LES of turbulent shear flow and pressure driven flow on shallow continental shelves.

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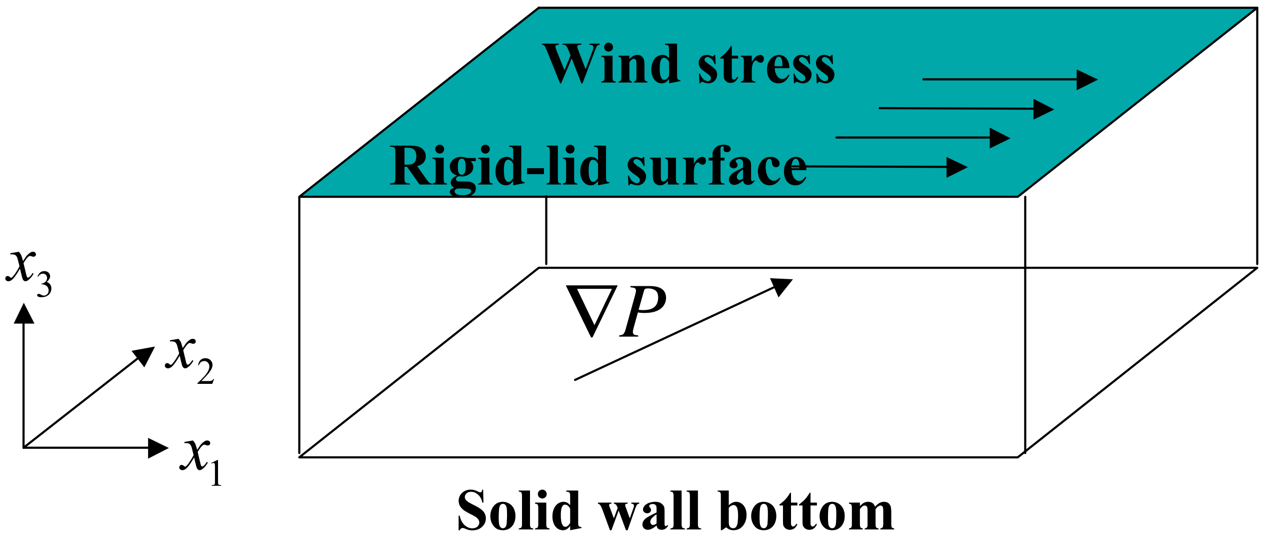
What is the effect of a wind stress, a pressure gradient and wave forcing or a combination of the three on the dynamic of a turbulent flow ?



- Simple geometry : make abstraction of interaction between the flow and the geometry.

- Use of LES allows an accurate description of the flow.

Outline

- Description of the model
- Numerical method
- Base flows :
  - Pure wind driven flow
  - Pure tidal flow
  - Wind driven flow with wave forcing
- Combinated flows :
  - Colinear wind and tide driven flow
  - Colinear wind and tide driven flow with wave forcing
  - Normal wind and tide driven flow
  - Normal wind and tide driven flow with wave forcing
- Conclusion
- Prospects

Description of the model (1)

The filtered Navier-Stokes equations

- Continuity: $$\frac{\partial \overline{u}_i}{\partial x_i} = 0$$

- Momentum: $$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{\prod}}{\partial x_i} + \frac{1}{\mathrm{Re}}\frac{\partial^2 \overline{u}_i}{\partial x_i^2} + \frac{\partial \boldsymbol{\tau}_{ij}^{(r)d}}{\partial x_j} + \frac{1}{La_T^2}\varepsilon_{ijk} u_j^s \overline{\omega}_k$$

$$\overline{\prod} = \overline{\pi} - \frac{1}{3}\rho \boldsymbol{\tau}_{kk}^{(r)} \qquad \mathrm{Re} = \frac{U_\tau H}{2\nu} \qquad La_T = \sqrt{\frac{U_\tau}{U_S}}$$

$$u_1^s = \frac{\cosh[2k(x_3 - H)]}{2\sinh^2(kH)} \qquad u_2^s = u_3^s = 0$$

Description of the model (2)

SGS stress: $\boldsymbol{\tau}_{ij}^{(r)} = \overline{u}_i \overline{u}_j - \overline{u_i u_j}$

Smagorinsky model for the SGS stress:

$$\boldsymbol{\tau}_{ij}^{(r)d} \equiv \boldsymbol{\tau}_{ij}^{(r)} - \frac{1}{3}\boldsymbol{\tau}_{kk}^{(r)}\delta_{ij} = 2\nu_T \overline{S}_{ij} \qquad \overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$$

Eddy viscosity: $\nu_T = (C_S \overline{\Delta})^2 \,|\, \overline{S} \,|$ $\qquad |\, \overline{S} \,| = \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}$

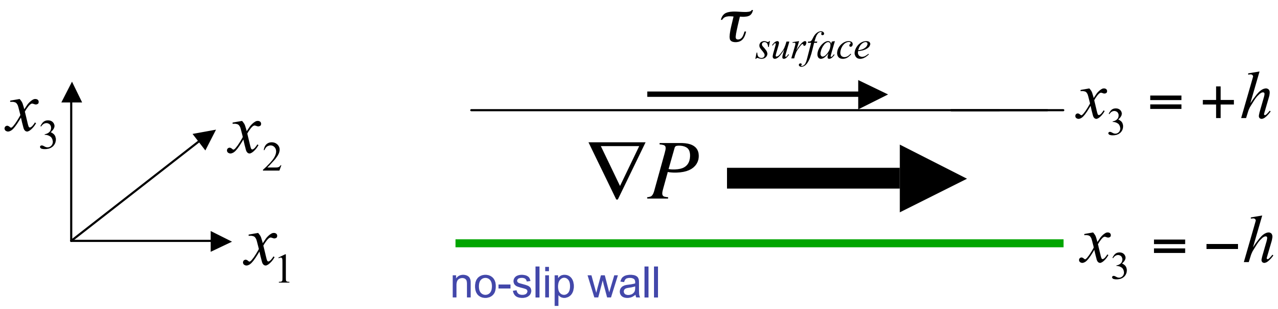
$(C_S \overline{\Delta})^2$ is computed dynamically using the Germano identity (Germano et al., Phys. Fluids, 1991)

Numerical method

- Time integration is by 2^{nd} order time-accurate fractional step scheme
  - Momentum eqns. are solved first followed by a pressure Poisson eqn.
  - The Poisson's equation for pressure enforces continuity
- Spatial discretization uses hybrid spectral/finite-difference approach
  - Horizontal directions are discretized spectrally
  - Vertical direction is discretized via 5^{th} and 6^{th} order compact finite-differences
- Code is parallelized using Message Passing Interface (MPI)
  - Enables use of multiple processors making computations faster
  - Typical runs last for about 2-3 month using 24 processors for 1000 seconds of physical time

Base Flows

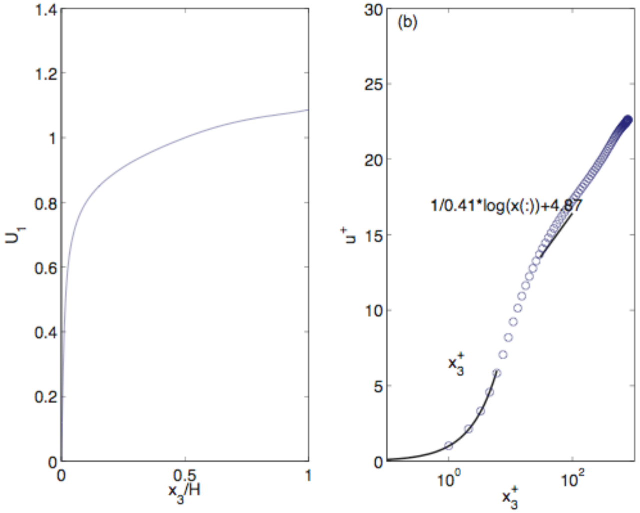
- Wind driven flow
- Tidal flow
- Wind driven flow with wave forcing



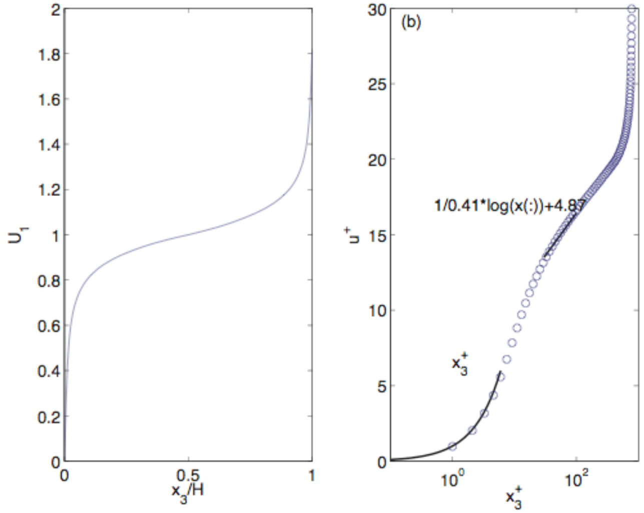
$L_1 = 4\pi h$ $\quad$ $L_2 = (8/3)\pi h$ $\quad$ $H = 2h$

- Pressure gradient or surface stress are applied such that $\mathrm{Re}_\tau \equiv u_\tau h/\nu = 395$
- Grid is chosen with 96x96x96 nodes
- La_t in case of wave forcing is chosen such that : $La_t = \sqrt{u_\tau / u_S} = 0.7$
- Mesh refinement ensures proper resolution of the boundary layer

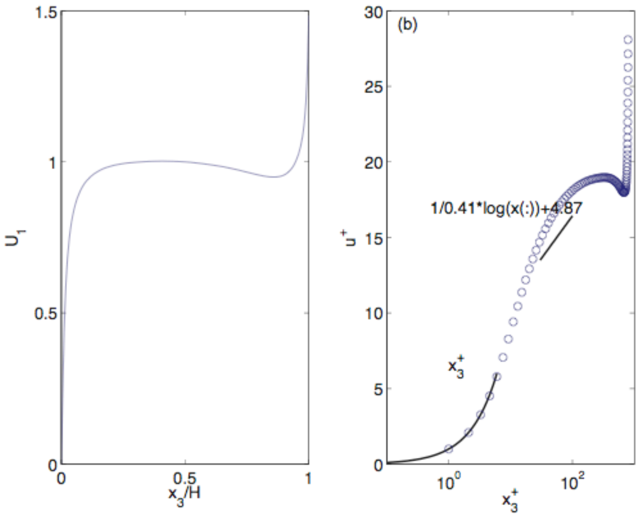
Average velocity profiles



Tidal flow



Wind driven flow

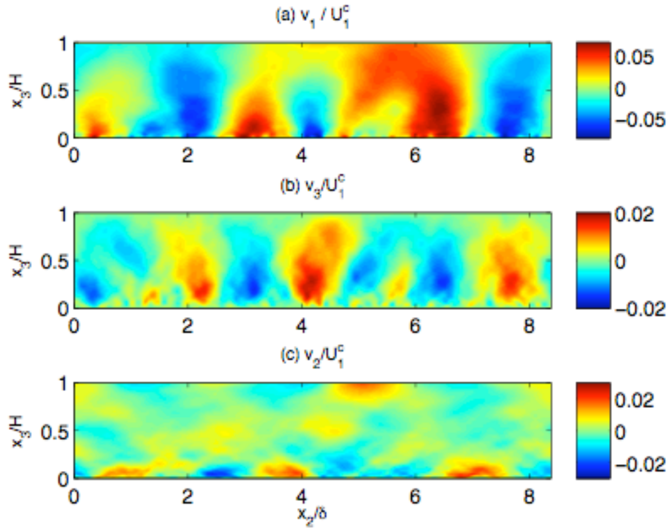


Wind driven flow with Langmuir circulation

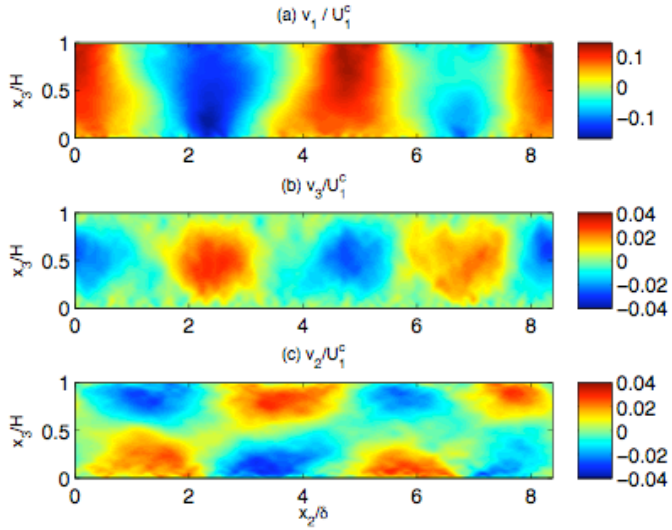
$$u^+ = \frac{\overline{u} \cdot h}{\nu \cdot \mathrm{Re}_\tau}$$

$$x_3{}^+ = {}^{x_3}\!/_{h} \cdot \mathrm{Re}_\tau$$

Visualization



Tidal flow



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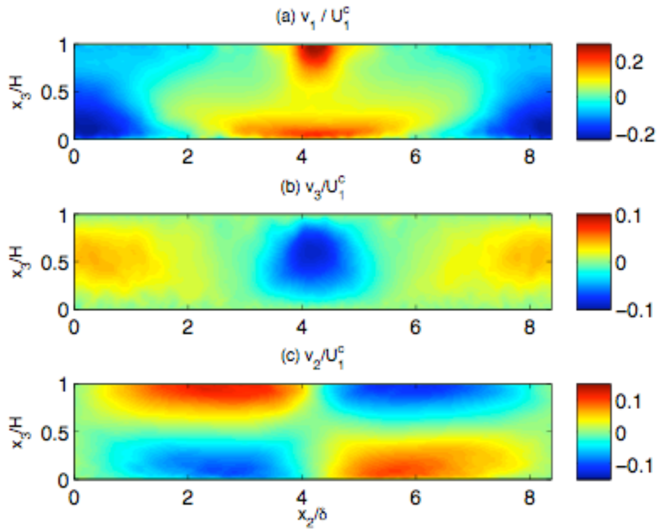
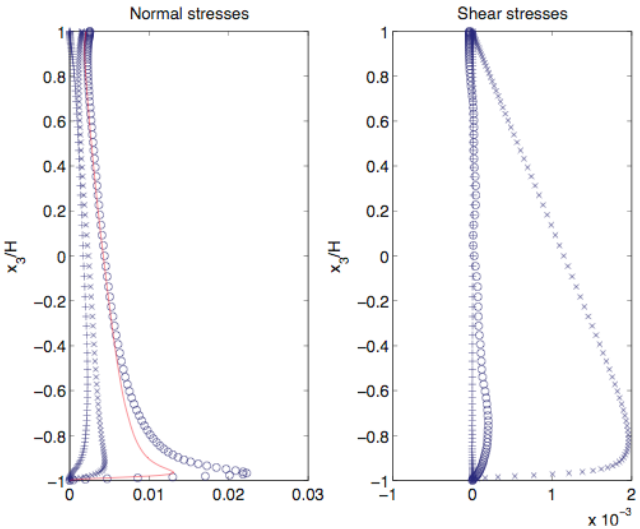
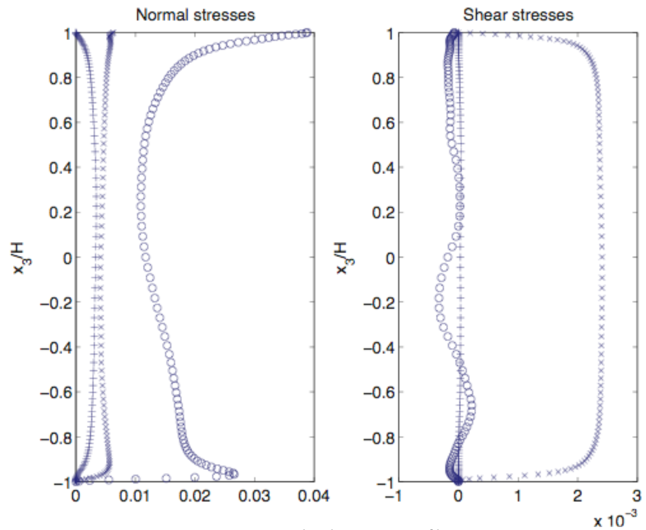


Wind driven flow with Langmuir circulation

Normal and shear stresses



Tidal flow



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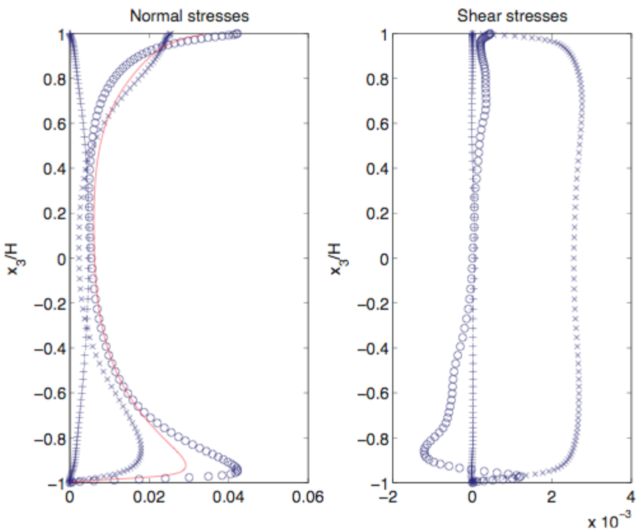

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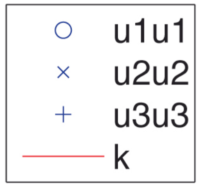
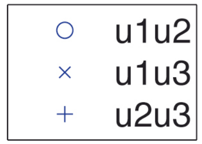
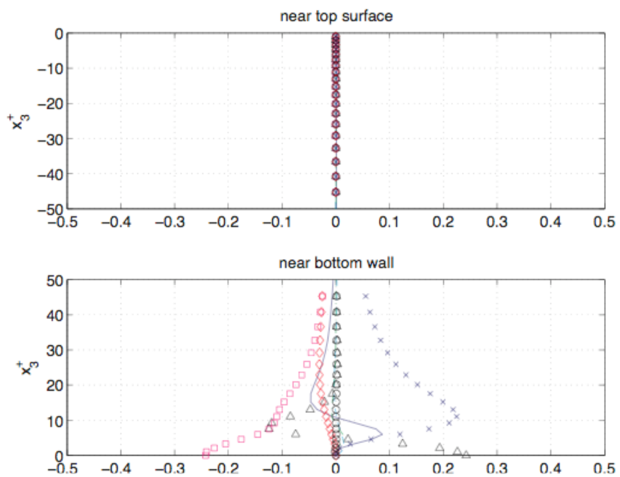


Normal stresses

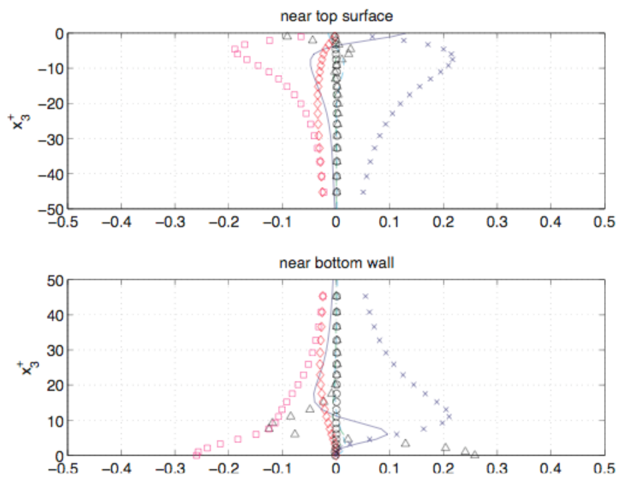


Shear stresses

Turbulent kinetic energy budgets



Tidal flow



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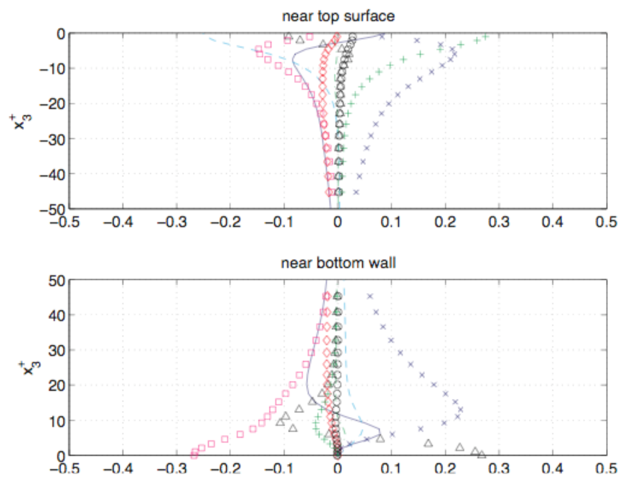
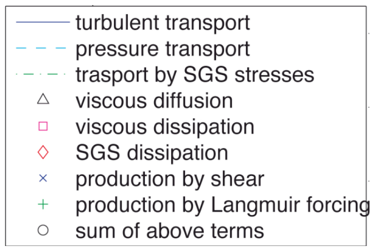




Wind driven flow with Langmuir circulation

Lumley Triangle

The Reynolds-stress anisotropy tensor has six independant components b_{ij}

$$b_{ij} = \frac{\langle \overline{u}_i \overline{u}_j \rangle}{\langle \overline{u}_i \overline{u}_i \rangle} - \frac{1}{3}\delta_{ij}$$

The *principal invariants* of **b** (Lumley, 1978) are :

$$I = b_{ii} \equiv 0, \qquad II = \frac{1}{2} b_{ij} b_{ji}, \qquad III = \frac{1}{6} b_{ij} b_{ji} b_{ki}$$

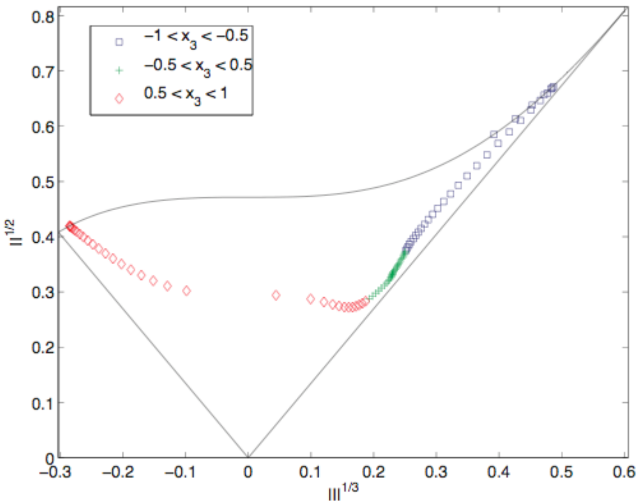
The characteristic equation of the matrix $[b_{ij}]$ is :

$$\lambda^3 - I\lambda^2 + II\lambda - III = 0$$

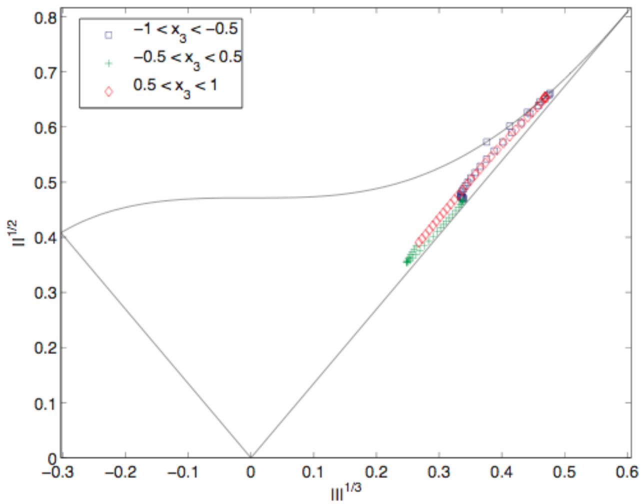
Because $I = b_{ii} = \text{trace}([b_{ij}]) = 0$, $\lambda_1 + \lambda_2 + \lambda_3 = 0$ and only two of the eigenvalues are independant. Pope (2000) defines another pair of invariant :

$$\eta^2 = \frac{1}{6} b_{ij} b_{ji}, \qquad \xi^3 = \frac{1}{6} b_{ij} b_{ji} b_{ki}$$

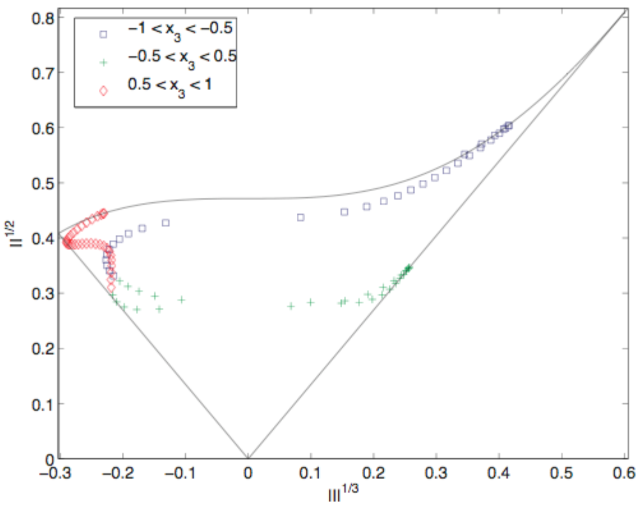
Lumley triangle



Tidal flow



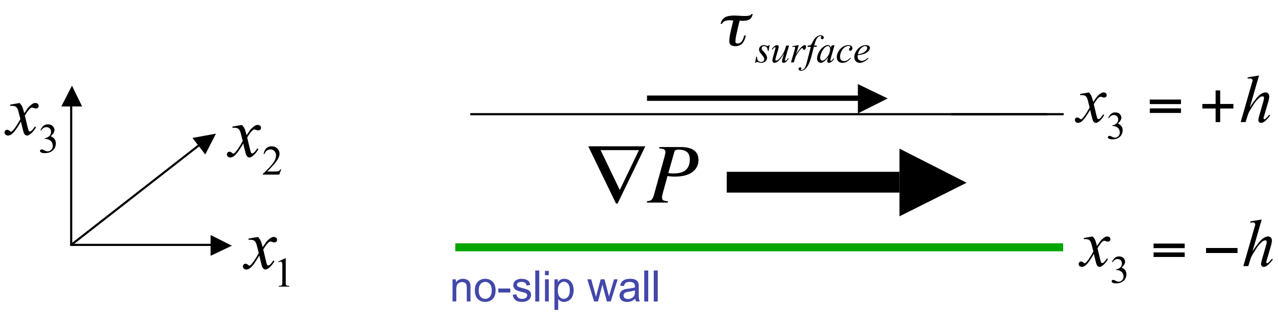
Wind driven flow



Wind driven flow with Langmuir circulation

Complex flows

- Wind and Tide flows colinear
- Wind and Tide flows normal
- Wind and Tide flows colinear with wave forcing
- Wind and Tide flows normal with wave forcing



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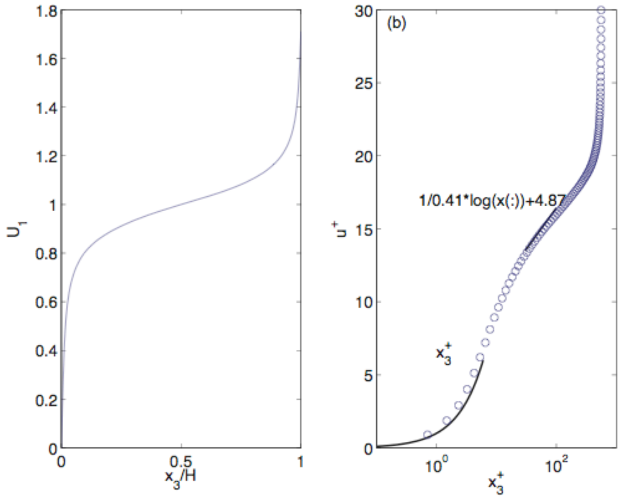
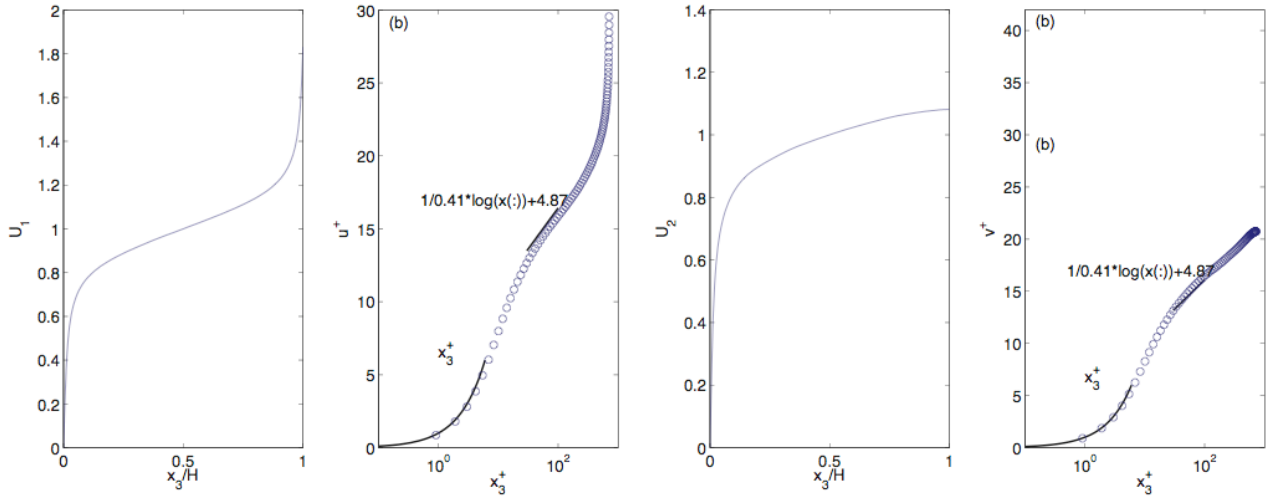
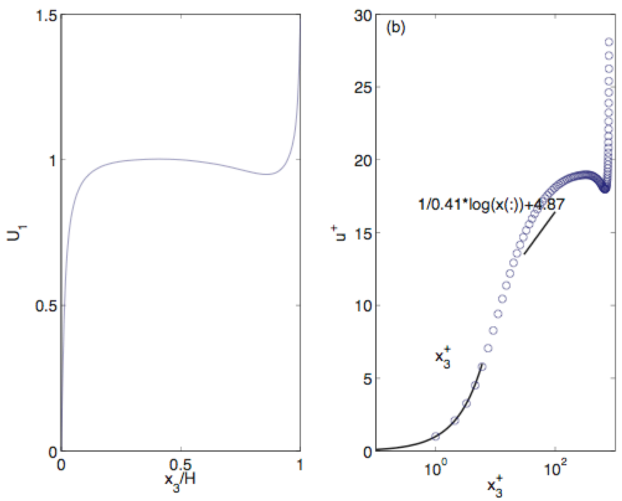


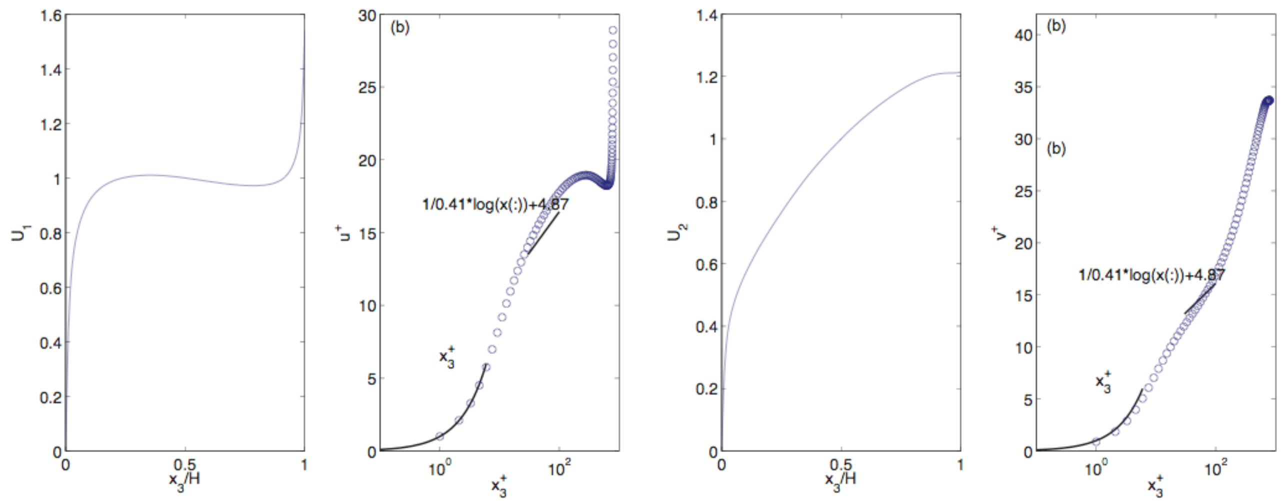
Tidal and wind driven flow colinear



Tidal and wind driven flow crossed

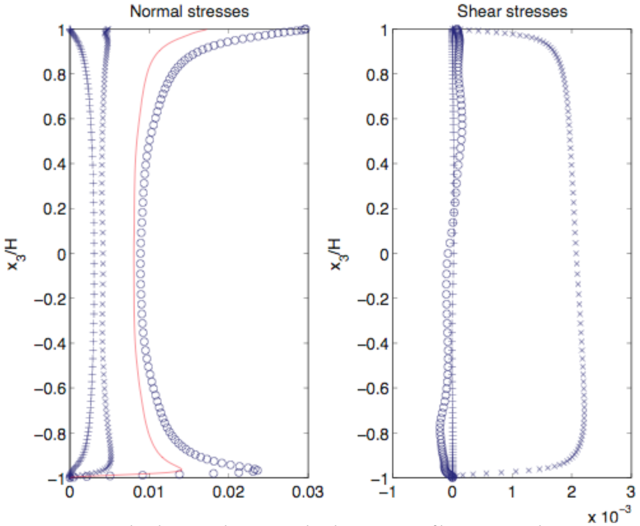


Tidal and wind driven flow colinear with Langmuir circulation

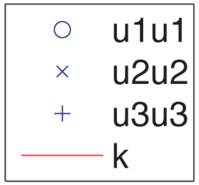


Tidal and wind driven flow crossed with Langmuir circulation

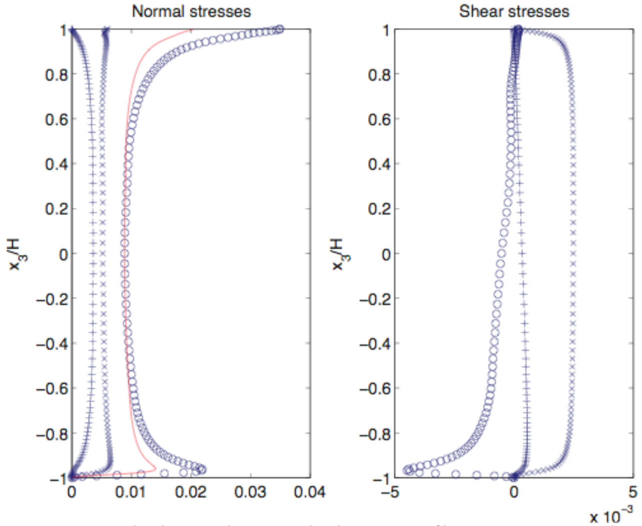
Normal and shear stresses



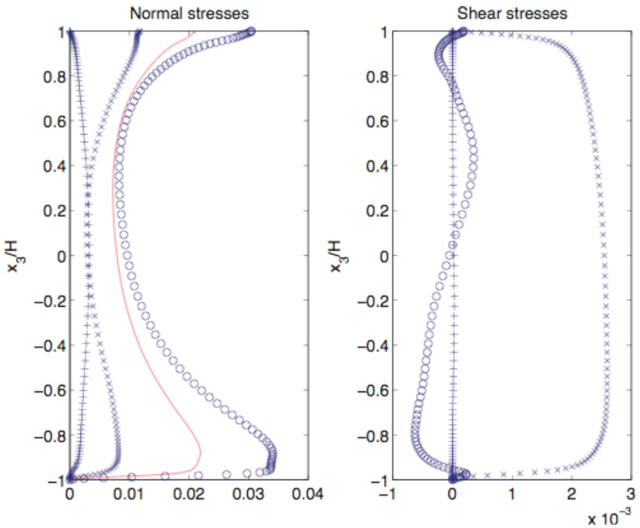
Tidal and wind driven flow colinear



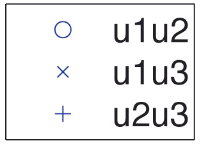
Normal stresses



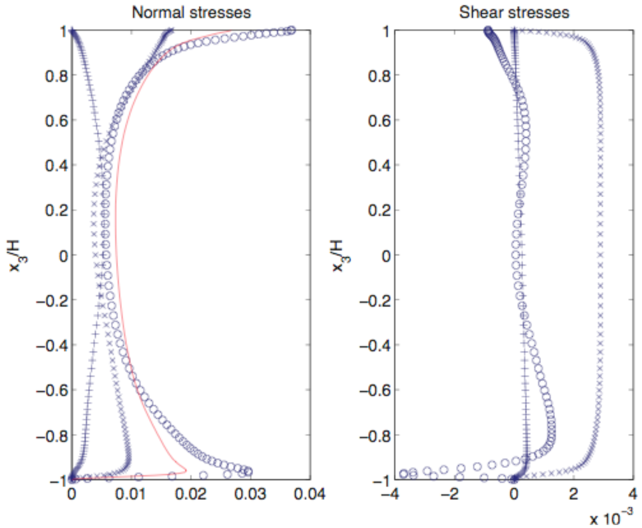
Tidal and wind driven flow crossed



Tidal and wind driven flow colinear with Langmuir circulation

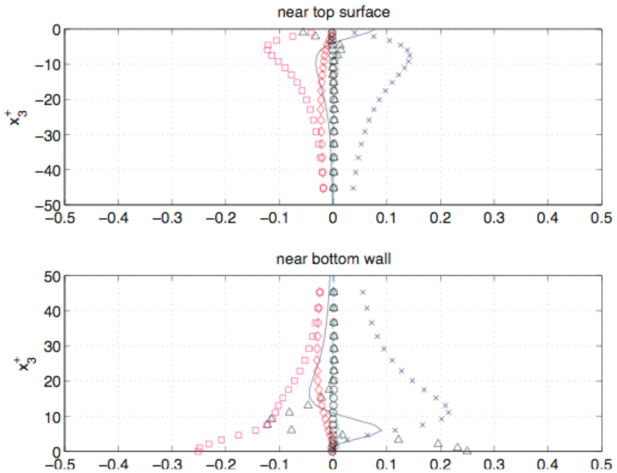


Shear stresses

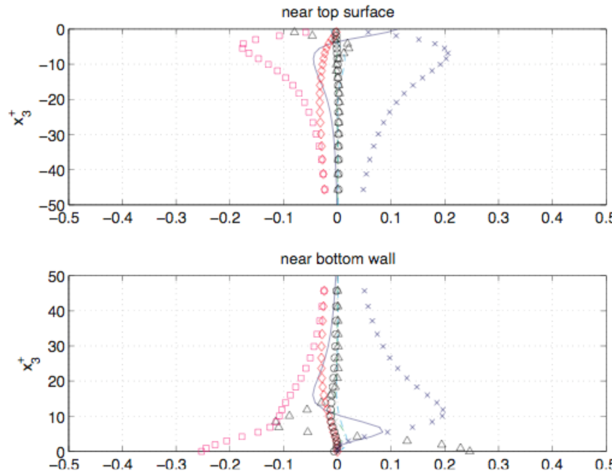


Tidal and wind driven flow crossed with Langmuir circulation

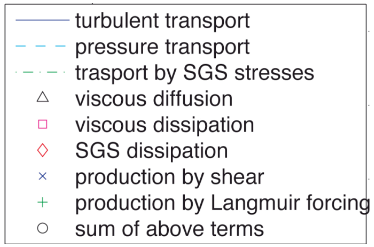
Turbulent kinetic energy budgets

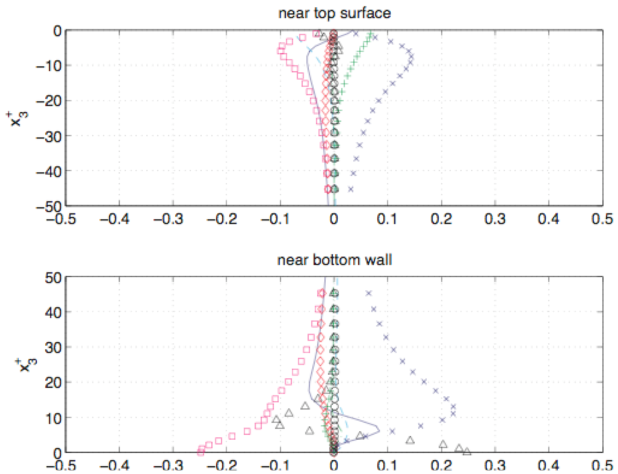


Tidal and wind driven flow colinear

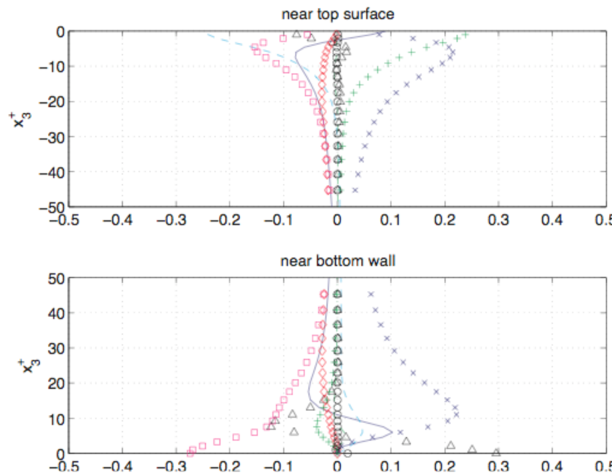


Tidal and wind driven flow crossed



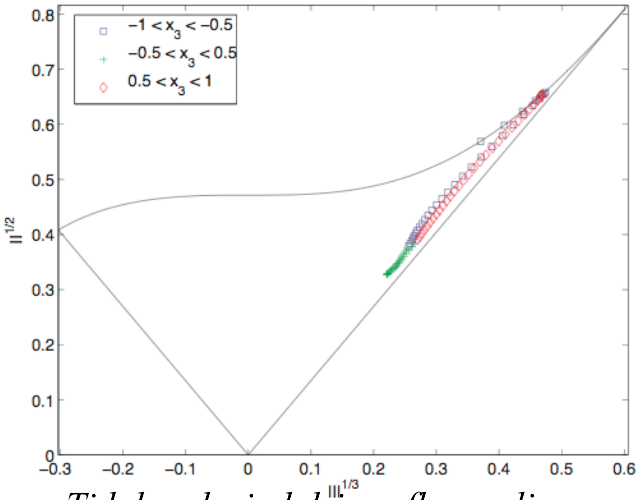


Tidal and wind driven flow colinear with Langmuir circulation

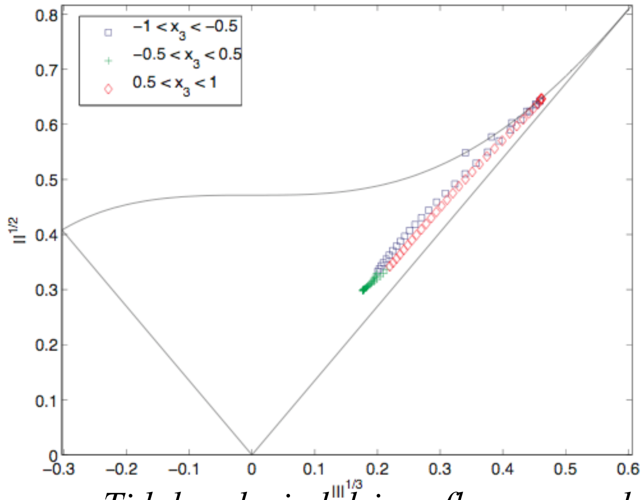


Tidal and wind driven flow crossed with Langmuir circulation

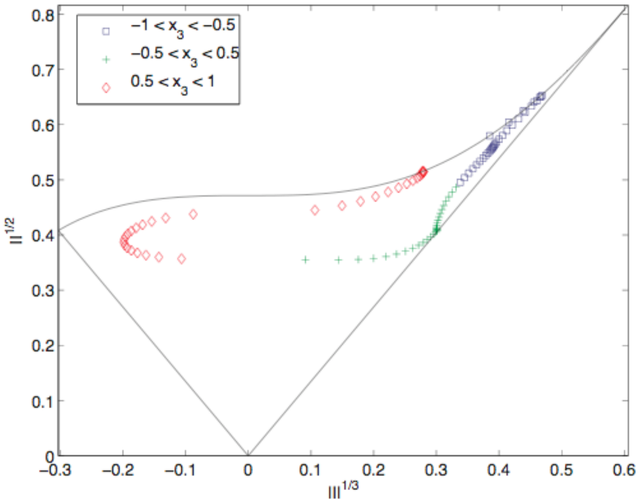
Lumley triangle



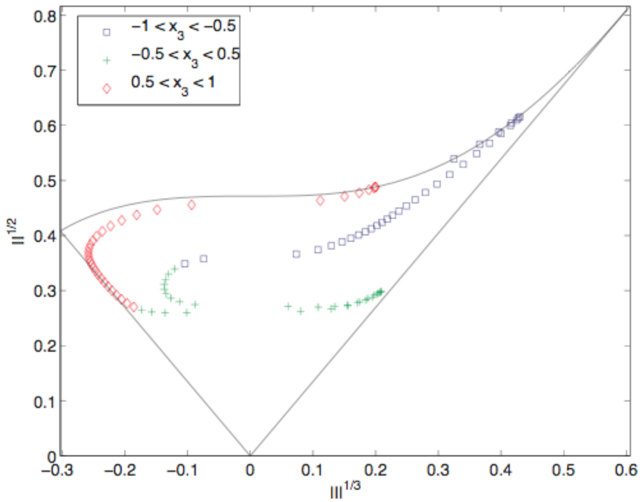
Tidal and wind driven flow colinear



Tidal and wind driven flow crossed



Tidal and wind driven flow colinear with Langmuir circulation



Tidal and wind driven flow crossed with Langmuir circulation

Conclusion

- Wave forcing creates highly coherent streamwise turbulent cells
- Those coherent structures induce an increase of normal stresses near the bottom and the surface.
- Langmuir circulation works as a mixing mechanism acting to homogeneize the water column.
- In case of Langmuir circulation, logarithmic law near wall is not observed.
- Balance of turbulence kinetic energy budget is substantially different with Langmuir circulation. For example :
  - The langmuir production term becomes one of the major terms of the balance
  - The pressure-strain rate which is usually negligible also becomes important

  This has major implication for RANS type modelling
- Crossed Tidal and Wind driven flow make $u_1 u_2$ Reynolds-stress non negligible
- Lumley triangle is a sensitive indicator of the structure of turbulence. It shows a more three dimensional structure of the flow in case of Langmuir ciculation

Extensions

- We are studying the impact of a strong stratification on wind and pressure driven flows.

- We are studying an extension to a secondary finer grid to transport a passive scalar in order to simulate gas transfer.

Thank You

Questions ?

The Craik-Leibovich Langmuir force

$$\mathbf{F} = \bar{\mathbf{U}}^{\mathbf{s}} \times \bar{\omega}$$

Here $\bar{\mathbf{U}}^{\mathbf{s}}$ is the Stokes steady drift velocity caused by the gravity waves and $\bar{\omega}$ is the vorticity of the shear flow.

For a surface gravity wave with amplitude a, frequency σ, wavenumber k and surface elevation, ζ, given by

$$\zeta = a\cos\left(kx_1 - \sigma t\right)$$

with the dispersion relation

$$\sigma^2 = gk\tanh\left(kH\right)$$

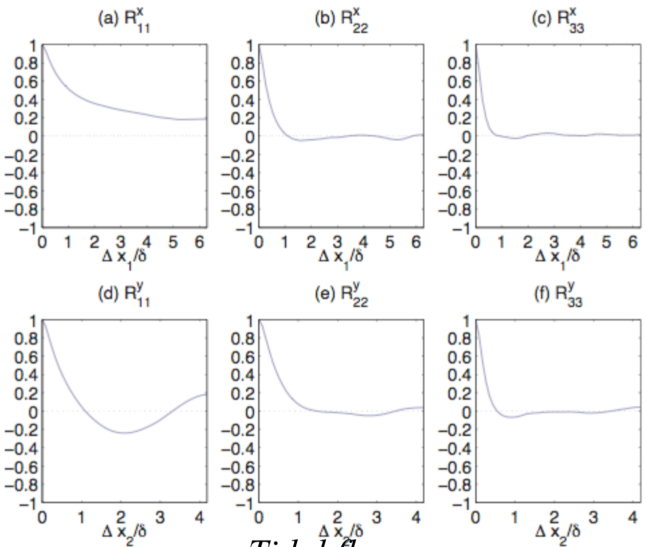
the (dimensional) Stokes drift velocity is

$$\bar{\mathbf{U}}^{\mathbf{s}} = u_S \quad \frac{\cosh 2k\left(z+H\right)}{2\sinh^2\left(kH\right)} \;\mathbf{e_1}\;; \qquad u_S = \sigma k a^2$$

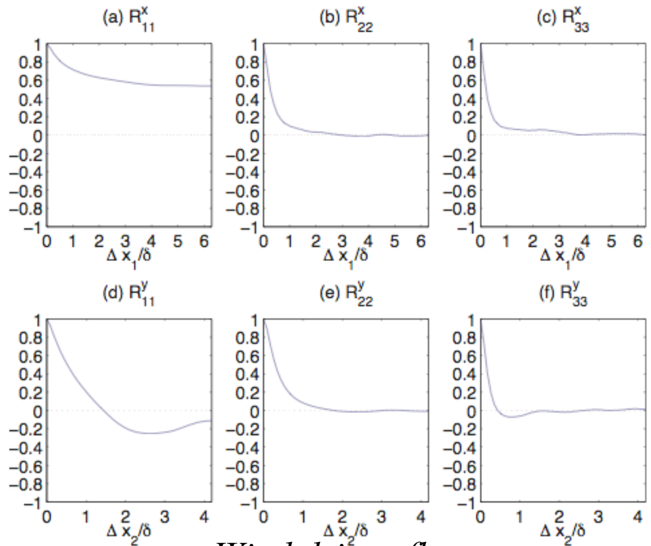
Mean resolved Reynolds stress tensor: Terms

$$
\begin{aligned}
P_{ij} &= -\langle u'_i u'_k \rangle \frac{\partial \langle \bar{u}_j \rangle}{\partial x_k} - \; u'_j u'_k \; \frac{\partial \langle \bar{u}_i \rangle}{\partial x_k} && \text{(shear prod. rate),} \\
Q_{ij} &= \frac{1}{La_T^2} \; \langle \epsilon_{jlk} u_l^s \omega'_k u'_i \rangle + \; \epsilon_{ilk} u_l^s \omega'_k u'_j && \text{(Langmuir forcing prod. rate),} \\
T_{ij} &= -\frac{\partial}{\partial x_k} \; u'_i u'_j u'_k && \text{(turbulent transport rate),} \\
T_{ij}^{\text{sgs}} &= \frac{\partial}{\partial x_k} \quad u'_i \tau'_{jk} \; + \; u'_j \tau'_{ik} && \text{(SGS transport rate),} \\
D_{ij} &= \frac{1}{\mathrm{Re}} \frac{\partial^2}{\partial x_k^2} \; u'_i u'_j && \text{(viscous diffusion rate),} \\
\Pi_{ij} &= -\frac{\partial}{\partial x_k} \; \delta_{jk} \; P' u'_i \; + \delta_{ik} \; P' u'_j && \text{(pressure transport rate),} \\
\phi_{ij} &= 2 \; P' S'_{ij} && \text{(pressure-strain redist. rate),} \\
\epsilon_{ij} &= -\frac{2}{\mathrm{Re}} \; \frac{\partial \bar{u}'_i}{\partial x_k} \frac{\partial \bar{u}'_j}{\partial x_k} && \text{(viscous dissipation rate) and} \\
\epsilon_{ij}^{\text{sgs}} &= - \; \tau'_{ik} \frac{\partial \bar{u}'_j}{\partial x_k} - \; \tau'_{jk} \frac{\partial \bar{u}'_i}{\partial x_k} && \text{(SGS dissipation rate).}
\end{aligned}
$$

Correlation Functions



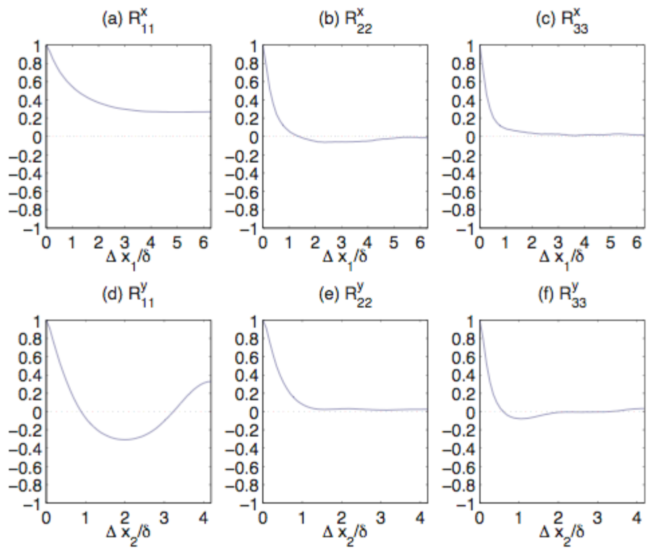
Tidal flow



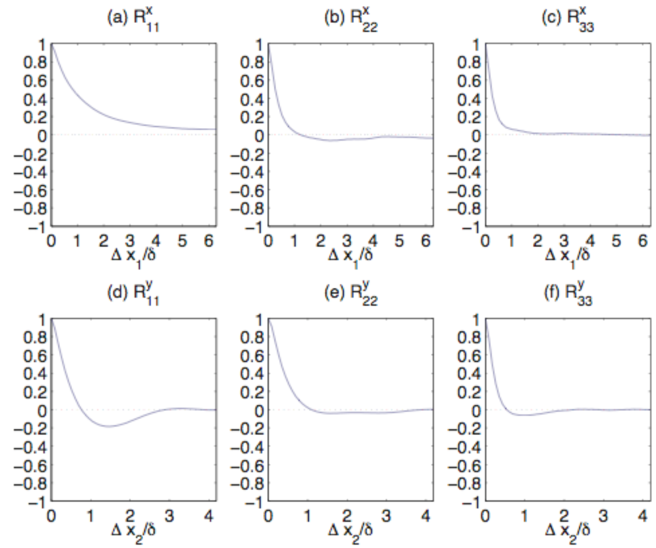
Wind driven flow

Wind driven flow with Langmuir circulation

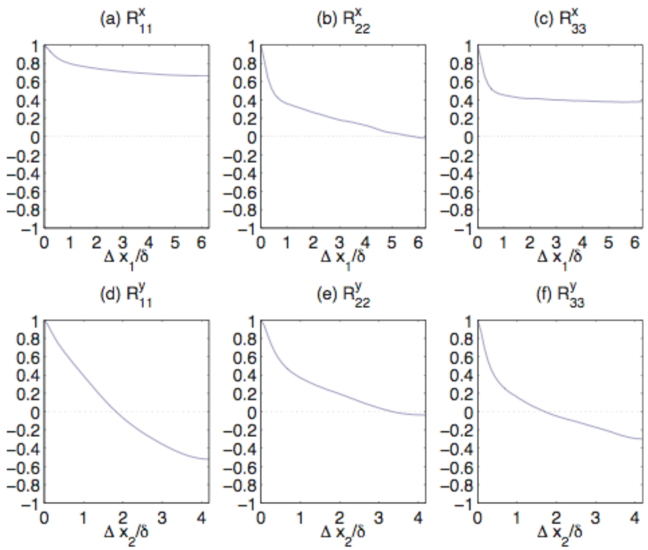
Correlation Functions



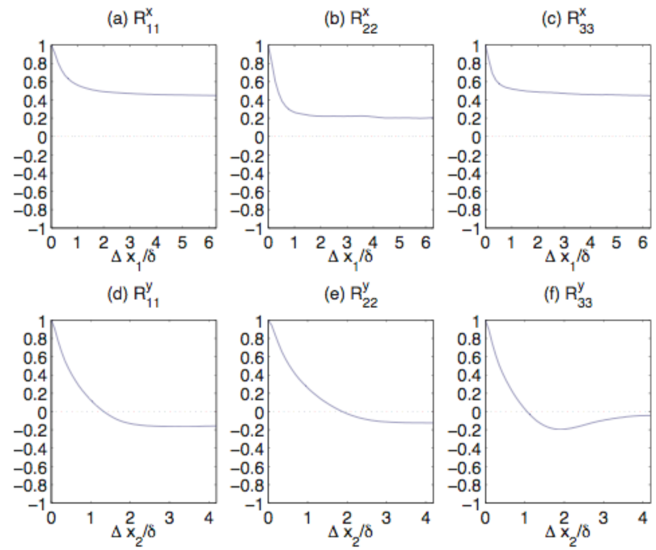
Tidal and wind driven flow colinear



Tidal and wind driven flow crossed



Tidal and wind driven flow colinear with Langmuir circulation



Tidal and wind driven flow crossed with Langmuir circulation