Assessing the Vertical Solution Algorithm in ECOM/POM

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Objectives

- 1. Assessing the Stability of Vertical Solution Algorithm in ECOM
- 2. Assessing the Limitation on Vertical Solution Algorithm of ECOM
- 3. Suggest a surrogate accuracy criterion for vertical mixing algorithm, which is $\frac{K_H \Delta t}{\Delta z^2} \leqslant \frac{1}{2}$

ECOM vertical mixing algorithm

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}(K_H \frac{\partial T}{\partial z}) + advection, 0 \le z \ge H, t > 0$$

$$\frac{T_k^{n+1} - T_k^{n-1}}{\Delta t} = K_{H_{k-1/2}} \frac{T_{k-1}^{n+1} - T_k^{n+1}}{\Delta z^2} - K_{H_{k+1/2}} \frac{T_k^{n+1} - T_{k+1}^{n+1}}{\Delta z^2} + adv^{n-1} (1)$$

$$-K_{H_{k-1/2}} \frac{\Delta t}{\Delta z^2} T_{k-1}^{n+1} + (K_{H_{k-1/2}} \frac{2\Delta t}{\Delta z^2} + K_{H_{k+1/2}} \frac{2\Delta t}{\Delta z^2} + 1) T_k^{n+1} - K_{H_{k+1/2}} \frac{2\Delta t}{\Delta z^2} T_{k+1}^{n+1} = T_k^{n-1} + Adv^{n-1}$$

$$a_k = -K_{H_{k-1/2}} \frac{\Delta t}{\Delta z^2}; c_k = -K_{H_{k-1/2}} \frac{\Delta t}{\Delta z^2}; b_k = a_k + c_k - 1$$

$(1) turn..in..to (2)$

$$-a_k T_{k-1}^{n+1} + b_k T_k^{n+1} - c_k T_{k+1}^{n+1} = -T_k^{n-1} + Adv^{n-1}$$

$$\begin{pmatrix} b_1 & c_1 & 0 \\ & \ddots & \\ 0 & a_{kb-1} & b_{kb-1} \end{pmatrix} \begin{pmatrix} T_1 \\ . \\ . \\ T_{kb-1} \end{pmatrix}^{n+1} = - \begin{pmatrix} T_1 \\ . \\ . \\ T_{kb-1} \end{pmatrix}^{n-1}$$

Applying..Thomas..Algorithm

$$c_k' = \begin{cases} \frac{c_1}{b_1}; k = 1 \\ \frac{c_k}{b_k - c_{k-1}' a_k}; k = 2,3,..kb-1 \end{cases} ......d_k' = \begin{cases} \frac{d_1}{b_1}; k = 1 \\ \frac{d_k - d_{i-1}' a_k}{b_k - c_{k-1}' a_k}; k = 2,3,..kb-1 \end{cases}$$

$$T_{kb-1} = d_{kb-1}'; T_k = d_k' - c_k' T_{k+1}$$

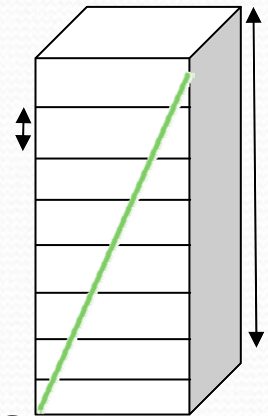
Method

- 1. A simplified heat equation with Newman Boundary Condition is solved by ECOM in a 1D vertical model.

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}(K_H \frac{\partial T}{\partial z}), 0 \leq z \geq H, t > 0$$

$$\frac{\partial T}{\partial z}|_{z=0} = 0; \frac{\partial T}{\partial z}|_{z=H} = 0$$

$$T(z,0) = f(z)$$



- 2. An Analytic Solution of the heat equation is computed via Fourier Method
- 3. The modeled and analytic solution is compared and analyzed.

Analytic Algorithm

- Applying Separate Variables and Fourier Series

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}(K_H \frac{\partial T}{\partial z}), -H \le z \ge 0, t > 0$$

$$\frac{\partial T}{\partial z}|_{z=0} = 0; \frac{\partial T}{\partial z}|_{z=-H} = 0$$

$$T(z,0) = f(z)$$

$$T(z,t) = \sum_{n=1}^{+\inf} D_n \cos(\frac{n\pi z}{H}) e^{-\frac{n^2 \pi K_H t}{H^2}} + a_0$$

$$a_0 = \int_{-H}^{0} f(z) dz$$

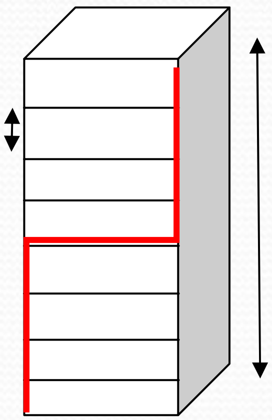
$$D_n = \frac{2}{-H} \int_{0}^{-H} f(z) \cos(\frac{n\pi z}{H}) dz = \frac{2}{H} \int_{-H}^{0} f(z) \cos(\frac{n\pi z}{H}) dz$$

Experiment I

- Model Setup

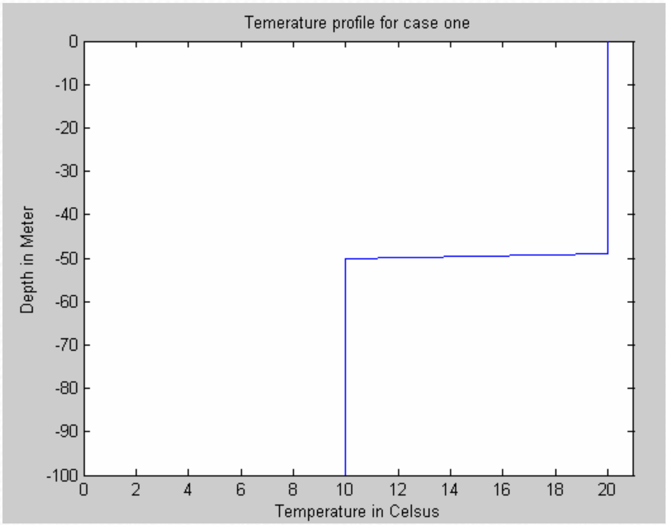
  H=100m, sigma layer#=100; K_H=0.01 m^2/s

  Time Step $\Delta t = 50\,\text{sec}$

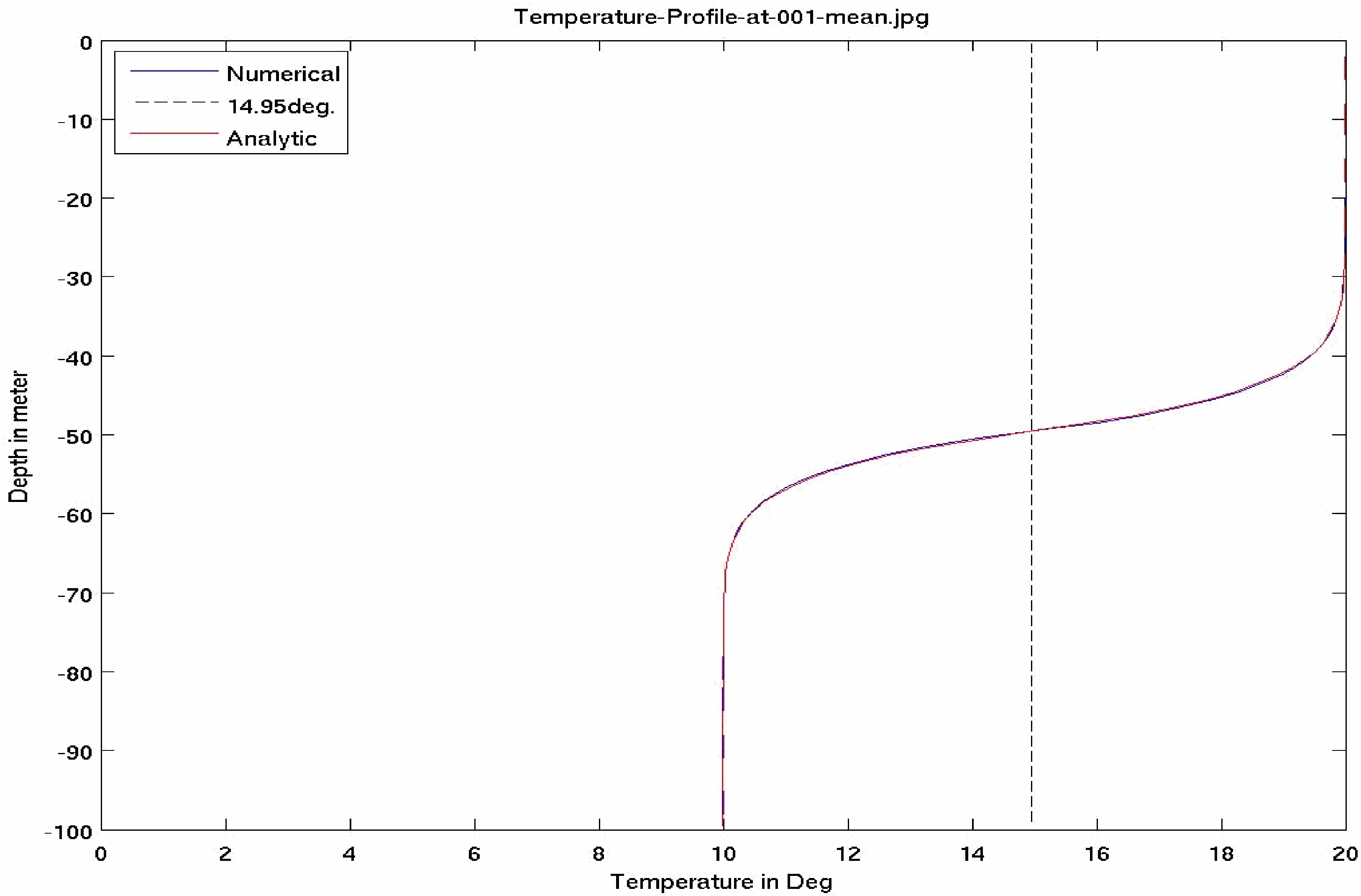


Initial Temperature Profile

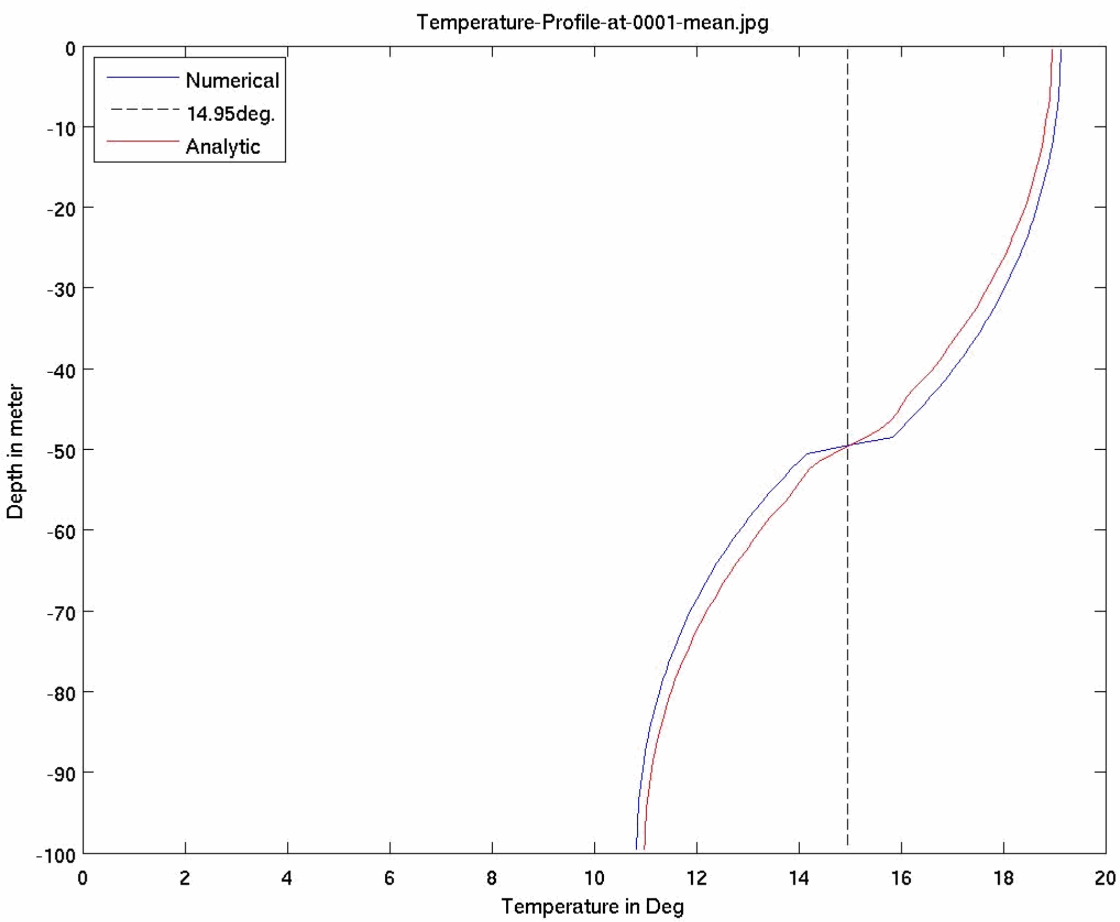
$$f(z) = \begin{cases} 20, when -50 < z \le 0 \\ 10, when -100 < z \le -50 \end{cases}$$

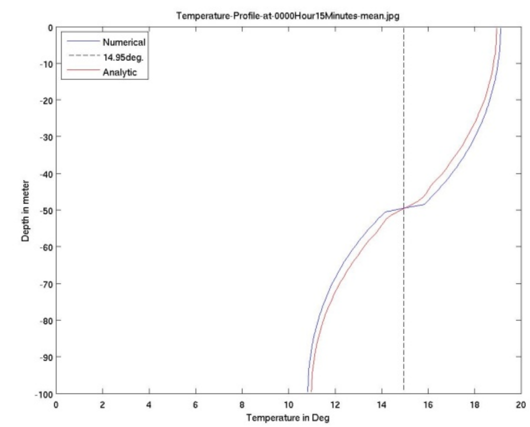


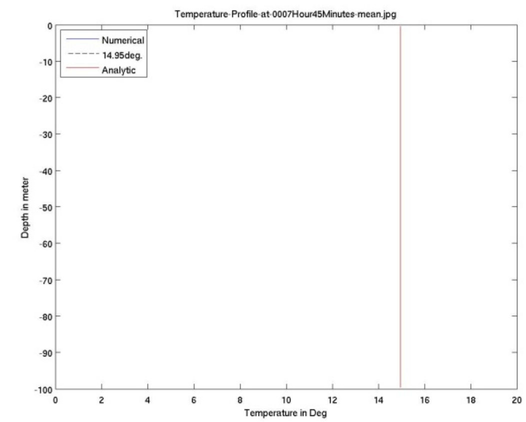
Results—Comparison between modeled and analytic



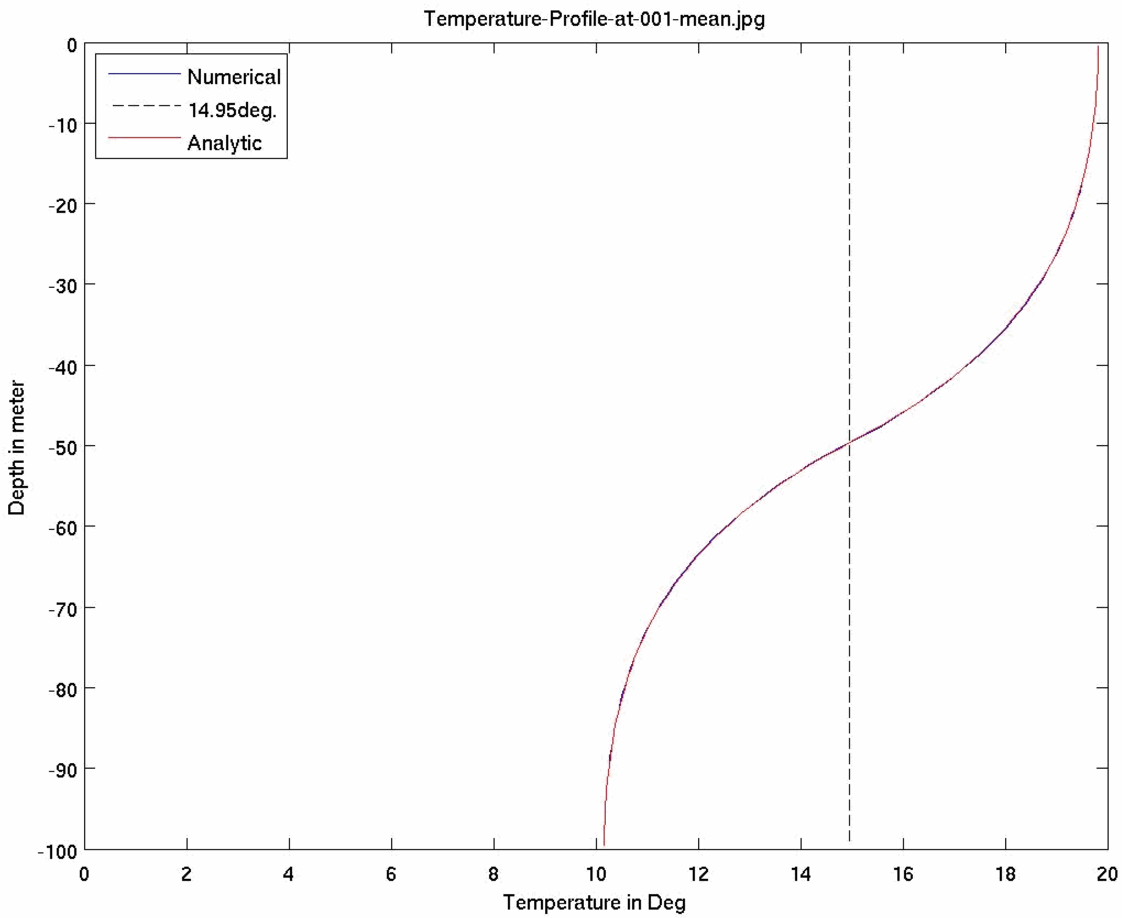
When K_H is Changed to $1m^2/s$







How to fix it?



K_H=1 m²/s
Dt=0.5 second

Accuracy Criterion $\frac{K_H \Delta t}{\Delta z^2} \leqslant \frac{1}{2}$

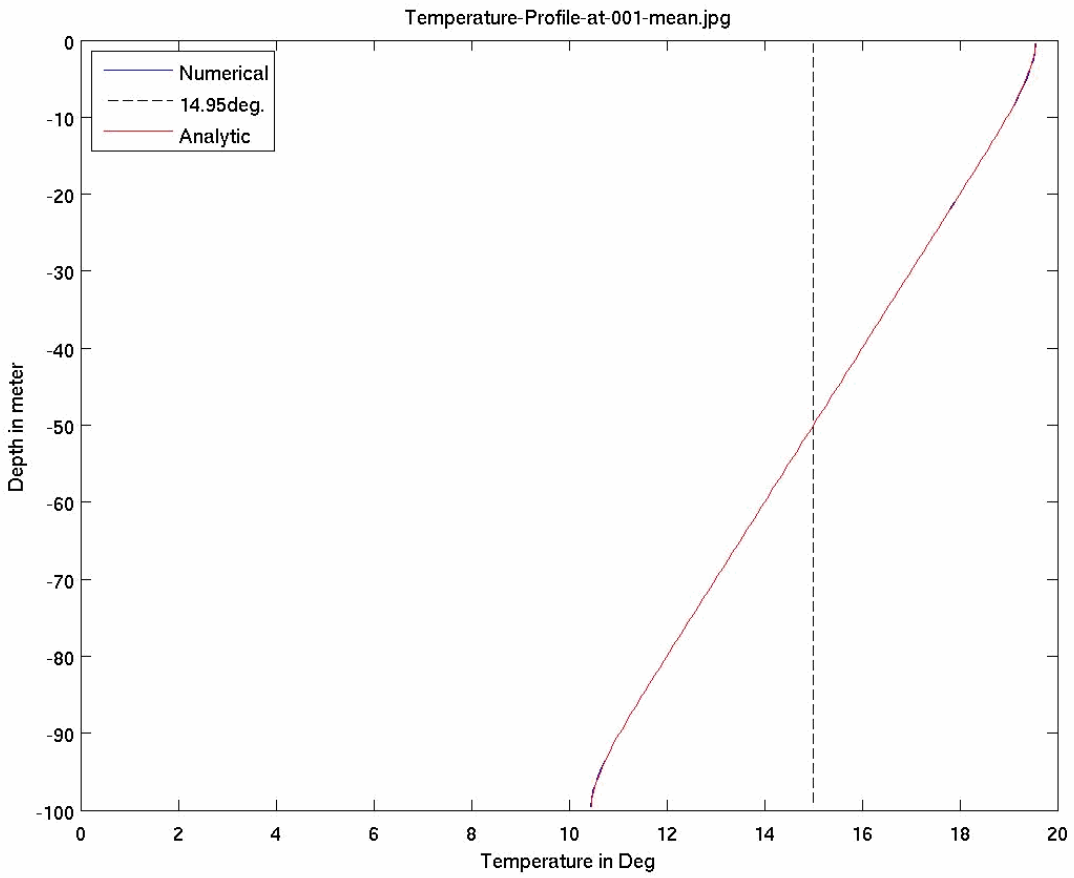
- Case I. $\frac{K_H \Delta t}{\Delta z^2} = \frac{0.01 m^2 s^{-1} 50 \sec}{(1m)^2} = \frac{1}{2}$

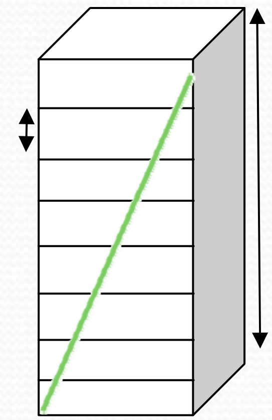
- Case II. $\frac{K_H \Delta t}{\Delta z^2} = \frac{1 m^2 s^{-1} 50 \sec}{(1m)^2} = 50$

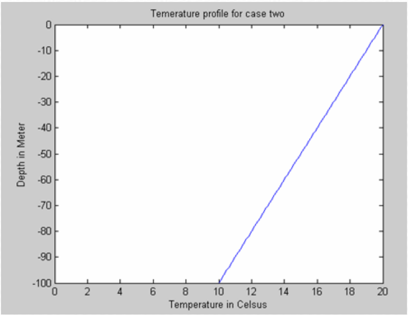
Case Sensitive?-- No

$$\frac{K_H \Delta t}{\Delta z^2} = \frac{1m^2 s^{-1} 0.5\sec}{(1m)^2} = \frac{1}{2}$$

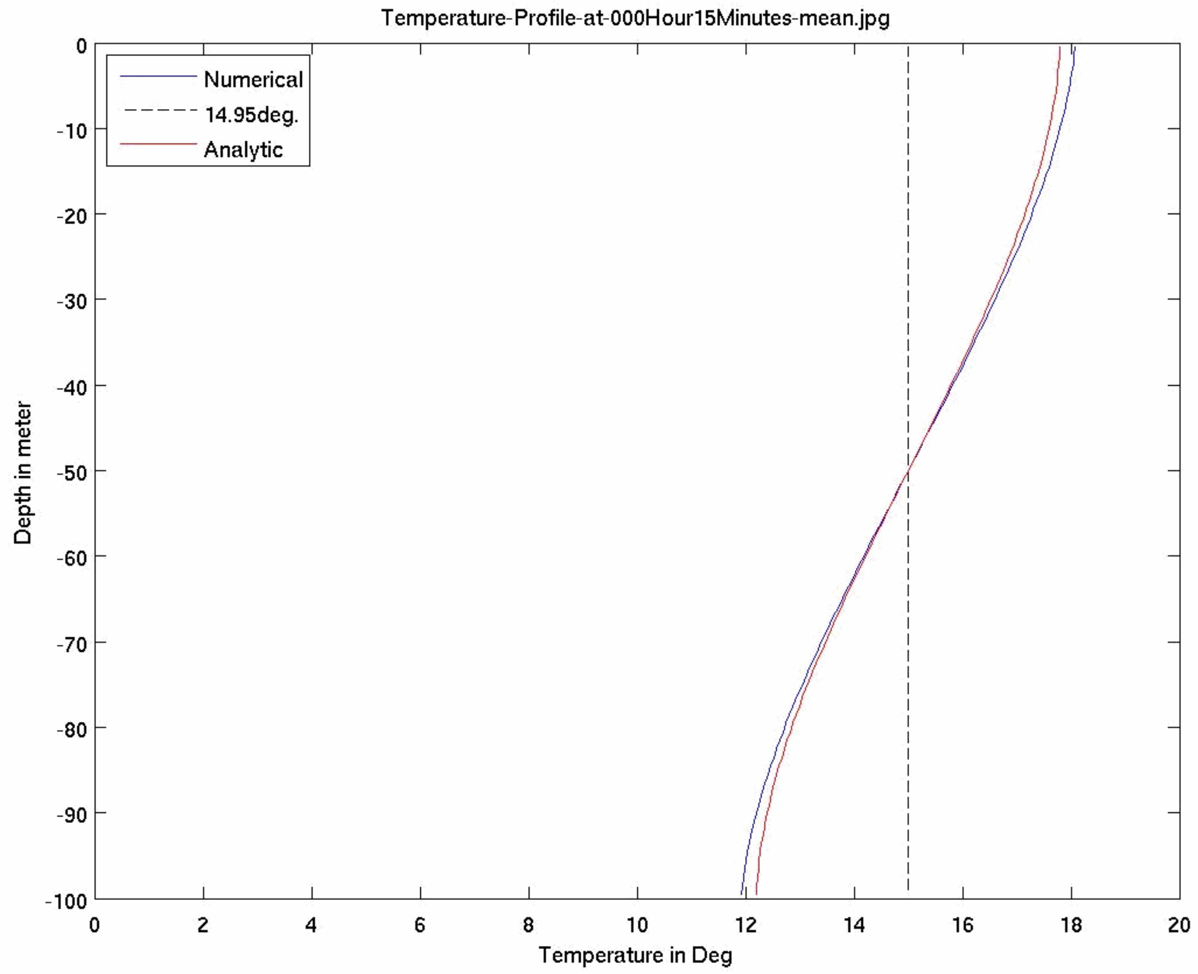
K_H=1 m²/s
Dt=0.5 second







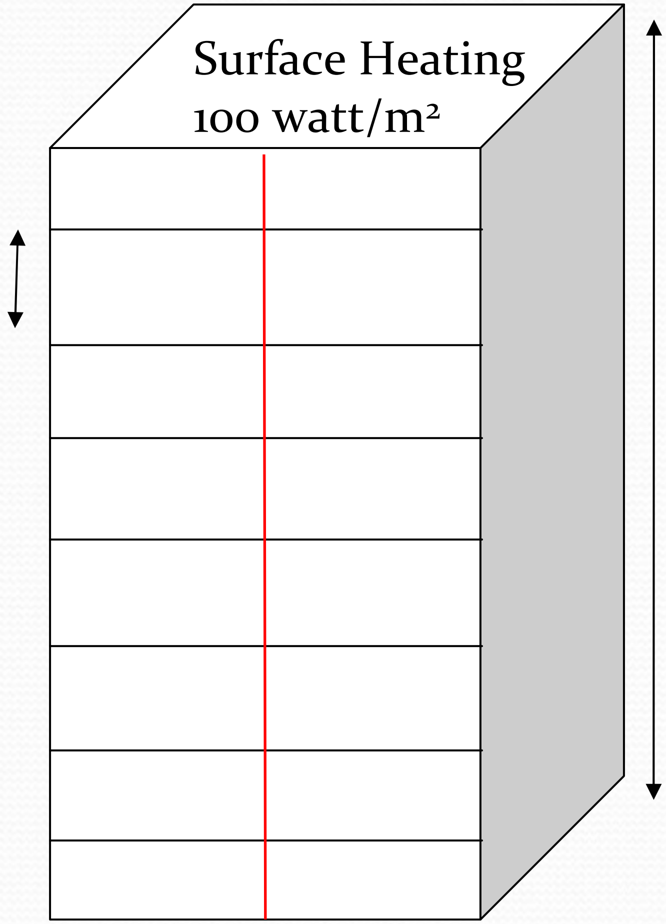
Time Step is Changed to 60 sec



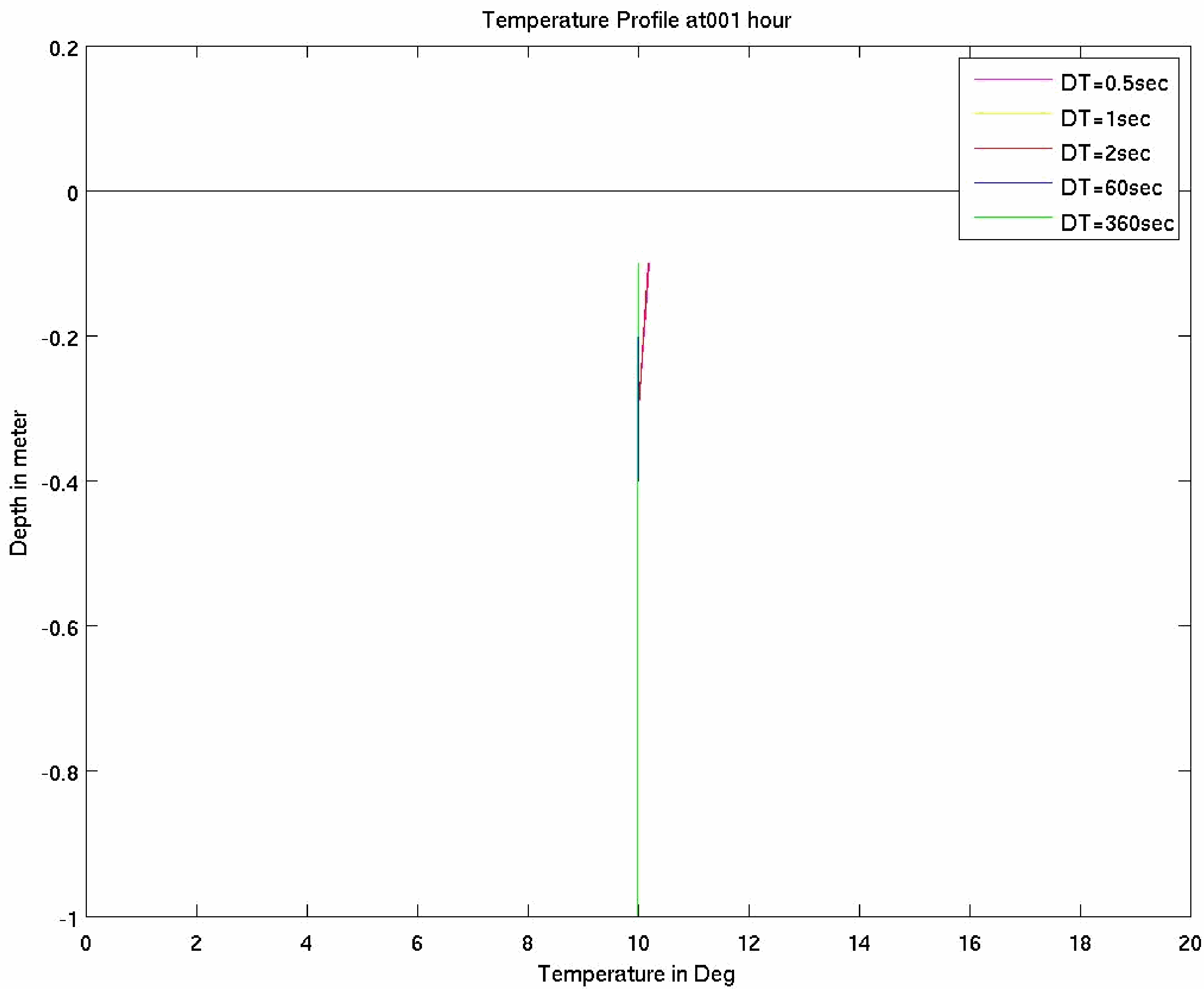
K_H=1 m²/s
Dt=60 second

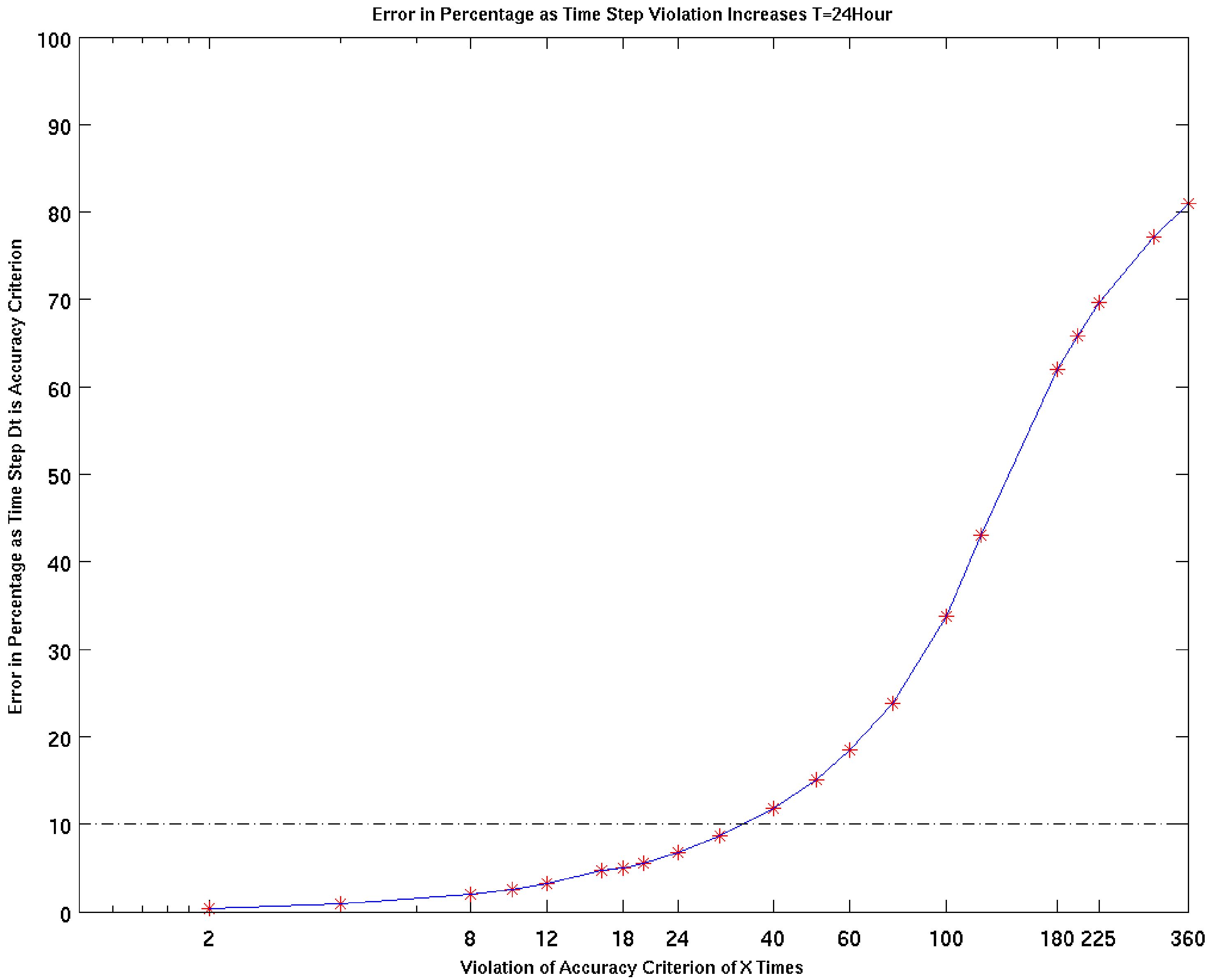
$$\frac{K_H \Delta t}{\Delta z^2} = \frac{1m^2 s^{-1} 60\sec}{(1m)^2} = 60$$

Heating on Surface



K_H=0.01m²/s
T(z)=10 °C


Temperature Profile at001 hour
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Depth in meter
0
2
4
6
8
10
12
14
16
18
20
Temperature in Deg
DT=0.5sec
DT=1sec
DT=2sec
DT=60sec
DT=360sec



Error in Percentage as Time Step Violation Increases T=24Hour
Error in Percentage as Time Step Dt is Accuracy Criterion
Violation of Accuracy Criterion of X Times
0
10
20
30
40
50
60
70
80
90
100
2
8
12
18
24
40
60
100
180 225
360

Conclusion

- 1. The Vertical Mixing Algorithm in ECOM/POM is ROBUST and STABLE.
- 2. However, there is Accuracy Limitation with regarding to this Algorithm
- 3. $\frac{K_H \Delta t}{\Delta z^2} \leqslant \frac{1}{2}$ can work as a surrogate for vertical mixing stability criterion.
- 4. When this criterion is violated greater than 30 times, the accuracy problem becomes apparent.

Thank You