Anticyclonic eddies trapped on the continental shelf by topographic irregularities

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Abstract. Nonlinear effects produced during the scattering of a barotropic shelf wave (BSW) by a spatially varying mean current are studied using a primitive equation numerical model. Both the BSW phase and the mean current propagate in the same (positive) direction along shelf/slope topography which is uniform everywhere except for a localized topographic irregularity, e.g., a submarine canyon. The mean current is specified at the upstream boundary and adjusts to the topography, closely following isolabths through the model domain. The incident BSW signal is then introduced at the upstream boundary either as a harmonic wave or as a pulse of finite duration. The BSW signal scatters its energy into other available wave modes when it encounters the topographic irregularity. The scattered wave field is dominated by evanescent modes which are trapped at the topographic irregularity and appear as intense mesoscale flows between the coast and the mean current. Nonlinear dynamics transform these large-amplitude evanescent modes into persistent eddy-like features on the shelf. The nonlinear interaction is much stronger when the current on the shelf associated with the BSW is opposite to the mean current direction (i.e., negative), so anticyclonic eddies are preferentially generated at the topographic irregularity. For a harmonic BSW, an anticyclonic eddy periodically appears when the negative current phase passes and disappears when the positive current phase passes. A BSW pulse with negative velocity at the coast produces a strong anticyclonic eddy which persists, after the pulse has passed, for a time period substantially longer than the pulse duration. A pulse with positive velocity at the coast does not generate any persistent features on the shelf. The anticyclonic eddies produce mass exchange between the shelf and the mean current and could contribute significantly to cross-shelf exchange on continental shelves.

1. Introduction

Enhanced mesoscale variability over continental shelves often occurs near topographic irregularities where the shelf and slope topography deviates substantially from its typically smooth along-shelf form. In numerous cases, anticyclonic flow features have been observed on the shelf between the coast and a mean current which flows cyclonically over the outer shelf and mid-to-upper slope (i.e., with the coast on its right in the northern hemisphere). These anticyclones are typically of order 10-100 km in diameter and persist for several weeks or longer. For example, Schumacher et al. [1993] described anticyclonic eddies trapped in the Shilikof sea valley downstream of Kodiak Island, Alaska. Royer et al. [1979] presented evidence of anticyclones in the northern Gulf of Alaska, between Kayak and Montague Islands. Eide [1979] reported an anticyclone trapped over Halten Bank off the Norwegian coast. Oguz et al. [1996] observed persistent mesoscale shelf flows in the northwestern part of the Black Sea, downstream of the Crimean peninsula, where both shelf and slope topographies change dramatically. Similar features have also been reported by Bray and Greengrove [1993] and Largier et al. [1993] over the northern California coast in the vicinity of Point Arena, where the shelf narrows rapidly.

Numerous studies have demonstrated the generation of mesoscale flows by the resonant interaction of an anticyclonic mean current with topographic irregularities, flows which might be further amplified by the instability of the mean current [e.g., Mitsudera and Grimshaw, 1991]. However, this mechanism does not operate in the case of a cyclonic mean current [e.g., Freeland, 1990]. In fact, there are only a few examples in which mesoscale flows are generated either by the adjustment of a cyclonic mean current over a topographic feature [e.g., Willmott and Collings, 1994; Gjevik and Moe, 1994] or by localized instabilities which are triggered by a topographic feature [e.g., Viera and Grimshaw, 1994].

Recently, we have proposed an alternative mechanism for mesoscale flow generation at topographic irregularities.
ties involving the scattering of coastally trapped waves [Yankovsky and Chapman, 1995, 1996] (hereinafter referred to as YC95 and YC96, respectively). The basic idea is as follows. The shelf/slope region acts as a waveguide for coastally trapped waves, which are essentially the shelf flow response to atmospheric (or other) forcing at subinertial frequencies. The waves propagate their phases cyclonically, i.e., in the same direction as the mean currents mentioned above. As a wave encounters a topographic irregularity, it scatters into other available wave modes. In the presence of a sheared mean current, only a few propagating modes may be available to carry energy downstream, so evanescent modes of substantial amplitude can be generated. The evanescent modes do not propagate energy in the along-shelf direction but rather decay exponentially both upstream and downstream from the scattering region. When combined with the mean current, the evanescent modes can appear as intense closed mesoscale flows on the shelf between the coast and mean current. They gradually propagate through the scattering region and disappear downstream. New features develop during the next wave period.

Both of our previous studies were highly idealized. The steady (mean) current was assumed to adjust to the irregular topography, exactly following isobaths, so the steady state dynamics did not contain any mesoscale phenomena. The wave dynamics were assumed linear and purely harmonic in time. In addition, the models were barotropic, so the incident waves were the unstratified version of coastally trapped waves, i.e., barotropic shelf waves (BSWs). Clearly, many questions were left unanswered regarding these assumptions and the applicability of the results to more general situations. For instance, how does a steady current adjust to variable topography in a fully nonlinear model? Does the flow closely follow isobaths as assumed in the previous models? Do nonlinear processes alter the wave scattering and the interaction of the wave with the mean current? What is the response to a nonperiodic wave forcing, such as a single wave pulse? Are mesoscale flows still generated, and if so, how persistent are they?

To address these issues, we have extended our ideas (YC95, YC96) to fully nonlinear dynamics which include wave and current interactions as well as mean current adjustment to irregular topography. Wave forcing still consists of BSWs (i.e., the fluid is unstratified), but the forcing may be either harmonic or a pulse of finite duration. We use the semispectral primitive equation numerical model (SPEM) developed by Haidvogel et al. [1991]. The model configuration and application are presented in section 2. The results appear in section 3, followed by a discussion and summary in section 4.

2. Model Description

We use SPEM [Haidvogel et al. 1991] to solve the nonlinear momentum and continuity equations for an unstratified, hydrostatic flow on a rotating $f$ plane:

\begin{align}
\frac{u_t + uu_x + vv_y + ww_z - f v}{\rho} &= \frac{1}{\rho} \nabla^2 u \\
\frac{v_t + uu_x + vv_y + ww_z + f u}{\rho} &= \frac{1}{\rho} \nabla^2 v \\
p_x &= -g \rho \\
ux + vy + wz &= 0
\end{align}

Here $(u, v, w)$ are the along-shelf $(x)$, cross-shelf $(y)$, and vertical $(z)$ velocities; $p$ is the pressure; $f$ is the Coriolis parameter; $\rho$ is the water density; and $g$ is the acceleration due to gravity. Subscripts denote partial differentiation with respect to time $t$ and the spatial coordinates. Some lateral mixing of momentum is required for numerical stability of the nonlinear flows, so Laplacian diffusion is applied along sigma-coordinate surfaces (see Haidvogel et al. [1991] for details) with a very small diffusion coefficient of $A = 10^{-7}$ m$^2$ s$^{-1}$. The lateral mixing is small enough that it does not affect the solutions in any important ways. Further, there is basically no vertical structure to the flows presented here, so the orientation of the lateral mixing is immaterial.

The numerical domain is a channel (Figure 1) with a wall at $y = 0$ representing the coast with depth $h_0$ and a wall at $y = W = 60$ km where the shelf/slope topography joins the deep sea with depth $H$. The cross-shelf boundaries at $x = 0$ and $x = L = 400$ km are open and are described below. The bottom topography is uniform along the channel everywhere except a region of finite length which we call the scattering region. Outside the scattering region, the depth increases exponentially away from the coast as

$$h = h_x = h_0 e^{\lambda y} \quad x < X_u, x > X_d$$

where $X_u$ and $X_d$ are the upstream and downstream edges of the scattering region, respectively, (denoted by asterisks along the coast in Figure 1) and $\lambda$ is a constant. Inside the scattering region, the depth is given by

$$h = \frac{h_x + h_d}{2} + \frac{(h_x - h_d)}{2} \cos \left[ \frac{2\pi(x - X_u)}{X_d - X_u} \right]$$

$$X_u \leq x \leq X_d$$

**Figure 1.** Standard model geometry with parameters given in Table 1. The downstream sponge layer (350 km $< x < 400$ km) is not shown. The mean current is indicated by vectors at $x = 0$. Solid curves are depth contours. The scattering region is located between the asterisks on the coastline ($y = 0$).
The cross-shelf depth profile at the center of the scattering region. This topography is similar to that used by YC96 except that the slope at the center of the scattering region is linear here, which introduces greater deviations of the isobaths than in YC96. Unless otherwise noted, standard parameter values used here are listed in Table 1. Values of \( X_0 \) and \( X_d \) are chosen to provide the desired width of the scattering region. The horizontal model grid is rectangular with typical resolution of \( Ax = 5 \text{ km} \) and \( A y = 0.94 \text{ km} \). Only five Chebyshev polynomials are used in the vertical because there is basically no vertical structure.

A rigid lid is assumed at the surface. Free-slip boundary conditions are applied at the channel walls and at the bottom. A sponge layer is applied over the last 50 km of the domain with a radiation condition at the downstream open boundary (\( x = L \)) as described by Gawarkiewicz and Chapman [1992]. Although not perfect, this boundary condition allows both mean currents and propagating waves to exit the model domain with little reflection or other contamination of the flow in the channel.

The forcing is applied at the upstream open boundary, \( x = 0 \) (Figure 1). At time \( t = 0 \), a mean along-shelf current is imposed with a Gaussian cross-shelf profile:

\[
u_m = U e^{-\delta^2(y-y_c)^2}
\]

where \( U \) is the maximum velocity, \( \delta \) is the inverse width scale, and \( y_c \) is the offshore location of the current maximum, typically over the upper slope. Standard parameter values (Table 1) are chosen so that the current profile (7) is stable; that is, the cross-shelf derivative of the potential vorticity of the mean current does not change sign. The mean current is stable over the exponential depth profile provided that \( U < 0.31 \text{ m s}^{-1} \). The imposed mean current inflow is held fixed until the flow in the channel has adjusted to the topography and has reached a steady state. This provides a test for whether or not the topographic irregularity causes the mean flow to become locally unstable and also provides a test of our previous assumption (in YC95, YC96) that the mean flow follows the isobaths exactly.

After the mean current has become steady, an incident BSW is imposed at \( x = 0 \), while maintaining the mean inflow. The structure of the BSW is determined by solving for the free BSW modes over the depth profile (5) in the presence of the sheared mean current (7) as described by Narayanan and Webster [1987] but using the same staggered grid as in SPEM. The amplitude of the incident BSW along-shelf currents is denoted by \( u_w \). The incident BSW may be harmonic in time with frequency \( \omega \) (section 3.3), or it may consist of a pulse of finite duration (section 3.4). In most cases, the incident wave is introduced as a first-mode wave, and its velocity amplitude is typically chosen to be comparable to the mean current maximum \( U \) (Table 1). This does not generally cause nonlinear effects when a single mode is present, i.e., upstream of the scattering region, because the BSW along-shelf currents are largest at the coast and decay rapidly offshore, becoming quite small where they overlap with the mean current. Thus the use of a linear BSW solution for the incident wave is reasonable.

3. Results

3.1. Uniform Channel

We begin by examining the mean flow development and the dispersion properties of the BSWs in a uniform channel (i.e., with no scattering region) with depth profile (5), mean current profile (7), and the standard parameter values (Table 1). This provides a test of the numerics and is useful for the interpretation of later results. The free-wave dispersion curves are shown in Figure 2. There are three propagating BSW modes with real frequencies and wavenumbers and with phase speeds greater than the maximum velocity of the mean current (\( U = 0.3 \text{ m s}^{-1} \)). All higher BSW modes are eliminated by the sheared mean current (see YC95 for more details). With so few propagating modes, we may expect that scattering will generate evanescent modes as found by YC95 and YC96.

When the mean current (7) is introduced at the upstream boundary, a shelf wave front is immediately generated and moves along the channel with a train of shorter dispersive waves following (as found by Hsieh and Gill [1984]). The front passes through the downstream boundary after about 10 days, and the flow reaches a steady state after about 100 days. The long adjustment time (compared to the duration of the shelf wave frontal passage) is caused by shelf waves with group velocity close to zero. The resulting steady state current follows the isobaths almost exactly because of the lack of bottom friction and the weak lateral diffusion of momentum.

At \( t = 100 \) days, a first-mode BSW with frequency \( \omega = 0.2f \) (period of 3.64 days) is introduced at the upstream boundary while maintaining the steady flow. Several wave periods are required to establish a purely

<table>
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<th>Parameter</th>
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<td>( W )</td>
<td>60 km</td>
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<td>( L )</td>
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</tr>
<tr>
<td>( H )</td>
<td>2100 m</td>
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<tr>
<td>( h_0 )</td>
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<tr>
<td>( \lambda )</td>
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<tr>
<td>( U )</td>
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<tr>
<td>( \delta )</td>
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<tr>
<td>( \rho )</td>
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<tr>
<td>( u_w )</td>
<td>0.3 m s^{-1}</td>
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<tr>
<td>( T )</td>
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<td>( t_b )</td>
<td>40 days</td>
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<tr>
<td>( A )</td>
<td>10 m^2 s^{-1}</td>
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3.2. Comparison With Linear Results

Before examining cases of strong scattering with associated nonlinear dynamics, it is useful to test the performance of the numerical model for weaker scattering and to compare the results with the equivalent linear problem (YC96). Therefore we use the model geometry described in section 2 but with the depth profile in the middle of the scattering region used by YC96:

$$h_0(y) = H - (H - h_0) \cos(\pi y/2W)$$

The scattering region is located between $X_u = 72$ km and $X_d = 240$ km, a rather gentle change in topography. The mean current with maximum velocity $U = 0.3$ m s$^{-1}$ is allowed to adjust for 60 days, by which time it becomes steady and closely follows the isobaths. A first mode BSW is then introduced at $x = 0$ with frequency $\omega = 0.2 \times 10^{-4}$ s$^{-1}$ (i.e., $\omega = 0.2f$) and velocity amplitude $u_w = 0.1$ m s$^{-1}$.

Figure 3a shows the solution of the linear problem of YC96, using their nondimensional parameters which correspond to the present case: $Ro = 0.05$, $\lambda = 3.9$, $X_u = 1.2$, and $X_d = 4$. Figure 3b shows instantaneous stream function fields of the wave part of the numerical solution (i.e., having subtracted the mean current at day 60 from the full solution) after five wave periods. The BSW velocity field from the numerical solution is purely periodic and nearly harmonic in time, indicating the linearity of the dynamics. The stream function fields in Figure 3, though not identical, are quite similar in structure, showing that the two methods produce consistent results.

3.3. Periodic Forcing

In our previous studies (YC95, YC96) we considered the scattering of periodic (harmonic in time) BSWs in terms of linearized equations. However, the localized
amplification of the wave velocity field and the remarkable increase in the spatial structure associated with evanescent modes might cause substantial nonlinear dynamics at the scattering region. To investigate this possibility, we extend the dynamics of YC96 to the fully nonlinear regime. There are two ways to enhance the nonlinear dynamics of BSWs at the scattering region: (1) by increasing the amplitude of the incident wave or (2) by increasing the strength of the scattering. Therefore we have examined the numerical model response to forcing by a periodic (harmonic) incident BSW for various wave amplitudes, wave frequencies, mean current amplitudes, and topographies. The procedure in each case is as described above; the mean current is applied for 60 days, and then the incident wave is introduced. Solutions are examined after several wave periods to allow higher-frequency transients to pass out of the model domain.

First, the effect of increasing wave amplitude is examined. The topography is given by (5) and (6) with the standard values in Table I and X = 60 km and Xa = 240 km. The scattering region is actually slightly longer than that used for the comparison with the linear solution (section 3.2). However, the depth contours deviate more within the scattering region, so scattering should be stronger.

The mean current again follows the isobaths very closely and could be approximated with minimal error as flowing exactly parallel to the isobaths. As in the previous subsection, a first-mode BSW with frequency co = 0.2f is introduced with velocity amplitude of uw = 0.1 m s^-1. The instantaneous wave stream function (mean current at 60 days has been subtracted) after five wave periods is shown in Figure 4a. The presence of more small-scale features than in Figure 3, especially downstream of the scattering region, supports the contention that this topography scatters more than that used by YC96. Comparison with the results from the linear model (not shown) again indicates good agreement for this small-amplitude wave.

An incident wave with a larger velocity amplitude (uw = 0.3 m s^-1) generates much stronger mesoscale circulations in the wave field (Figure 4b). The smaller scales of Figure 4a are greatly amplified; that is, many more closed contours appear within the features with 50-70 km along-shelf scales. The most intense circulations appear within and downstream of the scattering region. The strongest feature is located at about x = 100 km and decays upstream, indicating the presence of an evanescent mode as in YC96.

The total velocity field (including both mean and wave currents) is shown in Figure 4c corresponding to the stronger incident wave (Figure 4b). The effect of the scattered wave pattern downstream of the scattering region creates the general picture of a highly variable and meandering current. Scattering, in this case, can excite only three propagating modes with wavelengths of 490, 210, and 100 km for the first, second, and third modes, respectively. It appears from Figure 4c that the third mode dominates the response downstream, as a result of increased nonlinearity, generating flow structures with along-shelf scales of about 50 km. The largest velocity occurs at the coast near the upstream edge of the scattering region at about x = 80 km. When the wave crest passes this location and has a negative value (directed toward -x), the velocity at the coast reaches 0.7 m s^-1,

Figure 4. Scattering of a BSW with frequency ω = 0.2f over the standard topography (5) and (6) and parameters from Table 1. Figures 4a and 4b show instantaneous wave stream function (total solution minus the mean current at 60 days) for an incident wave amplitude of uw = 0.1 m s^-1 (contour interval is 5 x 10^4 m^3 s^-1) (Figure 4a), and uw = 0.3 m s^-1 (contour interval is 10^5 m^3 s^-1) (Figure 4b). Solid curves are positive stream function contours, and short-dashed curves are negative stream function contours. Figure 4c shows the total velocity field corresponding to Figure 4b. Vectors are plotted at every other grid point. Long-dashed curves are the 200-, 1000-, and 1800-m isobaths.
which is substantially greater than either the mean current or the wave amplitude or their combination. The result is a strong anticyclone at the upstream edge of the scattering region between the coast and the mean current.

The strength of this anticyclone increases dramatically as the incident wave frequency is decreased. Figure 5 shows the velocity field at 1/8 period intervals through half of a wave period for the same case as shown in Figure 4c ($U = u_w = 0.3 \text{ m s}^{-1}$), except that the incident wave frequency is reduced to $\omega = 0.1f$. The wavelengths of the propagating modes are now roughly twice those at the higher frequency because the waves are nearly nondispersive in this frequency range (see Figure 2). Consequently, the along-shelf spatial scales are longer and the flow is less energetic downstream of the scattering region in Figure 5a than in Figure 4c. However, the anticyclone which appears near the upstream edge of the scattering region now has velocities which exceed 1 m s$^{-1}$. Its spatial scale is roughly the same as in Figure 4c.

Figure 5 also shows that the response to a harmonic incident wave is no longer harmonic, a clear indication of a nonlinear response. After three or four wave periods, the response throughout the channel becomes periodic, but important asymmetries occur through the wave period. For example, when the negative velocity

wave crest (measured at the coast) passes the scattering region (Figure 5a), the anticyclone forms. Currents are weak within and downstream of the scattering region. As the wave velocity slows and reverses, the anticyclone gradually disappears (Figures 5b and 5c), but a cyclone does not develop (Figures 5d and 5e). Instead, the current near the coast is amplified within and downstream of the scattering region without any noticeable feature upstream. As a result, the currents in Figures 5a and 5e, which are exactly 1/2 period apart with respect to the incident wave, are remarkably different in both current structure and strength.

The importance of the anticyclone for cross-shelf exchange may be demonstrated by following neutrally buoyant (i.e., Lagrangian) floats through the flow field

Figure 6. Lagrangian particle trajectories during three wave periods of the scattering shown in Figure 5. A particle was released at the upstream end (toward $x = 0$) of each curve.
Figure 7. Instantaneous velocity fields as in Figure 4c but with shorter scattering regions: (a) $X_u = 60$ km and $X_d = 150$ km and (b) a canyon with $X_u = 90$ km and $X_d = 135$ km. Vectors are plotted at every other grid point in Figure 7a and at every fourth grid point in Figure 7b.

The method of float tracking is described by Hedstrom [1990]. Typical trajectories of floats released at several locations between the mean current and the coast are illustrated in Figure 6. Each float is released at the time corresponding to Figure 5a (after the response to the incident BSW has become periodic) and is followed for three wave periods (21.8 days). As expected from Figure 5, the behavior is very different upstream (in the region where the anticyclone develops) and downstream of the scattering region. Floats released upstream are quickly injected into the mean current by the anticyclone, where they rapidly move along the shelf and leave the domain. Even floats released very near the coast are swept across and then along the shelf in this manner. In contrast, floats released downstream primarily oscillate in place with a small along-shelf drift which eventually carries them out of the model domain. Thus the anticyclone upstream of the scattering region greatly enhances cross-shelf motion.

The strength of scattering may be increased by shortening the length of the scattering region. Two different topographies are considered here; one with $X_u = 60$ km and $X_d = 150$ km and the other more "canyon-like" with $X_u = 90$ km and $X_d = 135$ km. The canyon-like topography requires increased along-shelf resolution, so twice the number of along-shelf grid points are used ($\Delta x = 2.5$ km) with the same channel length ($L = 400$ km). Otherwise, all parameters have their standard values (Table 1). The incident wave frequency is $\omega = 0.2f$, and both the mean current and wave amplitudes are 0.3 m s$^{-1}$.

In both cases, the adjusted mean current closely follows the depth contours. There is some minor spreading of mean current stream function contours over the canyon "corners," but there is no eddy development at the scattering region associated with the adjustment process. Figure 7 shows the velocity fields after the response has become periodic (as in Figure 4c), at the moment when the negative velocity crest nears the upstream edge of the scattering region and the anticyclone is at its maximum intensity. Qualitatively, the response is the same as before, but there are some quantitative differences. Compared to the previous cases, the anticyclones in Figure 7 (1) are more intense (...their maximum velocities exceed 1 m s$^{-1}$), (2) are slightly more compact (...they decay faster both upstream and downstream), and (3) are more complicated in their structure. These features may be seen more definitively by comparing the along-shelf velocities at the coast (Figure 8a) and across the shelf through the anticyclone (Figure 8b). In addition, a relatively weak but noticeable anticyclone develops at the downstream edge of the scattering region for the canyon topography (Figure 7b). All of these changes result from the enhanced generation of evanescent modes. As the along-shelf scale of the scattering region is reduced, the incident wave adjustment through the scattering region occurs over a shorter distance. This excites higher evanescent modes with more complex cross-shelf structure, shorter along-shelf scales, and more rapid along-shelf decay.

As in the previous calculations, the response is periodic but not harmonic in time. The development of the anticyclone is followed by the amplification of currents in the middle of the scattering region. A cyclone counterpart to the anticyclone does not form. The downstream response is again highly variable owing to the excitation of all available propagating modes.

So far, the mean current has been stable. To demonstrate that this is not essential to the above results, the calculation shown in Figure 4c is repeated with a stronger mean current, $U = 0.45$ m s$^{-1}$, for which the cross-shelf derivative of potential vorticity changes sign. Thus the mean current could be unstable. However, the mean current adjusts with no signs of unstable (growing) waves, and the resulting steady state flow has the same spatial structure as the previous cases but with larger velocities appropriate for the stronger inflow. A linear stability analysis of the inflow current (over the exponential depth profile) shows that the most unstable wave has a period of 25.3 hours, a wavelength of 39.7 km, and an $e$-folding time of 97.1 hours. Clearly this wave, if generated within the model domain, would leave the domain before its amplitude could grow substantially. For completeness, we have introduced a BSW with frequency $0.2f$ and amplitude $u_w = 0.3$ m...
Figure 8. Instantaneous along-shelf velocity for the cases shown in Figure 4c (dashed line) and Figure 7b (solid line): (a) along-shelf distribution at the coast and (b) cross-shelf distribution at $x = 65$ km (case in Figure 4c) and $x = 70$ km (case in Figure 7b).

These last calculations serve to justify some of the assumptions made in our previous linear studies (YC95, YC96). In particular, they show that a slightly unstable mean current (i.e., one whose velocity slightly exceeds the threshold of instability) may keep its shape for a long time if unstable modes are not explicitly excited by external forcing to have substantial amplitudes within the model domain. Therefore the examples considered by YC96 with $Ro=0.07$ are reasonable, and the conclusion that energy is extracted from the mean current when multiple BSW modes are present seems to be valid. To be absolutely convinced, we have calculated the scattering of a second-mode BSW corresponding to the case in YC96 and find no development of localized instabilities at the scattering region where, in the linear problem, energy is transferred from the mean current to the wave field (i.e., the resulting flows are purely harmonic with the frequency of the incident wave).

3.4. Pulse Forcing

A harmonic incident BSW is a highly idealized choice of forcing. Somewhat more realistic is an incident wave pulse of finite duration which could be thought of as representing the coastal ocean response to an atmospheric forcing event, e.g., a storm of finite scale and duration. Therefore we examine next the response to an idealized wave pulse which is given the cross-shelf structure of the first mode BSW with frequency $\omega = 0.1f$. At this frequency, the first mode is nearly nondispersive, so the pulse can propagate along the shelf while maintaining its shape, provided that little energy is produced at higher frequencies. The time dependence of the pulse is Gaussian:

$$u_p = u_c(y) e^{-(t-t_p)^2/T^2} \quad t_b < t < t_b + 3 \text{ days}$$

$$u_p = 0 \quad \text{otherwise} \quad (9)$$

where $u_c$ is the cross-shelf structure of the pulse, $t_p = t_1 + 1.5$ days, $T = 1$ day, and $t_1$ is the time when the pulse enters the model domain. The maximum velocity of the pulse is $\pm 0.3$ m s$^{-1}$, the sign corresponding to the direction of the current at the coast relative to the direction of the mean current. The topography and mean current are identical to those of the canyon case (Figure 7b).

The response to forcing by a wave pulse is dramatically different for positive and negative pulses. Figures 9 and 10 show velocity vectors at various times for the two cases. Only the sign of the incident pulse has been changed. The pulse travels through the 350-km domain shown in exactly 2.5 days, so the crest of the pulse reaches the downstream end of the domain at $t_b + 4$ days. The positive pulse (Figure 9) moves through the scattering region with little disruption. The velocities within the scattering region are temporarily amplified, but the pulse basically continues through the model domain, leaving behind little evidence of its passage. In contrast, the negative pulse (Figure 10) generates an
Figure 9. Instantaneous velocity fields generated by the passage of a positive BSW pulse given by (9) with parameters listed in Table 1 and maximum velocity of 0.3 m s⁻¹ over the canyon topography with \( X_0 = 90 \) km and \( X_d = 135 \) km. Times, in days after \( t_b \), are (a) 1, (b) 2, (c) 4, (d) 6, and (e) 8. Vectors are plotted at every fourth grid point. Dashed curves are the 200-, 1000-, and 1800-m isobaths.

Figure 10. Same as Figure 9, but for a negative BSW pulse.
anticyclonic eddy near the upstream edge of the scattering region which remains for a long time after the pulse has left the domain. The velocities within the anticyclone reach $1 \text{ m s}^{-1}$, far exceeding either the mean current maximum or the pulse maximum. This anticyclone is clearly the counterpart of the anticyclone which appeared during the harmonic forcing each time a negative wave velocity passed the scattering region (section 3.3). Furthermore, if the harmonic forcing is terminated after several wave periods, a similar anticyclone persists for a time equivalent to many wave periods thereafter.

A different view of pulse scattering and anticyclone formation is obtained from a phase plot in which the along-shelf velocity at the coast is contoured in time and space. Figures 11 and 12 show the results for positive and negative incident pulses, respectively. The thick solid lines represent free-wave phase speeds. In both figures, the pulse can be seen approaching the scattering region at constant speed corresponding to the first-mode BSW (i.e., straight contours parallel to the free-wave speed). Upon encountering the topographic irregularity, the current at the coast intensifies. The lack of activity near the inflow boundary ($x = 0$) after the pulse passes ($t > 80$ hours) confirms that virtually no reflection occurs at the scattering region. Most of the positive pulse (Figure 11) continues through the model domain at the first-mode wave speed. In addition, a much weaker second-mode pulse is excited and moves more slowly through the model domain. This process is qualitatively similar to the linear model of BSW pulse scattering described by Wilkin and Chapman [1987]. After 180 hours, the entire disturbance has passed with virtually no residual current at the coast.

The behavior of the negative pulse (Figure 12) is quite different. The pulse seems to stop and then back up slightly after encountering the topographic irregularity. This persistent feature is the anticyclone shown in Figure 10. A shadow zone with nearly zero velocity occurs for about 30 km downstream of the scattering region ($135 \text{ km} < x < 165 \text{ km}$). A portion of the incident pulse energy passes through the model domain as two small-amplitude pulses with opposite signs. The first pulse represents the continuation of the incident pulse, moving with about the first-mode (nondispersive) wave speed. The second pulse is generated during the anticyclone formation and moves with about the second-mode wave speed. These are followed by some very weak transient negative currents associated with second- and third-mode waves. After 180 hours, only the anticyclone remains, trapped upstream of the scattering region and decaying rather slowly with time.

The response downstream of the scattering region is somewhat reminiscent of the response of a coastal ocean to a localized storm of finite duration considered by Carton [1984]. In his examples, coastally trapped waves are generated at the edges of the storm and propagate away at the phase speed corresponding to each mode. Far downstream of the storm, the modes have separated so each mode passes as a distinct entity. In Carton's examples, there are two sources of coastally trapped waves: the upstream and downstream edges of the forcing region. This results in the maximum response occurring downstream of the forcing region. Further, an infinite number of propagating modes exist because there is no sheared mean current. In our case of a negative pulse, the strong anticyclonic eddy develops as the pulse enters the scattering region, and two pulses with opposite signs propagate downstream as first- and second-mode BSWs. The anticyclone might be thought of as a source for the second pulse. Higher modes are
not available (beyond the third), so radiation cannot continue as in Carton's examples. Of course, the details of the dynamics are quite different in the two problems, so the comparison is only suggestive.

The anticyclone is generated by the nonlinear interaction of the mean current and the scattered wave field. When the wave pulse encounters the canyon, the velocities increase at the upstream edge owing to the generation of evanescent modes as the pulse scatters. For a positive pulse, the combination of the pulse current with the mean current does not create substantial cross-shelf shear in the along-shelf velocity because both velocities are in the same direction. In contrast, the combination of the negative pulse current with the mean current creates strong cross-shelf shear, and the nonlinear terms become of leading order in the momentum balance. This is illustrated in Figure 13, which shows the largest terms in the along-shelf momentum balance (1) near the scattering region at the moment when the negative pulse maximum has just passed the scattering region. This is the time of maximum strength of the anticyclone (t = t6 + 3 days). The nonlinear terms dominate near the coast within the anticyclone, requiring all other terms in order to balance momentum. The nonlinear terms are dominated by ∂u/∂y. Additional calculations show that the nonlinearity decreases for smaller maximum pulse velocities. The resulting anticyclone is not only weaker, but its spatial scale is reduced and its center is closer to the scattering region. For the positive pulse, the momentum balance at the same time (not shown) just exhibits the geostrophic balance within the "meander" of the mean current over the canyon and a minor nonlinearity in the center of scattering region at the coast.

An obvious question arises as to what effect a positive pulse has on the anticyclonic eddy generated by a previous negative pulse. Does it simply destroy the eddy? Figure 14 shows the velocity vectors at various times after a positive pulse, with the same amplitude as the previous negative pulse, enters the model domain. Figure 14a shows the positive pulse as it meets the anticyclone. Subsequently, the positive pulse seems to carry the anticyclone into the scattering region where it is greatly reduced in magnitude. A much weaker anticyclone emerges from the scattering region (Figure 14c) and begins to propagate along the shelf and eventually out of the model domain. As it moves it also becomes noticeably longer. Figure 15 reveals that the weaker anticyclone (negative contours downstream of the scattering region) is moving downstream with the phase speed of the third-mode BSW. Its cross-shelf size is most consistent with the third-mode and the lengthening results from dispersive effects. The point is that the positive pulse does not completely destroy the anticyclone. Instead, it carries the anticyclone past the topographic irregularity before rapidly passing out of the model domain. The remaining anticyclone is no longer trapped, so it can move along the shelf. If it encounters another topographic irregularity, it presumably can become trapped and amplified again.

3.5. Wider Shelf

In all of the examples considered above, the topographic irregularity reduced the shelf width substantially within the scattering region, to the point that the shelf nearly vanished at the center of the scattering region. Consequently, the mean current flows very close to the coast because it follows the isobaths so closely. Thus the mean current easily interacts with the wave field (either harmonic or pulse forcing) because the maximum wave velocities occur at the coast. This may not be the case if the topographic irregularity does not extend so close to the coast. To illustrate, we consider a topography exactly like the canyon case above, but with a 20-km-wide strip of uniform shelf added, across which the depth decreases exponentially from 45 to 13 m (i.e., the new coastal depth is h0 = 13 m). The mean current is imposed for 60 days in order to reach a steady state (i.e., t6 = 60 days). All other parameters are identical to those in the standard calculations described above (Table 1).

We first consider the scattering of a first-mode harmonic BSW with frequency ω = 0.2f. Mesoscale features are again generated shoreward of the mean current by the scattering process. The horizontal scales are now smaller than the total shelf width, so several eddies exist between the mean current and the coast. The flows are periodic after several wave periods, but they are again not harmonic. Figure 16 shows the velocity vectors averaged over a wave period. Eddy-like patterns appear within the scattering region with velocities in the range of 0.1-0.2 m s⁻¹ even after averaging. (Instantaneous
velocities are much stronger, reaching 0.6 m s\(^{-1}\).) As in the previous examples, they are enhanced after the passage of the wave crest with negative velocity at the coast.

The passage of a negative pulse (as in section 3.4) reveals that the response is much more complicated than the narrow shelf case. The cross-shelf structure of the incident pulse corresponds to a free-wave structure at \(\omega = 0.08f\); the choice of a slightly lower frequency than the previous example ensures that the pulse is nondispersive. Figure 17 shows a time sequence of velocity vectors during and after the passage of the pulse. Numerous eddies, both anticyclonic and cyclonic, are formed during the scattering process. After the pulse

Figure 14. As in Figure 10, but for a positive pulse arriving 8 days after the negative pulse (i.e., immediately after Figure 9e). Times, in days after \(t_b\) for the positive pulse, are (a) 1, (b) 2, (c) 4, (d) 6, and (e) 8. Vectors are plotted at every fourth grid point. Dashed curves are the 200-, 1000-, and 1800-m isobaths.

Figure 15. As in Figure 11, but for the positive pulse following the negative pulse shown in Figure 14.

Figure 16. Velocity field resulting from a periodic incident BSW over the wider shelf with the canyon topography, averaged through a wave period. Incident wave amplitude is \(u_\omega = 0.3\) m s\(^{-1}\) and \(\omega = 0.2f\). Other parameters are listed in Table 1. Dashed lines are the 45-, 200-, and 1000-m isobaths.
passes, mesoscale features remain for a long time, but they continue to evolve while slowly weakening. They are not stationary as the single anticyclone was over the narrow shelf.

Figure 18 shows the space-time development of the along-shelf velocity at the coast for the same calculation. Most of the negative pulse passes through the scattering region and out of the model domain, while some energy propagates as a second-mode pulse with negative velocity at the coast; much like the positive pulse over the narrow shelf (Figure 10). There is some indication that energy may also escape as a third-mode pulse. The remaining eddies move slowly along the shelf, oscillating in strength as they move. The oscillations excite evanescent modes upstream of the scattering region with very short scales and reversing velocities. A single strong anticyclone is not formed in this case because the coastal waveguide is not blocked by the topographic irregularity to the extent that it was on the narrow shelf and the BSWs (propagating and evanescent) have different scales. Nevertheless, an anticyclone does persist for some time near the upstream edge of the scattering region.

The trajectories of Lagrangian floats during the pulse passage (not shown) demonstrate strong cross-shelf motion (i.e., lateral mixing) on the shelf within the scattering region associated with the eddy development. Floats released within the eddies move in complicated ways with substantial cross-shelf displacements. Floats released outside the eddy region tend to move in very small, nearly elliptical trajectories, basically remaining stationary.

4. Discussion and Summary

The numerical calculations described above illustrate some properties of the nonlinear dynamics of barotropic shelf waves which encounter topographic irregularities.
in the presence of a sheared mean current. We emphasized these effects by considering a mean current which is weak enough to maintain its structure (i.e., it does not develop exponentially growing unstable waves) and by choosing the incident wave (either harmonic or pulse) of small enough amplitude that its propagation in the uniform channel is almost linear.

Our previous calculations using linear models (YC95, YC96) revealed the generation of mesoscale flows within and near the scattering region with spatial scales of the order of topographic feature and amplitudes substantially exceeding that of the incident wave. These mesoscale flows were associated primarily with the generation of evanescent modes excited by the adjustment of the incident wave to the changing conditions of the coastal wave guide.

In the primitive equation numerical model, the mesoscale flows are transformed into eddies trapped at the topographic irregularity. The generation of eddies occurs due to nonlinear interaction of the mean current and the unsteady BSWs. The nonlinear interaction strongly depends on the phase of the wave passing through the scattering region. In particular, the cross-shelf shear of the along-shelf velocity is much greater for the wave phase for which the current at the coast is negative (i.e., opposite to the mean current) than when it is positive (i.e., in the same direction as the mean current). This leads to the preferential generation of anticyclones between the mean current and the coast.

When the incident signal is introduced as a harmonic wave, mesoscale eddies exist as a transient phenomenon, periodically appearing when the BSW phase with favorable velocity enters the scattering region and disappearing when the BSW phase with unfavorable velocity enters the scattering region. The eddies further modify the BSW scattering and produce downstream flow on the shelf which is not harmonic (apart from the incident wave). The more realistic representation of a BSW signal as a wave pulse shows the asymmetry of the response even more clearly. A shelf wave pulse with velocity at the coast opposite that of the mean current generates a strong anticyclone just upstream of the topographic irregularity. The anticyclone persists long after the pulse is gone, changing slightly with time and decaying slowly but on a much longer timescale than the duration of the wave pulse event. Currents within the anticyclone tend to be much larger than either the mean current or the maximum wave amplitude. In contrast, a shelf wave pulse with velocity at the coast of the same sign as the mean current does not generate any persistent flow trapped at the scattering region.

The response becomes more complicated over a wider shelf in which the topographic irregularity (and hence the mean current) remains farther offshore. That is, the shelf waveguide is only partially interrupted. In this case, eddies with smaller scales are formed and remain within the scattering region. These eddies are not steady, and they excite some evanescent and propagating modes creating highly variable mesoscale dynamics on the shelf. Their velocities tend to be weaker than those of the persistent single anticyclone on the narrower shelf.

The possible importance of these mesoscale eddies in cross-shelf exchange has been demonstrated by following Lagrangian floats through the flow fields. In general, particles released over the shelf downstream of the scattering do not penetrate far offshore, despite the fact that they may exhibit complicated motions owing to the presence of multiple free BSW modes. Their trajectories tend to be nearly closed with a slight downstream drift (e.g., Figure 6). On the other hand, particles released in (or nearby) the mesoscale eddies move rapidly across the shelf where they may be injected into the mean current and carried quickly hundreds of kilometers downstream, out of the model domain. Upon reaching the mean current, the particles are available for enhanced cross-shelf mixing owing to the presence of multiple modes superimposed on the sheared mean current, as described by Pratt et al. [1995].

The process described here may represent an effective mechanism for generating mesoscale variability on continental shelves under certain conditions, whether the mean current is stable or not. The requirements are (1) a sheared mean current over the outer shelf, flowing in the same direction as Kelvin wave propagation, (2) a relatively narrow shelf, (3) a substantial signal of propagating coastally trapped waves (generated either locally or remotely), and (4) some along-shelf topographic irregularities (which can be found along almost any coast). Geographically, these conditions are typical for eastern boundaries of the oceans as well as for many semiclosed seas. Some particular examples are discussed by YC95 and in the introduction section.

In the present calculations, the generation of the anticyclonic eddies trapped at the topographic irregularity requires two components: a sheared mean current and propagating BSWs. It is important to note that neither component, when considered separately, has the same effect. For instance, an unstable mean current might generate eddies, but they are not trapped at the topographic irregularity. Instead, they originate there and grow as they propagate downstream. We have demonstrated this by calculating the adjustment of a much stronger (and unstable) mean current, with maximum velocity of 1.2 m s\(^{-1}\) (not shown). The mean current was imposed gradually over a period of 10 days to avoid generating an anticyclone from the initial BSW front. Once the mean current reaches full speed, unstable waves grow rapidly as they propagate downstream of the scattering region. However, there are no localized instabilities associated with the scattering region that persist in the same location. If, on the other hand, the upstream part of the channel were long enough, then unstable waves could arrive at the scattering region with large amplitudes and scatter their energy in the same manner as the stable BSW, producing similar features. We have not investigated this case.

Purely oscillatory currents can also produce a mean circulation at topographic features through nonlinear rectification. For example, Verron et al. [1995a, b] and Haidvogel and Beckmann [1995] each considered periodic currents in a homogeneous flow past topographic features. For submarine canyon topography, qualitatively similar to our case, they also obtained anticy-
clonic eddies adjacent to the canyon. However, rectified flows operate in an entirely different manner than our proposed mechanism. They exist coincident with the oscillating currents and appear only in the time-averaged velocity field. Rectified velocities typically do not exceed the amplitude of the periodic current, and, in addition to producing mesoscale features, they tend to expand both upstream and downstream as an along-shelf current.

Finally, we have also computed the scattering of a first-mode BSW for the same case shown in Figure 4b, but without a mean current (not shown). To allow for the reflection of short BSWs, the model domain was extended by 300 km at the inflow end (i.e., the incident wave was introduced at $z = -300$ km). The reflected short shelf waves disturb the incident wave field, but they do not amplify the total velocity as much as evanescent modes do in the presence of a mean current ($0.4 \text{ m s}^{-1}$ versus $0.7 \text{ m s}^{-1}$). Further, they are not trapped at any particular location but propagate upstream while gradually decaying. And, most importantly, there is no enhanced nonlinear dynamics which can produce persistent mesoscale eddies.

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