Basic Fluid Mechanics

Summary of introductory concepts

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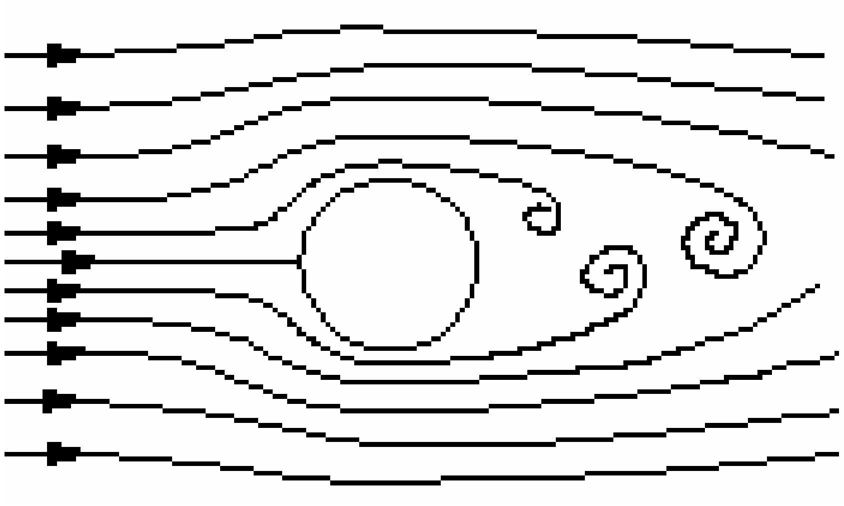
Introduction

Field of Fluid Mechanics can be divided into 3 branches:

- Fluid Statics: mechanics of fluids at rest
- Kinematics: deals with velocities and streamlines w/o considering forces or energy
- Fluid Dynamics: deals with the relations between velocities and accelerations and forces exerted by or upon fluids in motion

Streamlines

A streamline is a line that is tangential to the instantaneous velocity direction (velocity is a vector that has a direction and a magnitude)



Instantaneous streamlines in flow around a cylinder

Intro…con't

Mechanics of fluids is extremely important in many areas of engineering and science. Examples are:

- Biomechanics
  - Blood flow through arteries
  - Flow of cerebral fluid
- Meteorology and Ocean Engineering
  - Movements of air currents and water currents
- Chemical Engineering
  - Design of chemical processing equipment

Intro…con't

- Mechanical Engineering
  - Design of pumps, turbines, air-conditioning equipment, pollution-control equipment, etc.
- Civil Engineering
  - Transport of river sediments
  - Pollution of air and water
  - Design of piping systems
  - Flood control systems

Dimensions and Units

- Before going into details of fluid mechanics, we stress importance of units
- In U.S, two primary sets of units are used:
  - 1. SI (Systeme International) units
  - 2. English units

Unit Table

Quantity	SI Unit	English Unit
Length (L)	Meter (*m*)	Foot (*ft*)
Mass (m)	Kilogram (*kg*)	Slug (slug) = *lb*sec^2/ft*
Time (T)	Second (*s*)	Second (*sec*)
Temperature (Θ)	Celcius (^oC)	Farenheit (^oF)
Force	Newton *(N)=kg*m/s^2*	Pound (lb)

Dimensions and Units con't

- 1 *Newton* – Force required to accelerate a 1 *kg* of mass to 1 m/s^2
- 1 *slug* – is the mass that accelerates at 1 ft/s^2 when acted upon by a force of 1 *lb*
- To remember units of a Newton use F=ma (Newton's 2nd Law)
  - $[F] = [m][a] = kg*m/s^2 = N$

More on Dimensions

- To remember units of a slug also use F=ma => m = F / a
- [m] = [F] / [a] = *lb* / (*ft* / sec^2) = *lb**sec^2 / *ft*

- 1 *lb* is the force of gravity acting on (or weight of) a platinum standard whose mass is 0.45359243 *kg*

Weight and Newton's Law of Gravitation

- Weight
  - Gravitational attraction force between two bodies
- Newton's Law of Gravitation

$$F = G\, m_1 m_2 / r^2$$

  - G- universal constant of gravitation
  - m_1, m_2 - mass of body 1 and body 2, respectively
  - r- distance between centers of the two masses
  - F- force of attraction

Weight

- m_2 - mass of an object on earth's surface
- m_1 - mass of earth
- r - distance between center of two masses
- r_1 - radius of earth
- r_2 - radius of mass on earth's surface
- $r_2 << r_1$, therefore $r = r_1+r_2 \sim r_1$
- Thus, $F = m_2 * (G * m_1 / r^2)$

Weight

- Weight (W) of object (with mass m_2) on surface of earth (with mass m_1) is defined as

  $W = m_2 g$; $g = (Gm_1/r^2)$ gravitational acceleration

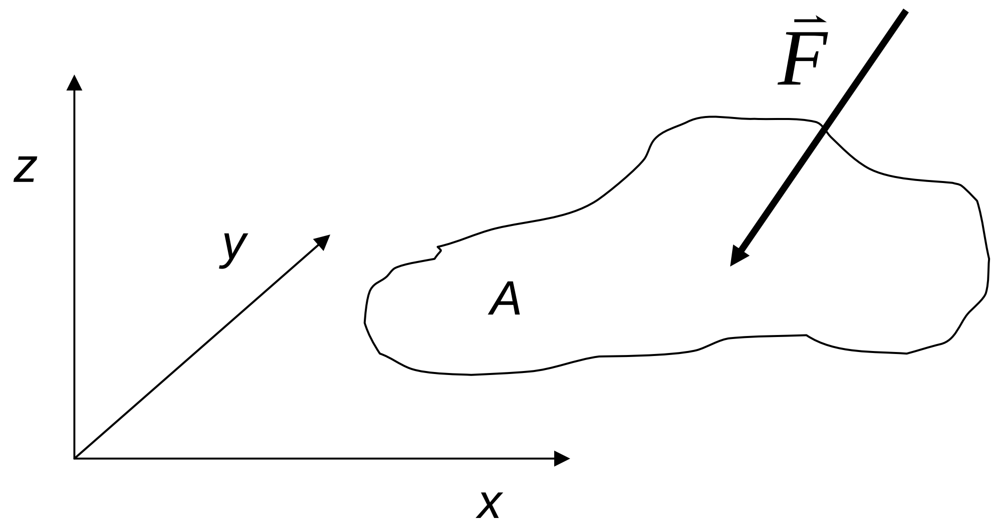
  $g = 9.31$ m/s^2 in SI units

  $g = 32.2$ ft/sec^2 in English units

- See back of front cover of textbook for conversion tables between SI and English units

Properties of Fluids - Preliminaries

- Consider a force, $\vec{F}$, acting on a 2D region of area *A* sitting on x-y plane



Cartesian components: $\vec{F} = F_x\,(\pm\hat{i}) + F_y\,(\pm\hat{j}) + F_z(\pm\hat{k})$

Cartesian components

$\pm\hat{i}$ - Unit vector in $\pm$*x-direction*

$\pm\hat{j}$ - Unit vector in $\pm$*y-direction*

$\pm\hat{k}$ - Unit vector in $\pm$*z-direction*

F_x - Magnitude of $\vec{F}$ in $\pm$*x-direction* (tangent to surface)

F_y - Magnitude of $\vec{F}$ in $\pm$*y-direction* (tangent to surface)

F_z - Magnitude of $\vec{F}$ in $\pm$*z-direction* (normal to surface)

- For simplicity, let $F_y = 0$

- Shear stress and pressure

$$\tau = \frac{F_x}{A} \quad (shear\ stress)$$

$$p = \frac{F_z}{A} \quad (normal\ stress\ (pressure))$$

- Shear stress and pressure at a point

$$\tau = \left(\frac{F_x}{A}\right)_{\lim A \to 0} \qquad p = \left(\frac{F_z}{A}\right)_{\lim A \to 0}$$

- Units of stress (shear stress and pressure)

$$\frac{[F]}{[A]} = \frac{N}{m^2} = Pa\ (Pascal)\ in\ SI\ units$$

$$\frac{[F]}{[A]} = \frac{lb}{in^2} = psi\ (pounds\ per\ square\ inch)\ in\ English\ units$$

$$\frac{[F]}{[A]} = \frac{lb}{ft^2} = pounds\ per\ square\ foot\ (English\ units)$$

Properties of Fluids Con't

- Fluids are either liquids or gases
- Liquid: A state of matter in which the molecules are relatively free to change their positions with respect to each other but restricted by cohesive forces so as to maintain a relatively fixed volume
- Gas: a state of matter in which the molecules are practically unrestricted by cohesive forces. A gas has neither definite shape nor volume.

More on properties of fluids

- Fluids considered in this course move under the action of a shear stress, no matter how small that shear stress may be (unlike solids)

Continuum view of Fluids

- Convenient to assume fluids are continuously distributed throughout the region of interest. That is, the fluid is treated as a continuum
- This continuum model allows us to not have to deal with molecular interactions directly. We will account for such interactions indirectly via viscosity
- A good way to determine if the continuum model is acceptable is to compare a characteristic length (L) of the flow region with the mean free path of molecules, λ
- If $L << \lambda$, continuum model is valid

- Mean free path (λ) – Average distance a molecule travels before it collides with another molecule.

1.3.2 Density and specific weight

Density (mass per unit volume): $\rho = \dfrac{m}{V}$

Units of density: $[\rho] = \dfrac{[m]}{[V]} = \dfrac{kg}{m^3} \quad (in\ SI\ units)$

Specific weight (weight per unit volume): $\gamma = \rho g$

Units of specific weight:

$$[\gamma] = [\rho][g] = \frac{kg}{m^3}\frac{m}{s^2} = \frac{N}{m^3} \quad (in\ SI\ units)$$

Specific Gravity of Liquid (*S*)

$$S = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\rho_{liquid}\, g}{\rho_{water}\, g} = \frac{\gamma_{liquid}}{\gamma_{water}}$$

- See appendix A of textbook for specific gravities of various liquids with respect to water at 60 ^{o}F

1.3.3 Viscosity (μ)

- Viscosity can be thought as the internal stickiness of a fluid
- Representative of internal friction in fluids
- Internal friction forces in flowing fluids result from cohesion and momentum interchange between molecules.
- Viscosity of a fluid depends on temperature:
  - In liquids, viscosity decreases with increasing temperature (i.e. cohesion decreases with increasing temperature)
  - In gases, viscosity increases with increasing temperature (i.e. molecular interchange between layers increases with temperature setting up strong internal shear)

More on Viscosity

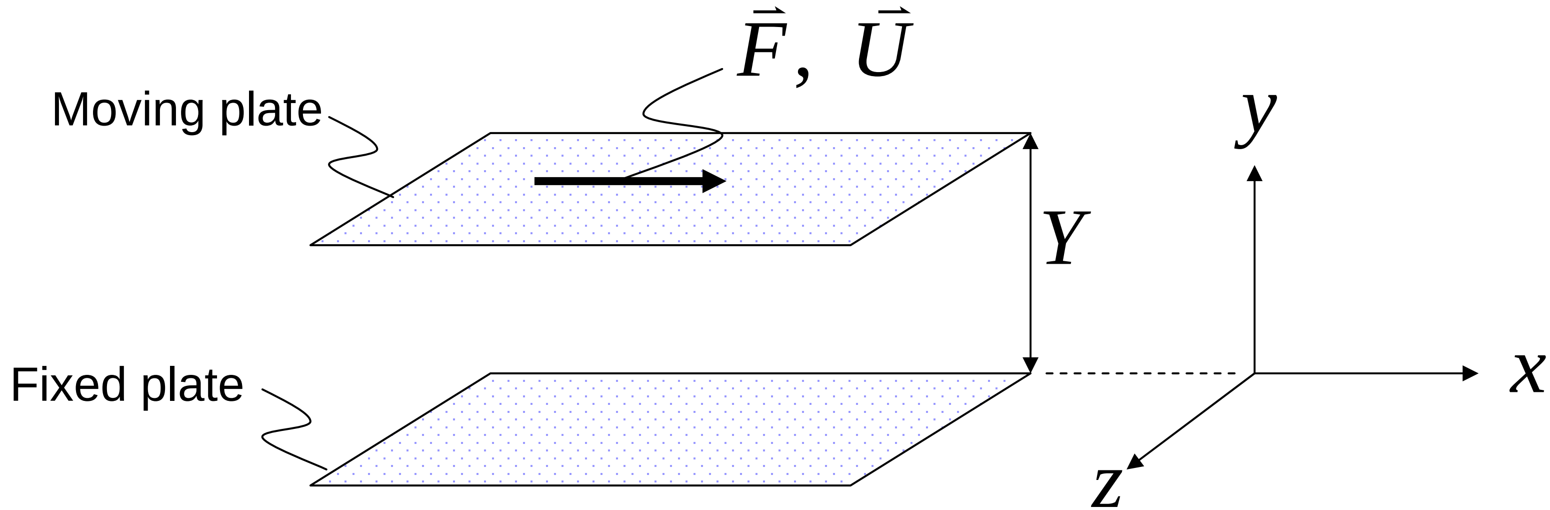
- Viscosity is important, for example,
  - in determining amount of fluids that can be transported in a pipeline during a specific period of time
  - determining energy losses associated with transport of fluids in ducts, channels and pipes

No slip condition

- Because of viscosity, at boundaries (walls) particles of fluid adhere to the walls, and so the fluid velocity is zero relative to the wall
- Viscosity and associated shear stress may be explained via the following: flow between no-slip parallel plates.

Flow between no-slip parallel plates

-each plate has area *A*

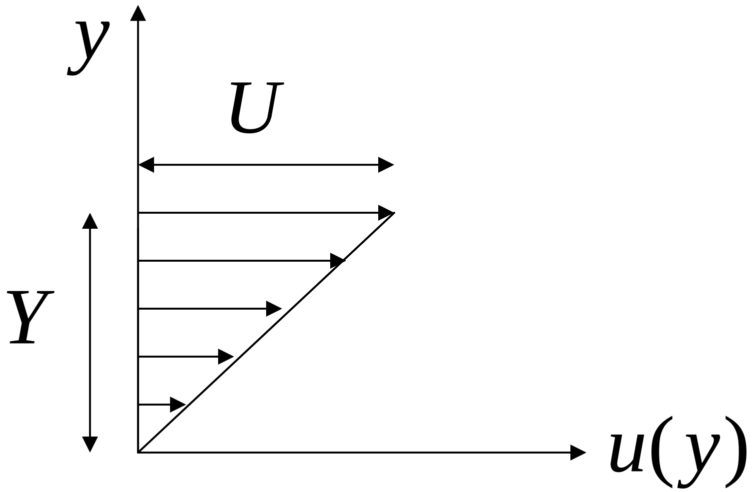


$$\vec{F} = F\hat{i} \qquad \vec{U} = U\hat{i}$$

Force $\vec{F}$ induces velocity $\vec{U}$ on top plate. At top plate flow velocity is $\vec{U}$

At bottom plate velocity is 0

The velocity induced by moving top plate can be sketched as follows:



$$u(y = 0) = 0$$

$$u(y = Y) = U$$

The velocity induced by top plate is expressed as follows:

$$u(y) = \left(\frac{U}{Y}\right) y$$

For a large class of fluids, empirically, $F \propto \frac{AU}{Y}$

More specifically, $F = \mu \frac{AU}{Y}$; μ *is coefficient of viscosity*

Shear stress induced by F is $\tau = \frac{F}{A} = \mu \frac{U}{Y}$

From previous slide, note that $\frac{du}{dy} = \frac{U}{Y}$

Thus, shear stress is $$\boxed{\tau = \mu \frac{du}{dy}}$$

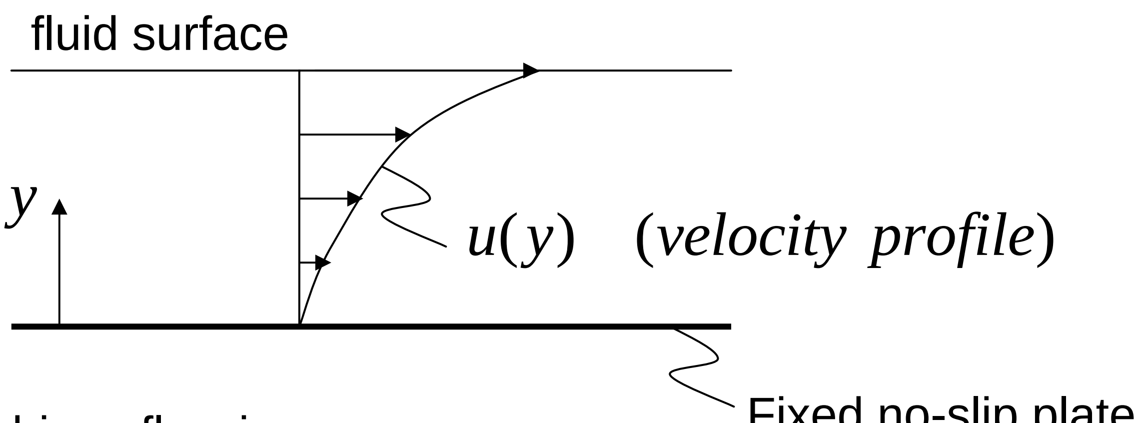
In general we may use previous expression to find shear stress at a point inside a moving fluid. Note that if fluid is at rest this stress is zero because $\frac{du}{dy} = 0$

Newton's equation of viscosity

Shear stress due to viscosity at a point: $\tau = \mu \dfrac{du}{dy}$

μ - viscosity (coeff. of viscosity)

$\nu = \dfrac{\mu}{\rho}$ - kinematic viscosity



e.g.: wind-driven flow in ocean

As engineers, Newton's Law of Viscosity is very useful to us as we can use it to evaluate the shear stress (and ultimately the shear force) exerted by a moving fluid onto the fluid's boundaries.

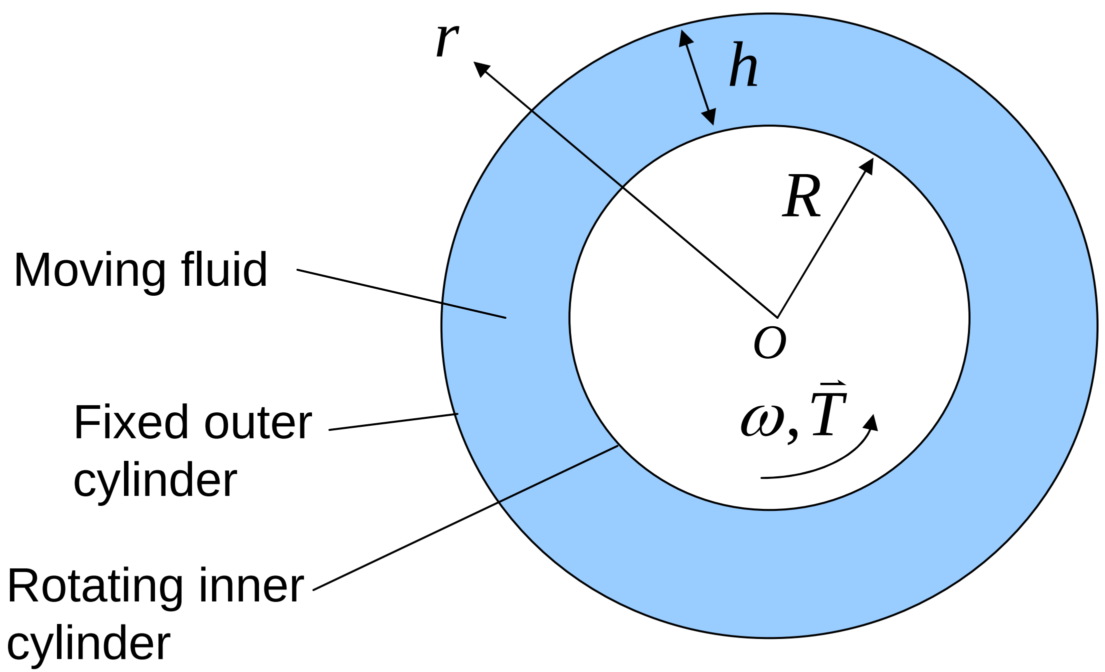
$$\tau \text{ at boundary} = \mu \left(\frac{du}{dy} \right)_{\text{at boundary}}$$

Note y is direction normal to the boundary

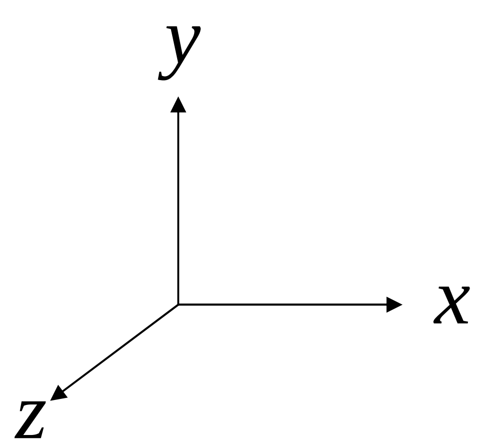
Viscometer

Coefficient of viscosity μ can be measured empirically using a viscometer

Example: Flow between two concentric cylinders (viscometer) of length L



r - radial coordinate



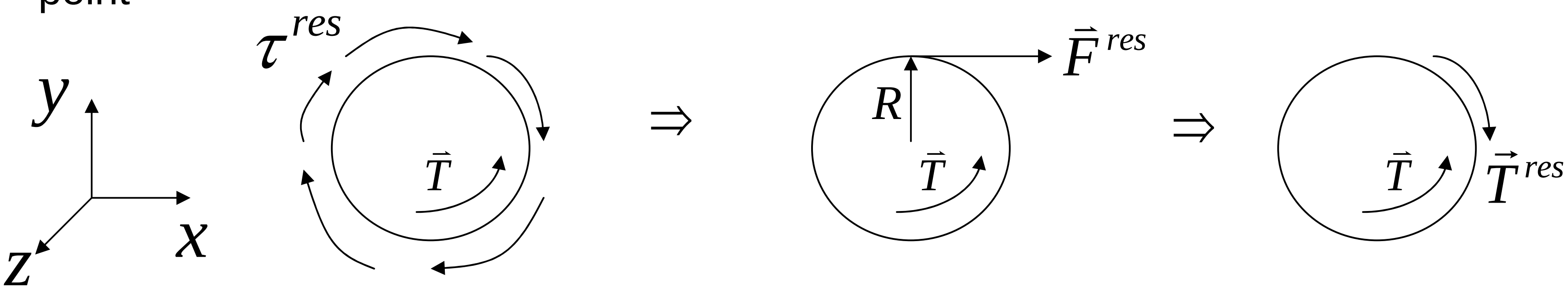
Inner cylinder is acted upon by a torque, $\vec{T} = T\,\hat{k}$, causing it to rotate about point O at a constant angular velocity ω and causing fluid to flow. Find an expression for $\vec{T}$

Because ω is constant, $\vec{T} = T\,\hat{k}$ is balanced by a resistive torque exerted by the moving fluid onto inner cylinder

$$T = T^{res}$$

$$\vec{T}^{res} = T^{res}(-\hat{k})$$

The resistive torque comes from the resistive stress τ^{res} exerted by the moving fluid onto the inner cylinder. This stress on the inner cylinder leads to an overall resistive force $\vec{F}^{res}$, which induces the resistive torque about point O



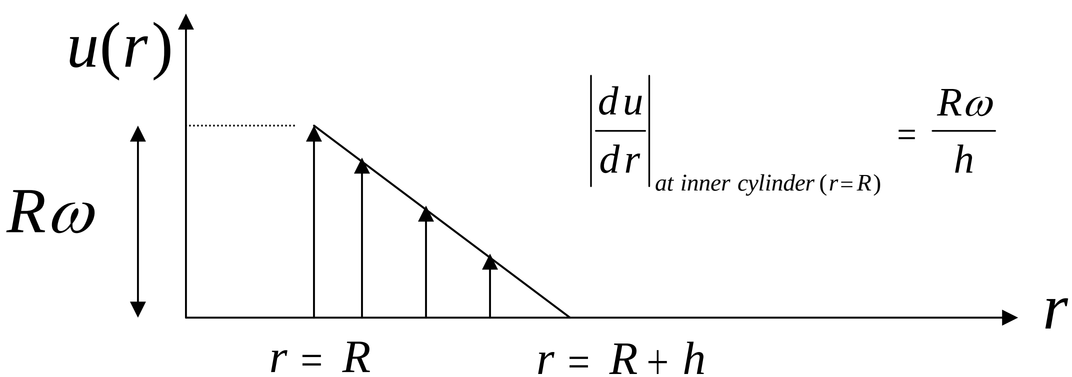
$$T = T^{res} = F^{res} R$$

$$F^{res} = \tau^{res} A = \tau^{res} (2\pi R L)$$ (Neglecting ends of cylinder)

How do we get τ^{res} ? This is the stress exerted by fluid onto inner cylinder, thus

$$\tau^{res} = \mu \left| \frac{du}{dr} \right|_{at\ inner\ cylinder\ (r=R)}$$

If h (gap between cylinders) is small, then



$$\left| \frac{du}{dr} \right|_{at\ inner\ cylinder\ (r=R)} = \frac{R\omega}{h}$$

Thus, $\tau^{res} = \mu \dfrac{R\omega}{h}$

$$T = T^{res} = F^{res} R$$

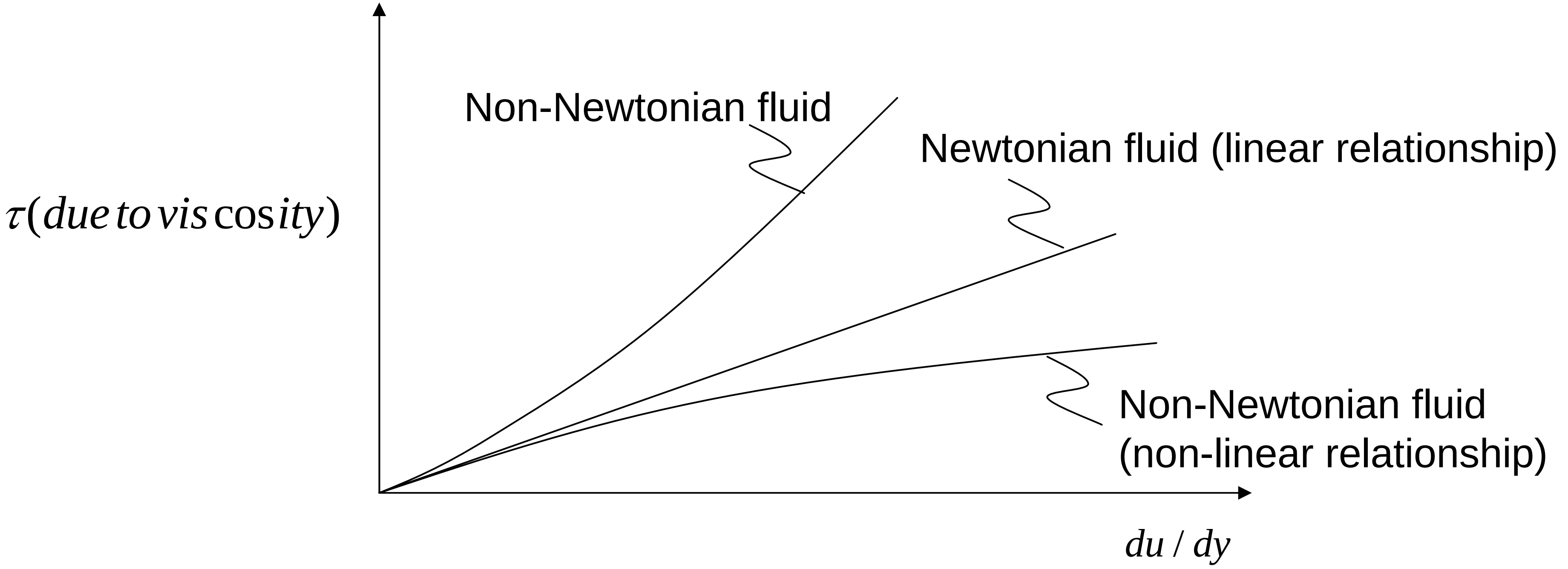
$$T = T^{res} = \tau^{res} AR = \tau^{res} (2\pi R L) R$$

$$= \mu \left(\frac{R\omega}{h} \right) (2\pi R L) R$$

$$T = \frac{R^3 \mu \omega 2\pi L}{h}$$

Given T, R, ω, L, h previous result may be used to find μ of fluid, thus concentric cylinders may be used as a viscometer

Non-Newtonian and Newtonian fluids



- In this course we will only deal with Newtonian fluids
- **Non-Newtonian fluids**: blood, paints, toothpaste

Compressibility

- All fluids compress if pressure increases resulting in an increase in density
- Compressibility is the change in volume due to a change in pressure
- A good measure of compressibility is the bulk modulus (It is inversely proportional to compressibility)

$$E_{\upsilon} = -\upsilon \frac{dp}{d\upsilon} \qquad \upsilon = \frac{1}{\rho} \quad (\textit{specific volume})$$

$$p \text{ is pressure}$$

Compressibility

- From previous expression we may write

$$\frac{(\upsilon_{final} - \upsilon_{initial})}{\upsilon_{initial}} \approx -\frac{(p_{final} - p_{initial})}{E_{\upsilon}}$$

- For water at 15 *psia* and 68 degrees Farenheit, $E_{\upsilon} = 320{,}000\ psi$

- From above expression, increasing pressure by 1000 *psi* will compress the water by only 1/320 (0.3%) of its original volume

- Thus, water may be treated as incompressible (density (ρ) is constant)

- In reality, no fluid is incompressible, but this is a good approximation for certain fluids

Vapor pressure of liquids

- All liquids tend to evaporate when placed in a closed container

- Vaporization will terminate when equilibrium is reached between the liquid and gaseous states of the substance in the container

  i.e. # of molecules escaping liquid surface = # of incoming molecules

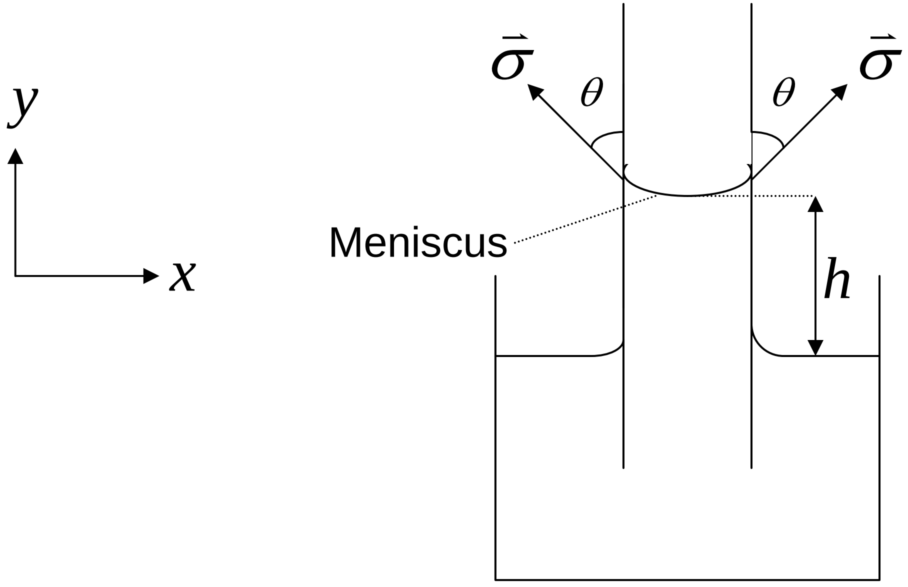
- Under this equilibrium we call the call vapor pressure the saturation pressure

- At any given temperature, if pressure on liquid surface falls below the the saturation pressure, rapid evaporation occurs (i.e. boiling)

- For a given temperature, the saturation pressure is the boiling pressure

Surface tension

- Consider inserting a fine tube into a bucket of water:



r - radius of tube

$\vec{\sigma}$ - Surface tension vector (acts uniformly along contact perimeter between liquid and tube)

Adhesion of water molecules to the tube dominates over cohesion between water molecules giving rise to $\vec{\sigma}$ and causing fluid to rise within tube

$\vec{\sigma} = \sigma \hat{n}$

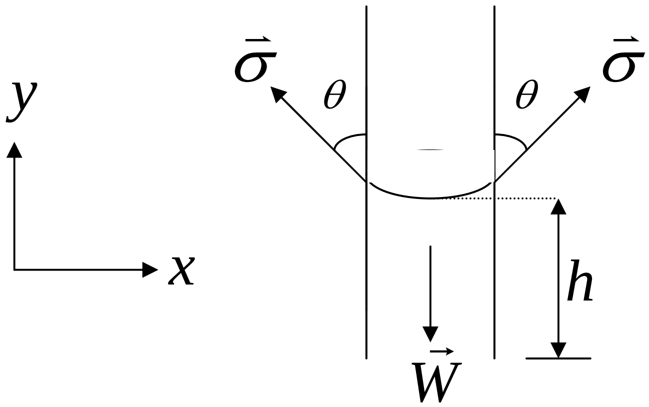
$\hat{n}$ - unit vector in direction of $\vec{\sigma}$

σ - surface tension (magnitude of $\vec{\sigma}$)

$$\vec{\sigma} = \sigma\,[\sin\theta(\hat{i}) + \cos\theta(\hat{j})]$$

$$[\sigma] = \frac{force}{length}$$

Given conditions in previous slide, what is σ ?



$$\vec{\sigma} = \sigma\,[\sin\theta(\hat{i}) + \cos\theta(\hat{j})]$$

$$\vec{W} = W(-\hat{j})$$ (weight vector of water)

Equilibrium in *y-direction* yields: $\sigma\cos\theta(2\pi r)\,(\hat{j}) + W(-\hat{j}) = 0\,\hat{j}$

Thus
$$\boxed{\sigma = \frac{W}{2\pi r\cos\theta}}$$

with $W = \gamma_{water}\,\pi r^2\,h$