3}$.

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Fig. 7. Streamfunction (m$^2$ s$^{-1}$) after 600 years for sloping ocean floor experiment.
5.3. Ice shelf flow

Since surges in the flow of ice streams are a possible consequence of a warming climate, we consider experiments in which the ice stream velocity is increased (to 0.75 or 1.0 km/yr) and/or the sidewall friction is reduced (to 75 kPa). In order to isolate the effects of interaction with the ocean, the initial configuration of the shelf is determined by running the ice shelf component to steady state (as in Section 4.1) for each combination of parameters. Overall ocean circulation in these simulations varies very little from the control experiment. The resulting equilibrium shelf profiles are summarized in Fig. 8. As might be expected, increasing the ice stream velocity results in a thicker shelf, while decreasing sidewall friction (which increases the velocity difference between the ice front and ice stream) results in increased steepness near the grounding line. As might further be expected, the faster-flowing ice shelves in these experiments require less time to come to equilibrium with the ocean. (For this study, we consider an ice shelf to reach equilibrium when its total mass varies by less than 0.5% in the following 100 years.) While by this _ad hoc_ definition the control experiment reaches equilibrium in 490 years, the fastest of these variations (1.0 km/yr, 75 kPa) takes only 200 years. Partly as a result, only 5.4% of the initial shelf mass is lost during this experiment, compared with 10.3% of the (smaller) initial shelf mass for the control experiment.

5.4. Seasonal temperature cycle

In order to test the consequences of assuming a constant restoring temperature profile at the ice front, we impose on this profile a sinusoidal variation with an amplitude of 0.05°C and a period of one year. (All other parameters remain as in the control experiment.) After 10 years, we find that the maximum melt rate (near the grounding line) varies between 90.1 and 96.4 cm/yr, lagging the variation at the ice front by eight to nine months, while the average melt rate varies between 40.7 and 43.1 cm/yr. Since the corresponding melt rates for the control experiment are 96.1 and 43.4 cm/yr, the seasonal variation produces slightly less basal melting. However, this effect is quite small, with the equilibrium profile of this variation containing only 0.6% more mass than the equilibrium profile of the control experiment. We can thus conclude that moderate seasonal

![Fig. 8. Equilibrium ice shelf drafts (m) for varying flow parameters.](image-url)
temperature cycles have relatively little effect on model results for the long experiments considered in this study.

5.5. Ocean warming

Starting from the equilibrium configuration of the control experiment, we impose constant perturbations on the potential temperature profile at the ice front and investigate the effects on basal melting and ice shelf mass balance. As seen in Fig. 9, the average melt rate initially shows a greater than linear response to ocean warming. For each temperature perturbation, however, the positive feedback between basal slope and melting (described in Section 4.3) results in decreased average melting over time as the ice shelf base flattens over most of its length. Since the speed of this process increases with ocean temperature, average melting decreases more rapidly for higher temperature perturbations, and the response to warming becomes sublinear after approximately one hundred years. However, due to the effect of increasing basal steepness near the grounding line, the maximum melt rate increases over time for all of the warming experiments (Fig. 10). The results of these experiments are summarized in Fig. 11. While in nature such strong melting would likely be at least partially compensated by acceleration of the ice shelf, these results still suggest considerable sensitivity of the ice shelf to ocean warming.

6. Conclusions

The initial results produced by this model suggest that interaction with the ocean significantly affects the equilibrium mass balance and morphology of an ice shelf. While for colder ice shelves these effects take several centuries to appear, experiments with ocean warming produce noticeable changes to the ice shelf profile within decades. Such scenarios are thus attractive candidates for simulation with a full three-dimensional model to test the results of this study; as part of future modeling efforts we will seek to perform direct comparison (for short integrations where feasible) of the two-dimensional model against the model of Holland and Jenkins (2001). Uncertainties in several physical parameters due to lack of observational data suggest that
our quantitative results should not be considered exact. However, the qualitative behavior of the model is unaffected by these uncertainties, indicating that the relevant physics is accurately simulated and that the results thus provide some insight as to the processes at work in the ice shelf–ocean system.

Fig. 10. Response of maximum melt rate (m/yr) to ocean warming.

Fig. 11. Loss of initial ice mass (%) due to ocean warming.
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